Banking and Shadow Banking

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Abstract

This paper incorporates shadow banking modeled as off-balance-sheet financing in a continuous-time macro-finance framework. In our model, regular banks pursue regulatory arbitrage via shadow banking, and they support their shadow banks with implicit guarantees. We show that the enforcement problem with implicit guarantees gives rise to an endogenous constraint on leverage for shadow banking. Our model captures that shadow banking is pro-cyclical and that shadow banking increases endogenous risk. Tightening bank regulation in our model increases the borrowing capacity of shadow banking and endogenous risk. In addition, we uncover novel channels through which bank regulation influences social welfare in the presence of shadow banking.

Keywords: shadow banking; implicit guarantee; bank regulation; endogenous risk

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Introduction

The 2007-09 global financial crisis brought prominence to shadow banking—those bank-like activities that unregulated financial entities and off-balance-sheet vehicles undertake—in both policy discussions and academic research. Although there are already many research papers on shadow banking, implicit guarantees that financial firms heavily use in the shadow banking system have not drawn enough attention in academia. Specifically, implicit guarantees refer to firms' unbinding promises that they will protect their off-balance-sheet entities in trouble.[1]

In this paper, we endogenize the borrowing capacity of shadow banking based on the enforcement problem with implicit guarantees. By exploring the dynamics of shadow banking, we highlight that the pro-cyclicality of shadow banking increases endogenous risk that the banking system generates and that strengthening regulation of regular banking raises the borrowing capacity of shadow banking, which in turn increases endogenous risk in the economy. In addition, our paper investigates welfare implications of bank regulation in the economy where banking activities can migrate to the unregulated shadow banking system.

Similar to Thomas and Worrall (1988) and Kehoe and Levine (1993), the enforcement problem with implicit guarantees gives rise to the maximum borrowing capacity of shadow banking, and this enforcement problem originates from institutional details of shadow banking. To be consistent with such details, we model regular banking as a regular bank's *on-balance-sheet financing* and shadow banking as the regular bank's *off-balance-sheet financing*. To regulatory authorities, a regular bank constructs its shadow bank (i.e., off-balance-sheet vehicle) as a legally separate entity to circumvent any regulation of financial activities that the shadow bank undertakes. To creditors, however, the regular bank extends the unbinding implicit guarantee to the shadow bank. Due to the absence of an authority, an enforcement problem arises, as the shadow bank's creditors

[1] Gorton and Souleles (2007) have an extensive discussion on the institutional details of securitization and off-balance sheet financing. They emphasize the enforceability problem of implicit guarantees for off-balance-sheet financing. Besides securitization, implicit guarantees are also used for short-term financial instruments, such as Asset-Backed Commercial Paper and Money Market Funds (MMFs). For instance, HSBC spent \$35 billion in order to bring assets of its off-balance-sheet structured investment vehicles (SIVs) onto its balance sheet in late 2007 (Goldstein, 2007); Citigroup moved \$37 billion assets in SIVs back to its balance sheet (Moyer, 2007). In the money market, Securities and Exchange Commission reported that at least 44 MMFs received support from their sponsors to avoid breaking the buck during the 2007-2009 financial crisis (McCabe, 2010).

cannot force its parent regular bank to protect them if the shadow bank is in trouble.

We next argue that tightening regulation of regular banking raises the borrowing capacity of shadow banking (i.e., increasing leverage that a regular bank can obtain via shadow banking). As in the limited enforcement literature, if a regular bank defaults on its shadow bank obligations, then creditors will stop lending to shadow banks affiliated with the regular bank, which deprives the regular bank of its regulatory arbitrage opportunity. Therefore, the opportunity cost for the regular bank to default amounts to the present value of the future benefits that shadow banking offers. Since more stringent regulation leads to greater regulatory arbitrage opportunities, the opportunity cost of default is larger in economies with tighter regulation, and the leverage of shadow banking is higher in such economies.

Because the leverage of shadow banking is endogenously determined, there exists an interesting feedback loop between the opportunity cost of default and the leverage of shadow banking. For instance, if the cost of default declines due to loosening bank regulation, then the incentive to default will rise, and the shadow banking channel will contract. This shrinking shadow banking channel offers less benefits to regular banks, which consequently leads to an even lower opportunity cost of default. This feedback loop could amplify the effect of the initial drop in the cost of default so significantly that even a small drop can rule out shadow banking completely. Hence, shadow banking is unsustainable when the regulation of regular banking is sufficiently lenient.

More importantly, our paper demonstrates that if there is no such feedback loop in a model, tightening bank regulation does not necessarily lead to the expansion of the shadow banking system.

We embed our modeling of banking and shadow banking in a continuous-time, macro-finance framework (Brunnermeier and Sannikov, 2014). In this framework, because of a constraint on outside-equity financing, banks can only use leverage to finance their investments. We choose this macro-finance framework because we can model financial instability as endogenous risk that the banking system generates through the balance sheet mechanism (Kiyotaki and Moore, 1997; Bernanke, Gertler and Gilchrist, 1999). Bank regulation that limits the use of bank leverage can improve social welfare because the excessive use of bank leverage leads to high endogenous risk and causes a pecuniary externality for the entire economy (Lorenzoni, 2008; Stein, 2012). As a result, shadow banking emerges as regular banks' response to bank regulation.

The solution of a continuous-time, macro-finance model characterizes the full dynamics of an economy. With this advantage, our model captures two salient dynamic features of shadow banking observed during the 2007-09 financial crisis: the pro-cyclicality of shadow banking and reintermediation by shadow banks conducting fire sales of assets to regular banks. By analyzing both leverage dynamics and endogenous risk dynamics, we uncover a general equilibrium channel through which shadow banking adds to endogenous risk.

We first explain why shadow banking is pro-cyclical in our model. In economic booms, high asset prices and the corresponding low rates of return from holding assets lower the profitability of banking. Hence, regular banks are not inclined to leverage up via shadow banking and default. As a result, the enforcement problem is less severe in economic upturns, and the leverage of shadow banking also tends to be high in upturns. In addition, the feedback loop between the cost of default and the leverage of shadow banking amplifies the shadow banking sector's expansion in economic booms.

Shadow banking increases endogenous risk as a general equilibrium effect in our model. Since shadow banking is immune to bank regulation and its borrowing capacity is pro-cyclical, shadow banks accumulate substantial amounts of assets in upturns. Conversely, when a negative macroeconomic shock hits the economy, the shrinking shadow banking channel forces shadow banks to sell assets to regular banks at fire-sale prices (i.e., reintermediation). This asset fire-sale occurs because regular banks are reluctant to acquire assets due to bank regulation. Thus, the decline in asset prices has to be large enough such that regular banks are willing to purchase those assets. Naturally, endogenous risk increases in these situations.

We next elaborate on the U-shaped relationship between bank regulation and financial instability. When regulation is loose enough, shadow banking is unsustainable. In this situation, tighter regulation of regular banking leads to lower financial instability. When regulation is sufficiently tight, the funding capacity of shadow banking expands and a considerable number of banking activities shift to the shadow banking sector. Hence, more stringent regulation, in this circumstance, gives rise to a larger shadow banking system and higher financial instability.

The welfare analysis of our framework highlights novel channels through which strengthening bank regulation affects households' welfare in the presence of shadow banking. Tightening regulation leads to the expansion of shadow banking and the increase in credit demand. This could improve households' welfare because *i*) the rise in credit demand raises the return that households earn from lending to the banking sector, and *ii*) the volatility of households' welfare declines as more aggregate risk is concentrated in the banking sector. Nevertheless, if bank regulation is overly stringent, households' welfare will deteriorate. The intuition is that when the shadow banking sector is enormously large, the entire banking sector's exposure to aggregate risk is too high. And, this in turn leads to high endogenous risk and low social welfare.

Related Literature. The literature on shadow banking is swiftly growing and diverse. Different papers model shadow banking in drastically different ways, and Adrian and Ashcraft (2012) provide a thorough survey of this growing literature. In our paper, we attempt to categorize a few models of shadow banking along two dimensions: the motive for shadow banking and the type of negative externalities that shadow banking causes.

The existence of shadow banking can be demand/preference driven. For example, in Gennaioli et al. (2013), infinitely risk-averse households only value securities' worst scenario payoffs, and shadow banking can increase such payoffs by pooling different assets together. Meanwhile, in Moreira and Savov (2014), the preference specification of households leads directly to a demand for the liquid securities that shadow banking generates.

The second motive for shadow banking is regulatory arbitrage, as we discuss in this paper. Luck and Schempp (2014), Ordonez (2013), and Plantin (2014) are papers that fall into this category.

Models of shadow banking differ with respect to the type of the externalities that shadow banking causes. The first category includes non-pecuniary externalities. In Plantin (2014), shadow banking exposes the real sector to counter-productive uncertainty. In both Luck and Schempp (2014) and Gennaioli et al. (2013), creditors of shadow banks suffer from unexpected default that bank runs or crises cause. Generally, investments financed by shadow banking in these models have worse or more volatile fundamentals than investments financed by regular banking.

Unlike the first group of papers discussed above, we, as well as, Moreira and Savov (2014) assume that shadow banking does not involve any investments of inferior quality. Instead, we focus on the pecuniary externality; that is, the leverage choices of individual shadow banks cause excessive endogenous risk because they do not take into account the price impact of their actions in the competitive equilibrium.

Although Plantin (2014) also touches upon the idea that tight regulation may have negative unintended consequences, our paper differs his work in three critical aspects. First, we focus on the class of bank regulations that restricts the use of bank leverage; in contrast, Plantin (2014) examines regulation that prohibits banks from issuing outside equity. Second, we recognize financial instability as the endogenous risk that the financial market generates. For Plantin (2014), however, the riskiness of outside equity reflects the instability that is counterproductive for the real sector. Finally, the dynamic general equilibrium setting of our framework allows us to characterize dynamic properties of shadow banking and to discuss the trade-off between economic growth and financial stability, for which the static setting in Plantin (2014) is not suitable.

This paper is also related to the literature on pecuniary externalities. One closely related paper is Bianchi (2011), whose quantitative examination of the pecuniary externality in a dynamic general equilibrium model highlights that raising borrowing costs can prevent excessive borrowing and improve welfare.

For our methodology in this paper, we follow the emerging literature (e.g., Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012b, 2013)) that considers economies with financial frictions in a continuous-time setting. The methodology employed in this literature captures the exact characterization of full equilibrium dynamics particularly well. In our model, the tractability allows us to endogenize the leverage constraint for shadow banking and to explicitly solver for the debt capacity of shadow banking.

The paper is structured as follows. We first establish our baseline model in Section 1. In Section 2, we then characterize the equilibrium of this baseline model and illustrate the main results with numerical examples. In Section 3 and 4, we explore the welfare and policy implications of the baseline model. Section 5 concludes.

1 The Baseline Model

To analyze how shadow banking affects the aggregate economy, in this section we model shadow banking and regular banking within a continuous-time macro-finance framework developed by Brunnermeier and Sannikov (2014). In addition, we will specify the dynamic portfolio choice problem of each agent in the model as well as market clearing conditions.

1.1 Model Setup

The general model setup is standard in the literature on financial frictions. As in Kiyotaki and Moore (1997), He and Krishnamurthy (2012b), and others, our model has heterogeneous agents: productive bankers and less-productive households. Bankers raise funds from households through both regulated regular banking, modeled as on-balance-sheet financing, and unregulated shadow banking, modeled as off-balance-sheet financing.

1.1.1 Technology and Preferences

We consider a continuous-time infinite-horizon economy with two types of goods: durable physical capital goods and non-durable consumption goods.

At any time $t \in [0, \infty)$, a banker holding capital k_t produces consumption goods y_t according to $y_t = ak_t$. Households also have a linear production technology $y_t^h = a^h k_t^h$, although they are less productive; that is, $a^h < a$. Both bankers and households have the investment technology that they can invert $g(\iota_t)k_t$ units of consumption goods into $\iota_t k_t$ units of new capital goods, where $g(\iota_t) = \iota_t + 0.5\phi\,(\iota_t - \delta)^2$ with δ denoting the depreciation rate.[2] Thus, the capital stock held by an agent grows at the rate $\iota_t - \delta$ in the absence of any shock.

The exogenous aggregate shock to the economy is driven by a Poisson process $\{N_t\}_{t=0}^{\infty}$ with intensity λ. Whenever the Poisson shock hits the economy, the capital stock held by each agent

[2]We choose the functional form of capital adjustment cost to be consistent with Christiano et al. (2005), He and Krishnamurthy (2012a), and many other quantitative macroeconomic models.

drops by a constant proportion, κ.[3] The law of motion of the aggregate capital K_t is

$$dK_t = K_{t-}\left(\iota_{t-} - \delta\right)dt - K_{t-}\kappa dN_t$$

conditional on all agents choosing the same investment rate ι_{t-}, where K_{t-} denotes $\lim_{s\uparrow t} K_s$; that is, the left limit of the process $\{K_s, s \geq 0\}$ in period t. For purposes of exposition, we interpret time $t-$ as the period right before time t.

We assume that bankers have logarithmic utility, households are risk neutral, and both types of agents have a time discount rate ρ. The expected discounted lifetime utility of a banker is

$$E_0\left[\int_0^\infty e^{-\rho t} u\left(c_t\right) dt\right], \tag{1}$$

where

$$u\left(c\right) = \begin{cases} \ln\left(c\right), & \text{if } c > 0, \\ -\infty, & \text{otherwise.} \end{cases} \tag{2}$$

We assume that households can have negative consumption. In Section 3.2, we modify the baseline model such that households have Epstein-Zin preferences.

1.1.2 Physical Assets

The market for capital goods has no friction. The market price of capital goods is in units of consumption goods, denoted by q_t. The law of motion of q_t is

$$dq_t = q_{t-}\mu^q_{t-}dt - q_{t-}\kappa^q_t dN_t,$$

where μ^q_t and κ^q_t are determined endogenously in equilibrium.

In period t, in the absence of a negative shock, the rate of return for a banker holding capital

[3]The capital in the model should be measured in efficiency units, and both the investment and the Poisson shock affect the quality of capital. The setup yields the tractability of the model, since the economy is scale-invariant with respect to the aggregate capital stock. Other recent papers that also use a macroeconomic capital quality shock include Gertler and Karadi (2011) and Gertler et al. (2012).

goods is

$$\frac{a - g(\iota_t)}{q_t} + \iota_t - \delta + \mu_t^q.$$

Other than the dividend yield $\left(a - g(\iota_t)\right)/q_t$, there are two sources of gains from holding capital: the growth in the banker's capital stock $\iota_t - \delta$ and the rise in the price of capital μ_t^q. Similarly, there are two types of risk for holding capital: exogenous and endogenous. Exogenous risk is the κ proportional decline in the banker's capital stock caused by the Poisson shock. Endogenous risk is the κ_t^q proportional change in the price of capital, which is the general equilibrium effect of the Poisson shock. Endogenous risk affects the banker's investment return through its impact on the $1 - \kappa$ proportion of capital goods left to the banker. Formally, the rate of return for bankers from holding capital is

$$\underbrace{\left[\frac{a - g(\iota_{t-})}{q_{t-}} + \iota_{t-} - \delta + \mu_{t-}^q\right]}_{\equiv R_{t-}} dt - \kappa_t^Q dN_t,$$

where $\kappa_t^Q \equiv \kappa + (1 - \kappa)\,\kappa_t^q$ is the overall investment risk. And, the rate of return for households holding capital is

$$\underbrace{\left[\frac{a^h - g(\iota_{t-}^h)}{q_{t-}} + \iota_{t-}^h - \delta + \mu_{t-}^q\right]}_{\equiv R_{t-}^h} dt - \kappa_t^Q dN_t.$$

1.1.3 The Financial Market and Regulatory Authority

The financial market is incomplete. The following four assumptions detail the incompleteness of the financial market.

Assumption 1 *Households do not hold equity issued by other agents.*

A banker can establish a regular bank. Via regular banking, bankers issue short-term debt and equity to finance their holdings of capital goods. The regulatory authority imposes the regulation in Assumption 2 on regular banks.

Assumption 2 *Regular banks' debt financing is taxed at rate τ_t in period t; tax revenues are instantly redistributed back to regular banks as lump-sum subsidies and the amount of the subsidy is proportional to bankers' wealth.*

Under the regulation, regular banks have to pay tax τ_t for each dollar they raise. Even though

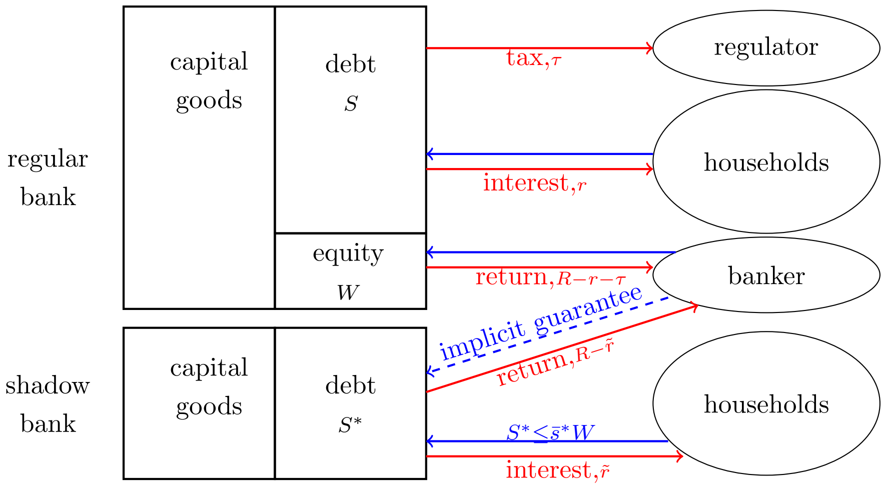

Figure 1: This figure details the financial side of the model. A regular bank's debt financing is taxed at rate τ. Households hold debt issued by regular banks and enjoy the rate of return r. Bankers earn regular banks' residual values at rate $R-r-\tau$ as their equity holders. Bankers also obtain shadow banks' residual value at rate $R-\tilde{r}$ as their guarantors, where $\tilde{r}$ is the rate of return that shadow banks promise to their household creditors. Bankers extend implicit guarantees to shadow banks. The maximum size of a shadow bank is the maximum leverage of shadow banking $\bar{s}$ times its banker's wealth W.

there is a tax rebate, the tax rate τ_t affects the optimal leverage of a regular bank because the rebate is distributed as a lump-sum subsidy.[4]

To circumvent the above regulation, a banker can sponsor a shadow bank that is not subject to bank regulation and earn its residual value as a management fee in each period. In practice, this activity is referred to as off-balance-sheet financing.[5]

Assumption 3 *The regulatory authority treats a shadow bank as a regular bank, if a banker holds the equity of the shadow bank.*

Shadow bank debt is also short term. Assumptions 1 and 3 imply that shadow banks are all debt financed. Any drop in the asset value of a shadow bank causes losses to its creditors unless the sponsor bails it out. Assumption 4 specifies the structure of the credit market for shadow banking and how households manage to secure the safety of their investments in shadow banks.

Assumption 4 *In any period t,*

i. each shadow bank offers a one-period debt contract $(\bar{s}^*_t, \tilde{r}_t)$;

[4]The lump-sum tax rebate set-up cancels the wealth effect of tax τ_t.

[5]Examples of off-balance-sheet financing in the real world are securitization, Asset-Backed Commercial Paper, and Money Market Funds.

ii. Given the contract, a shadow bank borrows up to $\bar{s}_t^*$ times the wealth of its sponsoring banker in period t and pays the principal and interest at rate $\tilde{r}_t$ in the following period.

iii. The market excludes bankers who default and allows their comebacks at rate ξ.

1.2 Problems for Bankers and Households

Suppose a banker's wealth is W_{t-} in period $t-$. S_{t-} denotes the value of debt that she issues via regular banking. The excess return from holding capital goods funded by regular banking is

$$S_{t-}\left(R_{t-}-r_{t-}-\tau_{t-}\right)dt-S_{t-}\kappa_t^Q dN_t,$$

where r_t is the risk-free rate.

The banker also manages a shadow bank. The size of the shadow bank is S_{t-}^* in dollar terms. The banker earns the difference between the return from the capital investment $R_{t-}S_{t-}^*$ and the interest $\tilde{r}_{t-}S_{t-}^*$ promised to creditors. The size of the shadow bank is limited by the leverage constraint specified by its debt contract $\left(\bar{s}_{t-}^*, \tilde{r}_{t-}\right)$

$$S_{t-}^* \leq \bar{s}_{t-}^* W_{t-}. \tag{3}$$

In addition, the banker needs to decide whether she would default or not if a Poisson shock hits the asset side of her shadow bank. This strategic choice is denoted by $\mathcal{D}_{t-}$. The subscript $t-$ means that the banker's default decision is based on the information realized prior to period t. Given that the Poisson shock hits the economy, if the banker does not default ($\mathcal{D}_{t-}=0$), she will bear the loss $S_{t-}^*\kappa_t^Q$ for creditors of her shadow bank; otherwise, she will not do so. Thus, a banker's dynamic budget constraint is

$$\begin{aligned} dW_t &= \left(W_{t-}R_{t-}+S_{t-}\left(R_{t-}-r_{t-}-\tau_{t-}\right)+S_{t-}^*\left(R_{t-}-\tilde{r}_{t-}\right)+\pi_{t-}W_{t-}-c_{t-}\right)dt \\ &\quad -\left(W_{t-}\kappa_t^Q+S_{t-}\kappa_t^Q+\left(1-\mathcal{D}_{t-}\right)S_{t-}^*\kappa_t^Q\right)dN_t, \end{aligned} \tag{4}$$

where π_t is the ratio of subsidy to wealth and c_t is the banker's consumption.

Taking $\{q_t, r_t, \tau_t, \tilde{r}_t, \pi_t, \bar{s}_t^*\}_{t=0}^{\infty}$ as given, the banker chooses $\{c_t, S_t, S_t^*, \iota_t, \mathcal{D}_t\}_{t=0}^{\infty}$ to maximize her expected lifetime utility (1) subject to the leverage constraint (3) and the dynamic budget constraint (4).

Households can invest in both capital goods and debt that both regular and shadow banks issue. S_t^h denotes the value of capital that household h holds, and n_t the value of shadow bank debt it holds. The wealth W_t^h of the household evolves according to

$$dW_t^h = \left(W_{t-}^h r_{t-} + S_{t-}^h \left(R_{t-}^h - r_{t-}\right) + n_{t-}\left(\tilde{r}_{t-} - r_{t-}\right) - c_{t-}^h\right) dt - \left(S_{t-}^h + \mathbf{d}_t n_{t-}\right) \kappa_t^Q dN_t, \quad (5)$$

where $\mathbf{d}_t$ denotes the fraction of shadow bank debt that defaults. Formally, a household chooses $\left\{c_t^h, n_t, S_t^h\right\}_{t=0}^{\infty}$ to maximize

$$U_0^h = E_0 \left[\int_0^{\infty} e^{-\rho t} c_t^h dt\right].$$

1.3 Equilibrium

We make the following assumption to guarantee that bankers' wealth share would not be large enough to undo all financial frictions.

Assumption 5 *Each banker retires independently at rate χ. If a banker retires, she can only save her wealth and earn the risk-free rate r_t*

$\boldsymbol{I} = [0,1]$ and $\boldsymbol{J} = (1,2]$ denote sets of bankers and households, respectively. Individual bankers and households are indexed by $i \in \boldsymbol{I}$ and $j \in \boldsymbol{J}$.

Definition 1 *Given the initial endowments of capital goods $\left\{k_0^i, k_0^j; i \in \boldsymbol{I}, j \in \boldsymbol{J}\right\}$ to bankers and households such that*

$$\int_0^1 k_0^i di + \int_1^2 k_0^j dj = K_0,$$

and a locally deterministic tax rate process $\{\tau_t\}_{t=0}^{\infty}$, an equilibrium is defined by a set of stochastic processes adapted to the filtration generated by $\{N_t\}_{t=0}^{\infty}$: the price of capital good $\{q_t\}_{t=0}^{\infty}$, the risk-free rate $\{r_t\}_{t=0}^{\infty}$, the maximum leverage of shadow banking $\{\bar{s}_t^*\}_{t=0}^{\infty}$, the interest rate on shadow bank debt $\{\tilde{r}_t\}_{t=0}^{\infty}$, the ratio of subsidy to wealth $\{\pi_t\}_{t=0}^{\infty}$, wealth $\left\{W_t^i, W_t^j\right\}_{t=0}^{\infty}$, capital holdings $\left\{k_t^i, k_t^j\right\}_{t=0}^{\infty}$, investment decisions $\left\{\iota_t^i\right\}_{t=0}^{\infty}$, default decisions $\left\{\mathcal{D}_t^i\right\}_{t=0}^{\infty}$, and consumption $\left\{c_t^i, c_t^j\right\}_{t=0}^{\infty}$

of banker $i \in \boldsymbol{I}$ and household $j \in \boldsymbol{J}$; such that

1. $\left\{W_0^i, W_0^j\right\}$ *satisfy* $W_0^i = q_0 k_0^i$ *and* $W_0^j = q_0 k_0^j$, *for* $i \in \boldsymbol{I}$ *and* $j \in \boldsymbol{J}$;
2. *bankers solve their problems given* $\{q_t, r_t, \tau_t, \tilde{r}_t, \pi_t, \bar{s}_t^*\}_{t=0}^{\infty}$;
3. *households solve their problems given* $\left\{q_t, r_t, \tilde{r}_t, \mathcal{D}_t^i, i \in \boldsymbol{I}\right\}_{t=0}^{\infty}$;
4. *the budget of the regulatory authority is balanced;*
5. *markets for both consumption goods and capital goods clear*

$$\int_0^1 c_i^i di + \int_1^2 c_t^j dj = \int_0^1 \left(a - g\left(\iota_t^i\right)\right) k_t^i di + \int_1^2 \left(a^h - g\left(\iota_t^j\right)\right) k_t^j dj, \tag{6}$$

$$\begin{aligned} \int_0^1 k_t^i di + \int_1^2 k_t^j dj &= K_t, \\ \textit{where } dK &= \left(\int_0^2 \left(\iota_t^i - \delta\right) k_t^i di\right) dt - \kappa K_t dN_t; \end{aligned} \tag{7}$$

6. *the credit market for shadow bank debt clears*

Given the definition, the credit market for regular bank debt clears by Walras' Law.

1.4 Financial Frictions

It is worthwhile to summarize three types of financial frictions in the baseline model. First, we do not allow for the issuance of outside equity (Assumption 1). The restriction on equity financing gives rise to the balance sheet amplification mechanism in the financial friction literature. Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012b) provide microfoundations for this constraint in continuous-time macro-finance models. Our model naturally inherits the amplification mechanism discussed in these two papers.

The second friction is the intervention of the regulatory authority (Assumption 2). Bank regulations are necessary since leverage chosen by bankers in a competitive market may not be socially optimal when there is a pecuniary externality, as discussed by Lorenzoni (2008) and

Stein (2012). Section 3 shows that the tax on regular banking in Assumption 2 can improve social welfare by adjusting the leverage choice of regular banks and diminishing the pecuniary externality. The tax rate can be interpreted as the shadow cost of bank regulation in the real world.[6]

The third financial friction is the leverage constraint on shadow banking (Assumption 4). Unlike other papers on shadow banking, our model is closely based on the institutional detail of off-balance-sheet financing that sponsoring banks typically offer implicit guarantees for creditors holding off-balance-sheet debt. And, these implicit guarantees are subject to an enforcement problem, which in turn gives rise to the leverage constraint on shadow banking.

2 Bank Regulation and Financial Instability

In this section, we use numerical examples to characterize equilibria of the baseline model. With the model characterization, we will present our main result that the relationship between endogenous risk and bank regulation displays a U shape.

2.1 Equilibrium Characterization

2.1.1 Production of Capital Goods

Households choose the investment rate ι to maximize the instant rate of return from holding capital goods R_t^h, that is, the optimal ι solves

$$\max_{\iota} \frac{-\iota - 0.5\phi\left(\iota - \delta\right)^2}{q_t} + \iota.$$

The first-order condition implies that the optimal investment rate ι_t^h chosen by bankers is a function of the price of capital q_t, that is, $\iota_t^h = \delta + (q_t - 1)/\,\phi$. Since bankers have the same

[6] Kisin and Manela (2014) provide an estimate of the shadow cost of major banks' capital requirement constraint.

investment technology as households do, in equilibrium

$$\iota_t = \iota_t^h = \delta + \frac{q_t - 1}{\phi}. \tag{8}$$

2.1.2 Households' Optimal Choices

Given that households are risk-neutral and that they are not financially constrained, in equilibrium the expected return households earn from holding any financial asset or capital goods must equal their time discount factor ρ. Therefore, we have following equilibrium conditions:

$$r_{t-} = \rho; \tag{9}$$

$$R_{t-}^h - \lambda \kappa_t^Q \leq r_{t-}, \text{with equality if } S_{t-}^h > 0, \tag{10}$$

$$\tilde{r}_{t-} - \lambda \mathbf{d}_t \kappa_t^Q \leq r_{t-}, \text{with equality if } n_{t-} > 0, \tag{11}$$

for all $t \geq 0$. Equation (10) and (11) indicate that if households hold capital goods or shadow bank debt in equilibrium, the expected return from holding such assets must equal the risk-free rate.

2.1.3 Bankers' Optimal Choices

In this section, we highlight the intuition underlying optimal conditions for bankers' choices rather than deriving these conditions rigorously. Readers can find relevant mathematical derivations in Appendix A.

To facilitate the illustration, we take advantage of two well-known properties of logarithmic preferences in the continuous-time setting: *i*) a banker's consumption c_t is ρ proportion of her wealth W_t in period t, that is,

$$c_t = \rho W_t, \tag{12}$$

and *ii*) a banker's expected life-time utility, that is, continuation value, in period t J_t satisfies

$$J_t \equiv E_t\left[\int_t^\infty e^{-\rho(u-t)}\ln(c_u)du\right] = \frac{ln(W_t)}{\rho} + h_t,$$

where h_t is an additive term that depends on market conditions and evolves endogenously according to $dh_t = h_{t-}\mu^h_{t-}dt - h_{t-}\kappa^h_t dN_t$. Similarly, if the banker defaults, her continuation value is $\hat{J}_t = ln(W_t)/\rho + \hat{h}_t$, where $\hat{h}_t$ follows $d\hat{h}_t = \hat{h}_{t-}\mu^{\hat{h}}_{t-}dt - \hat{h}_{t-}\kappa^{\hat{h}}_t dN_t$.

Since a banker with the access to shadow banking can always choose not to use shadow banking, the continuation value of a banker without the access to shadow banking must be less than the continuation value of a banker who has the access conditional on that they have the same amount of wealth, that is, $\hat{h}_t \leq h_t$ for any $t \geq 0$.

Intuitively, a banker's portfolio choice (S_{t-}, S^*_{t-}) and strategic default choice $\mathcal{D}_{t-}$ maximize the expected growth rate of her continuation value, that is, $E_{t-}[dJ_t/dt]$. If the banker does not intend to default $\mathcal{D}_{t-} = 0$, then Ito's Lemma implies that

$$\begin{aligned} E_{t-}\left[\frac{dJ_t}{dt}\right] = & \frac{1}{\rho}\Big(R_{t-} + \pi_{t-} + s_{t-}(R_{t-} - r_{t-} - \tau_{t-}) + s^*_{t-}(R_{t-} - \tilde{r}_{t-}) + \lambda \ln\Big(1 - (1 + s_{t-} + s^*_{t-})\kappa^Q_t\Big)\Big) \\ & + \lambda h_{t-}\big(1 - \kappa^h_t\big) + O, \end{aligned}$$

where $s_t = S_t/W_t$, $s^*_t = S^*_t/W_t$, and O denotes the sum of all other terms that are unrelated to s_{t-} and s^*_{t-}. First order conditions with respect to (s_{t-}, s^*_{t-}) are

$$R_{t-} - r_{t-} - \tau_{t-} \leq \frac{\lambda\kappa^Q_t}{1 - \big(1 + s_{t-} + s^*_{t-}\big)\kappa^Q_t}, \text{ with equality if } s_{t-} > 0, \tag{13}$$

$$R_{t-} - \tilde{r}_{t-} \geq \frac{\lambda\kappa^Q_t}{1 - \big(1 + s_{t-} + s^*_{t-}\big)\kappa^Q_t}, \text{ with equality if } s^*_{t-} < \bar{s}^*_{t-}. \tag{14}$$

If the banker intends to default to her shadow bank debt in the event of an adverse shock, then she does not bear the loss $S^*_{t-}\kappa^Q_t$ for creditors of her shadow bank but her continuation value shifts downward. Thus, if $\mathcal{D}_{t-} = 1$, then

$$E_{t-}\left[\frac{dJ_t}{dt}\right] = \frac{1}{\rho}\Big(R_{t-} + \pi_{t-} + s_{t-}(R_{t-} - r_{t-} - \tau_{t-}) + s^*_{t-}(R_{t-} - \tilde{r}_{t-}) + \lambda \ln\Big(1 - (1 + s_{t-})\kappa_t^Q\Big)\Big) + \lambda \hat{h}_{t-}\big(1 - \kappa_t^{\hat{h}}\big) + O.$$

Let $(\tilde{s}_{t-}, \tilde{s}^*_{t-})$ denote the optimal portfolio choice of the banker if she intends to default when her shadow bank is in trouble. The banker will always honor her shadow bank debt if doing so yields a higher expected growth rate of her continuation value, that is,

$$\begin{aligned} & \frac{1}{\rho}\Big(s_{t-}\left(R_{t-} - r_{t-} - \tau_{t-}\right) + s^*_{t-}\left(R_{t-} - r_{t-}\right) + \lambda \ln\Big(1 - \big(1 + s_{t-} + s^*_{t-}\big)\,\kappa_t^Q\Big)\Big) + \lambda h_{t-}(1 - \kappa_t^h) \\ \geq\ & \frac{1}{\rho}\Big(\tilde{s}_{t-}\left(R_{t-} - r_{t-} - \tau_{t-}\right) + \tilde{s}^*_{t-}\left(R_{t-} - r_{t-}\right) + \lambda \ln\Big(1 - \left(1 + \tilde{s}_{t-}\right)\kappa_t^Q\Big)\Big) + \lambda \hat{h}_{t-}(1 - \kappa_t^{\hat{h}}). \end{aligned} \tag{15}$$

Bankers, who cannot access shadow banking due to default, only choose the debt-to-equity $\hat{s}_{t-}$ of their regular banks in period $t-$. Similarly, these bankers choose $\hat{s}_{t-}$ to maximize

$$E_{t-}\left[\frac{dJ_t}{dt}\right] = \frac{1}{\rho}\Big(R_{t-} + \pi_{t-} + \hat{s}_{t-}(R_{t-} - r_{t-} - \tau_{t-}) + \lambda \ln\Big(1 - (1 + \hat{s}_{t-})\kappa_t^Q\Big)\Big) + O.$$

The first-order condition is

$$R_{t-} - r_{t-} - \tau_{t-} = \frac{\lambda \kappa_t^Q}{1 - \left(1 + \hat{s}_{t-}\right)\kappa_t^Q}. \tag{16}$$

2.1.4 Shadow Bank Debt Market and Enforcement Problem

Shadow banks compete on two dimensions of their debt contract $(\bar{s}^*_{t-}, \tilde{r}_{t-})$ in the credit market. The no-outside-equity financing constraint for bankers (Assumption 1) implies that shadow bank debt issued in equilibrium must be risk-free and that its interest rate $\tilde{r}_{t-}$ should equal the risk-free rate. Given that households refuse to hold equity issued by bankers (Assumption 1), no household would demand risky shadow bank debt because it is the combination of a risk-free debt component and an equity component. Therefore, shadow bank debt ought to be risk-free. The same logic implies that the maximum leverage of shadow banking $\bar{s}^*_{t-}$ should guarantee that bankers would not default in the event of an adverse shock. Thus, the enforcement constraint (15) must hold in equilibrium.

To interpret the enforcement constraint more clearly, we focus on the case where $s_{t-} > 0$. In this case, we can verify that $s_{t-} + s^*_{t-} = \tilde{s}_t$ and $s^*_{t-} = \tilde{s}^*_{t-} = \bar{s}^*_{t-}$. Then, the enforcement constraint (15) reduces to

$$s^*_{t-} \leq \frac{\rho\lambda\big(h_{t-}(1-\kappa^h_t) - \hat{h}_{t-}(1-\kappa^{\hat{h}}_t)\big)}{R_{t-} - r_{t-} - \tau_{t-}}.$$

The above inequality has a clear message that both the increased opportunity cost of default $(h_{t-}(1-\kappa^h_t) - \hat{h}_{t-}(1-\kappa^{\hat{h}}_t))$ and the decreased profitability of banking $(R_{t-} - r_{t-} - \tau_{t-})$ can alleviate the enforcement problem and raise the borrowing capacity of shadow banking.

We will impose a simplification assumption in Section 2.2 to ensure that the above simplification is always valid.[7] Thus, the maximum leverage of shadow banking in equilibrium satisfies

$$\bar{s}^*_{t-} = \frac{\rho\lambda\big(h_{t-}(1-\kappa^h_t) - \hat{h}_{t-}(1-\kappa^{\hat{h}}_t)\big)}{R_{t-} - r_{t-} - \tau_{t-}} \tag{17}$$

To fully specify $\bar{s}^*_{t-}$, we need to know the difference between h_t and $\hat{h}_t$, which is denoted by H_t. The following proposition characterizes H_t, whose interpretation is the opportunity cost for a banker to default on her shadow bank obligations in period t.

Proposition 1

$$H_t \equiv h_t - \hat{h}_t = E_t\left[\int_t^\infty \exp\left(-\left(\rho+\xi+\chi\right)\left(u-t\right)\right) f_u \, du\right], \tag{18}$$

where f_u equals

$$\frac{1}{\rho}\left(\begin{array}{c} \overbrace{\big(s_{u-} + s^*_{u-} - \hat{s}_{u-}\big)\left(R_{u-} - r_{u-} - \tau_{t-}\right)}^{\text{higher leverage benefit due to cheap credit}} + \overbrace{s^*_{u-}\tau_{u-}}^{\text{tax benefit}} \\ +\lambda\underbrace{\Big(\ln\left(1-\left(1+s_{u-}+s^*_{u-}\right)\kappa^Q_u\right) - \ln\left(1-\left(1+\hat{s}_{u-}\right)\kappa^Q_u\right)\Big)}_{\text{high risk due to high leverage}} \end{array}\right),$$

Proof. See Appendix B. ■

f_u is the tax benefit (i.e., regulatory arbitrage) that shadow banking offers, which is essentially

[7] See Assumption $2'$ and footnote 10.

the difference between the wealth growth rate of bankers with the access to shadow banking and the wealth growth rate of bankers without the access. The opportunity cost of default H_t is the present value of future tax benefits f_u that a banker will lose if she defaults in period t. The discount factor is the banker's time discount factor plus the "comeback" intensity ξ and the retirement rate χ. Once bankers return to the shadow banking sector or retire, the benefit of accessing shadow banking disappears.

We next highlight the feedback loop between the maximum leverage of shadow banking $\{\bar{s}_t^*\}_{t=0}^{\infty}$ and the cost of default $\{H_t\}_{t=0}^{\infty}$. First, the enforceability constraint (17) implies that the maximum leverage of shadow banking relies on a banker's cost of default to her shadow bank's obligations. Second, the probabilistic representation of the cost of default (18) indicates that the maximum leverage of shadow banking directly affects how costly default is for bankers.

This feedback loop gives rise to an equilibrium where shadow banking does not exist. Conjecture that $\{\bar{s}_t^* = 0\}_{t=0}^{\infty}$. The probabilistic representation (18) implies $\{H_t = 0\}_{t=0}^{\infty}$, and the enforceability constraint (17) confirms the conjecture. Thus, we have the following proposition.

Proposition 2 *In the baseline model, there exists an equilibrium where shadow banking does not exist, that is,* $\{\bar{s}_t^* = 0, H_t = 0\}_{t=0}^{\infty}$.

We label this degenerate equilibrium the "bad" equilibrium since productive bankers are unable to leverage up via shadow banking. On the contrary, there may also exist an equilibrium, where shadow banking exists. In this case, there exist a continuum of sunspot equilibria, in which the economy may suddenly switch to the "bad" equilibrium where shadow banking disappears. Since equilibrium selection is beyond the scope of this paper and the shadow banking sector is very important in reality, we will focus on the non-sunspot equilibrium, in which shadow banking exists and the shadow banking system never suddenly collapses.[8] We label this equilibrium the "good" equilibrium.

[8]Magnitudes of both endogenous risk κ_t^q and investment risk κ_t^Q are deterministic during each period in non-sunspot equilibria.

2.1.5 Miscellany

Equilibrium conditions 4 and 5 in Definition 1 are straightforward. The budget of the regulatory authority is balanced if it transfers all tax revenues back to bankers, that is, $\pi_t = s_t \tau_t$ for $t \geq 0$. Since households are risk-neutral and they can have negative consumptions, the market for consumption goods clears automatically. The market for capital goods clears if the fractions of capital held by bankers and households sum to 1. Let ψ_t denote the fraction of capital held by bankers, which equals $(1 + s_t + s_t^*)\omega_t$.

In our model, as in other continuous-time macro-finance papers, the wealth distribution matters for the dynamics of the economy. Later, we will capture the dynamics of an equilibrium with the bankers' wealth share $\omega_t \equiv \int_0^1 W_t^i di \Big/ q_t K_t$. Lemma 1 characterizes how ω_t evolves.

Lemma 1 *The law of motion of ω_t is*

$$\begin{aligned} d\omega_t &= \omega_{t-}\mu_{t-}^{\omega} dt - \omega_{t-}\kappa_t^{\omega} dN_t, && (19) \\ \textit{where } \mu_t^{\omega} &= R_t + s_t (R_t - r_t) + s_t^* (R_t - \tilde{r}_t) - \mu_t^q - \mu_t^K - \rho - \chi, && (20) \\ \textit{and } \kappa_t^{\omega} &= \frac{\left(s_{t-} + s_{t-}^*\right) \kappa_t^Q}{1 - \kappa_t^Q}. && (21) \end{aligned}$$

Proof. See Appendix B. ■

2.2 Markov Equilibrium

Equations (3) – (18) can characterize an equilibrium specified by Definition 1. Since our model has the property of scale-invariance with respect to K_t, we can characterize an equilibrium that is Markov in ω with a modification of Assumption 2.[9],[10]

Assumption 2′ *In period t, the tax rate τ_t equals $\min\{\tau, \tau s_t\}$, where τ is a positive constant and*

[9] A natural tax policy is that $\tau_t = \tau$ for $t \geq 0$. Given this policy, endogenous risk κ_t^q jumps as shadow banks become the marginal buyer of physical capital, which complicates the computation of an equilibrium. Our setup specified in Assumption 2′ makes the process that shadow banks become marginal buyers smooth and simplifies the computation.

[10] Given Assumption 2′, bankers would not have a corner solution $(s_{t-} = 0, s_{t-}^* > 0)$. If this is true, then the tax rate τ would be zero due to Assumption 2′, which contradicts the conjecture that $(s_{t-} = 0, s_{t-}^* > 0)$ is the conner solution. Therefore, the simplification of the enforcement condition (15) considered in Section 2.1.4 is general under Assumption 2′.

s_t denotes $\int_0^1 S_t^i \, di \Big/ \int_0^1 W_t^i \, di$; the tax rate is τ for bankers who cannot access shadow banking due to default.

Since the tax rate τ_t at any time t only depends on the aggregate variable, individual bankers take the tax rate as given.

In the Markov equilibrium, the dynamics of all endogenous aggregate variables can be fully described by the law of motion of the state variable and functions $q(\omega)$ and $H(\omega)$, whose domain is $(0, \bar{\omega}]$. Thanks to Ito's Lemma, we derive the law of motion of $\{q_t, H_t\}_{t=0}^{\infty}$.

$$\mu_t^q = \frac{q'(\omega_t)}{q(\omega_t)} \omega_t \mu_t^\omega, \tag{22}$$

$$\kappa_t^q = \frac{q(\omega_{t-}) - q(\omega_{t-}(1 - \kappa_t^\omega))}{q(\omega_{t-})}, \tag{23}$$

$$\mu_t^H = \frac{H'(\omega_t)}{H(\omega_t)} \omega_t \mu_t^H, \tag{24}$$

$$\kappa_t^H = \frac{H(\omega_{t-}) - H(\omega_{t-}(1 - \kappa_t^\omega))}{H(\omega_{t-})}. \tag{25}$$

The following proposition describes a system of delay differential equations and their boundary conditions, which define functions $q(\omega)$ and $H(\omega)$.

Proposition 3 *Given $(\omega, q(\omega'), H(\omega'), 0 < \omega' \leq \omega)$, we compute $(q'(\omega), H'(\omega))$ using the following procedure:*

1. *Conjecture that $\psi < 1$, we find $s + s^*$ such that*

$$\frac{a - a^h}{q} - \tau = \frac{\lambda \kappa^Q}{1 - (1 + s + s^*)\kappa^Q} - \lambda \kappa^Q,$$

   equations (21) and (23) hold. Next, we derive $(\kappa^\omega, \kappa^q, \kappa^Q, \kappa^H)$ according to Ito's Lemma and μ^q based on equation (13). In the end, we compute ψ to check if our conjecture is true.

2. *If $\psi < 1$ does not hold, then $\psi = 1$ and $s + s^* = 1/\omega - 1$. Similarly, we derive $(\kappa^\omega, \kappa^q, \kappa^Q, \kappa^H)$ according to Ito's Lemma and μ^q based on equation (13).*

3. *Given $s + s^*(= \tilde{s})$ and κ^Q, we compute $\bar{s}^*$ such that equation (17) holds and then derive s. In addition, we derive μ^ω according to equation (20).*

4. *We compute $q'(\omega)$ according to equation (22).*

5. *Finally, we compute f based on (18) and then derive $H'(\omega)$ according to*

$$(\rho+\xi+\chi)\,H(\omega)=f+\omega\mu^{\omega}H'(\omega)+\lambda\left(H(\omega\,(1-\kappa^{\omega}))-H(\omega)\right). \tag{26}$$

Boundary conditions are

$$\mu^{q}(\bar{\omega})=\mu^{H}(\bar{\omega})=\mu^{\omega}(\bar{\omega})=0,$$

$$\lim_{\omega\to 0}q(\omega)=\underline{q}\text{ and }\lim_{\omega\to 0}H(\omega)=0,$$

where $\underline{q}$ satisfies

$$a^{h}-\delta-\frac{\underline{q}^{2}-1}{2\phi}=\rho\underline{q}. \tag{27}$$

Proof. See Appendix B. ■

2.2.1 Equilibrium Uniqueness

Within the class of Markov equilibria, we can identify the condition under which the "bad" equilibrium is unique. To prove this result, we define mapping Γ which takes the cost of default function $H(\omega)$ as the input,

$$\Gamma H(\omega)=E_{t}\left[\int_{t}^{\infty}\exp(-(\rho+\xi+\chi)\,(u-t))f(\omega_{u})du\bigg|\,\omega_{t}=\omega\right]$$

where

$$f(\omega)=\frac{1}{\rho}\left(\begin{array}{c}(s+s^{*})\,(R(\omega)-r-\tau(\omega))+s^{*}\tau(\omega)-\hat{s}\,(R(\omega)-r-\tau)\\+\lambda\left(\ln\left(1-(1+s(\omega)+s^{*}(\omega))\,\kappa^{Q}(\omega)\right)-\ln\left(1-(1+\hat{s}(\omega))\,\kappa^{Q}(\omega)\right)\right)\end{array}\right)$$

and

$$s^{*}\leq\frac{\rho\lambda H(\omega)}{R(\omega)-r-\tau(\omega)},$$

where (s, s^*) are the portfolio weights of a banker who can access shadow banking given $\{q(\omega), \tau(\omega), \pi(\omega), \mu^\omega(\omega), \kappa^\omega(\omega)\}$ in an equilibrium and $\tilde{s}$ is the portfolio weight of a banker who cannot. Clearly, $H(\omega)$ in equilibrium is a fixed point of the mapping Γ. As we have noted, the mapping Γ allows for two possible fixed points: one leads to the "good" equilibrium, and the other yields the "bad" equilibrium. The following theorem provides a sufficient condition that justifies the uniqueness of the "bad" equilibrium, in which case the "good" equilibrium reduces to the "bad" one.

Theorem 1 *If $\tau < (\rho + \xi + \chi)\,\kappa$, the mapping Γ is a contraction mapping with the fixed point $H(\omega) = 0$ for all $\omega \in (0, \bar{\omega}]$.*

Proof. See Appendix B. ■

To show that Γ is a contraction mapping, we demonstrate that Γ satisfies Blackwell's sufficient conditions if $\tau < (\rho + \xi + \chi)\kappa$. The feedback loop illustrated earlier explains why Γ could be a contraction mapping. Suppose the comeback rate ξ increases permanently. Then, the opportunity cost of default drops because it becomes easier for a banker to escape from the punishment if she defaults, and thus the maximum leverage of shadow banking $\bar{s}^*$ declines (the enforceability constraint (17)). The decline in the leverage of shadow banking $\bar{s}^*$ reduces the opportunity cost of default H again (the probabilistic representation (18)). This cycle makes shadow banking unsustainable in equilibrium if ξ is large enough.

2.3 Numerical Example

In this section, we present main dynamic properties of the baseline model via numerical examples. Thanks to the global solution provided by the continuous-time approach, we are able to fully characterize dynamics of all endogenous variables because we can solve for values of these endogenous variables in each state as well as the dynamics of the state variable, bankers' wealth share ω.

We restrict the choice of parameter values by calibrating our model. Parameter values that we choose are $\rho = 3\%$, $\chi = 16\%$, $a = 22.5\%$, $a^h = 10\%$, $\delta = 10\%$, $\phi = 3$, $\lambda = 1$, $\kappa = 4\%$, $\tau = 3\%$, and $\xi = 7.5\%$. Appendix C contains the detail of our calibration.

2.3.1 Capital Misallocation, Endogenous Risk, and Pecuniary Externality

The constraint on issuing outside equity (Assumption 1) leads to capital misallocation and generates endogenous risk through the balance sheet amplification mechanism (He and Krishnamurthy, 2012b; Brunnermeier and Sannikov, 2014). As bankers' wealth share diminishes, they hold a declining fraction of capital goods and aggregate productivity deteriorates. The price of capital goods declines accordingly (Panel b in Figure 2). However, the excess rate of return for holding capital rises due to the low cost of purchasing capital (Panel d in Figure 2).

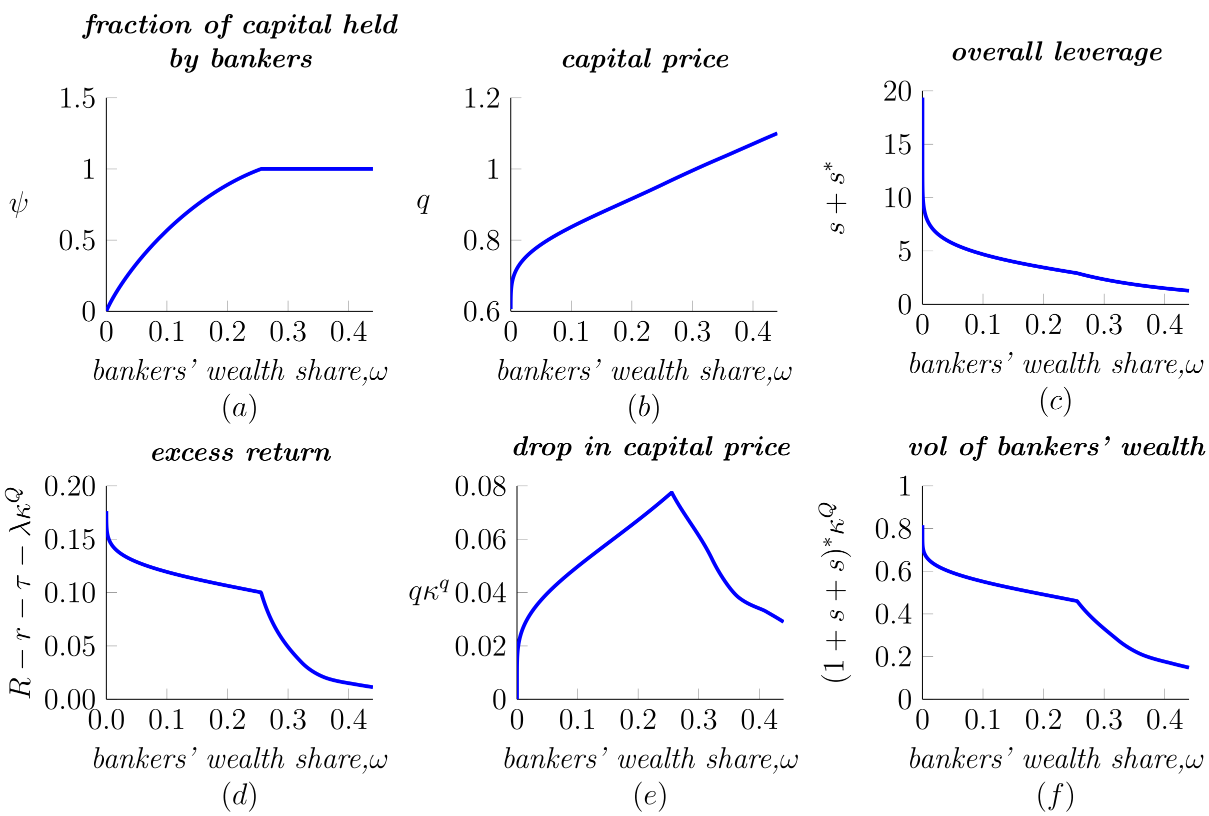

Figure 2: $\psi, q, s+s^*, R-r-\tau-\lambda\kappa^Q, q\kappa^q$, and $\left(1+s+s^*\right)\kappa^Q$ as functions of the state variable ω (i.e., bankers' wealth share). For parameter values, see Section 2.3.

A negative pecuniary externality exists in our model because bankers do not internalize how their leverage choice affects endogenous risk in the competitive economy. Therefore, the socially optimal leverage choice does not coincide with bankers' privately optimal choice. The tax on regular banking can twist bank leverage, lower endogenous risk, and improve social welfare. Welfare issues are discussed in detail in Section 3.

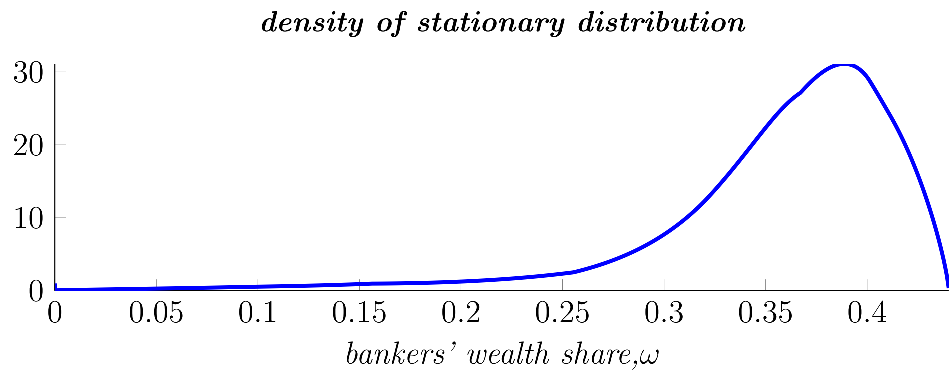

Figure 3: the density of stationary distribution. For parameter values, see Section 2.3.

2.3.2 The Dynamics of the State Variable.

The evolution of the state variable is described by Equation (19). Figure 3 shows that for most of the time bankers hold about 38% of the wealth in the economy. An economy is rarely in situations where bankers hold only a little wealth because the low price of capital goods and the high return from holding them help bankers to quickly build up their wealth and pull the economy out of recessions. Bankers' wealth share never exceeds the point $\bar{\omega}$, where $\mu^{\omega}(\bar{\omega}) = 0$ as bankers retire randomly.

2.4 The Feedback Loop in Shadow Banking

The feedback loop between the maximum leverage of shadow banking $\{\bar{s}_t^*\}_{t=0}^{\infty}$ and the cost of default $\{H_t\}_{t=0}^{\infty}$ is the driving force underpinning our main results: *i*) shadow banking is pro-cyclical; *ii*) shadow banking adds to financial instability (i.e., endogenous risk) through reintermediation; *iii*) bank regulation can raise or reduce financial instability under different circumstances. This section explains each of the three main results via dynamic and comparative statics analyses.

2.4.1 Dynamic Result: Pro-cyclical Leverage of Shadow Banking

We show that the leverage of shadow banking $\{s^*\}_{t=0}^{\infty}$ is pro-cyclical (Panel *a* in Figure 4). It is straightforward to see this from the expression for the maximum leverage of shadow banking (17). As the capital price increases in economic upturns (Panel *b* in Figure 4), the profitability of banking declines (Panel *c* in Figure 4) and the temptation to take high leverage declines. Therefore, the

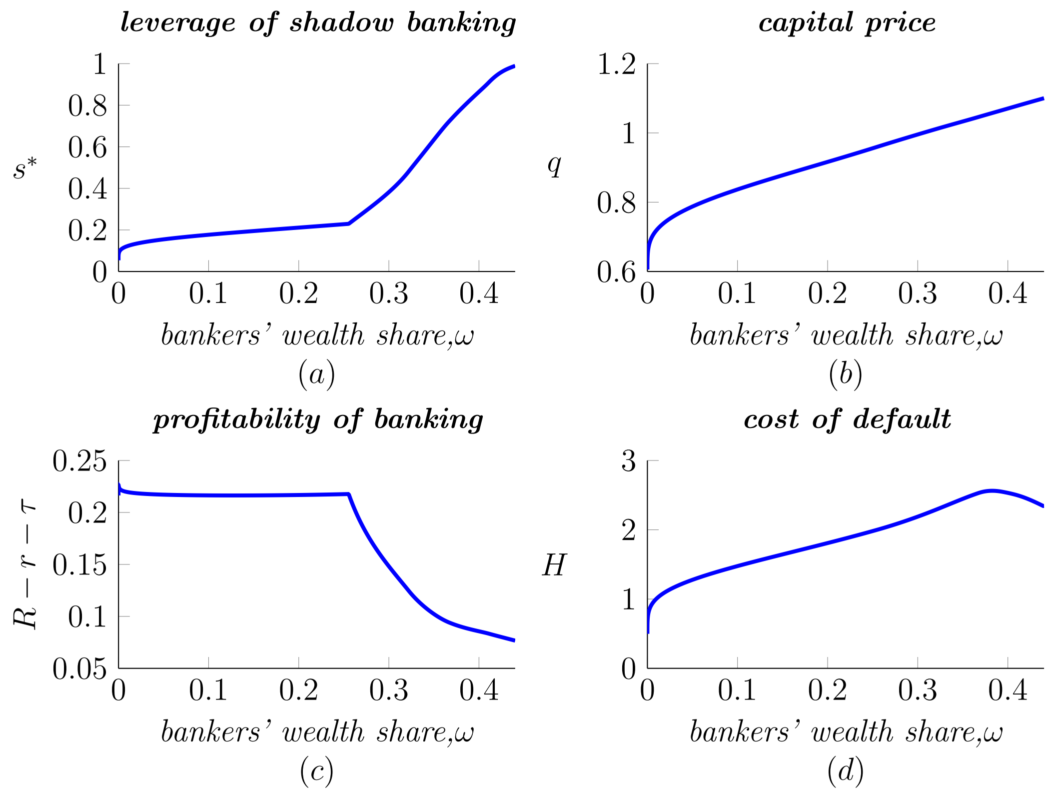

Figure 4: Pro-cyclical Shadow Banking
This figure presents the leverage of shadow banking s^*, capital price q, the profitability of banking $R - r - \tau$, and the cost of default H as functions of the state variable ω (i.e., bankers' wealth share). For parameter values, see Section 2.3.

constraint that prevents bankers' opportunistic behaviors becomes less tight in economic upturns. Thus, the leverage of shadow banking rises in upturns. In addition, the rising leverage of shadow banking increases the opportunity cost of default for bankers, which alleviates the enforcement problem further. Hence, the feedback loop between $\{s_t^*\}_{t=0}^{\infty}$ and $\{H_t\}_{t=0}^{\infty}$ contributes to the substantial expansion of shadow banking.

2.4.2 Comparative Statics

We next compare different economies and see how the presence of shadow banking changes the conventional understanding of financial instability and its connection to bank regulation. First, we examine economies with and without shadow banking and show that shadow banking increases financial instability. Next, we vary parameter τ and explain the U-shaped relationship between financial instability and the regulation of the traditional banking sector.

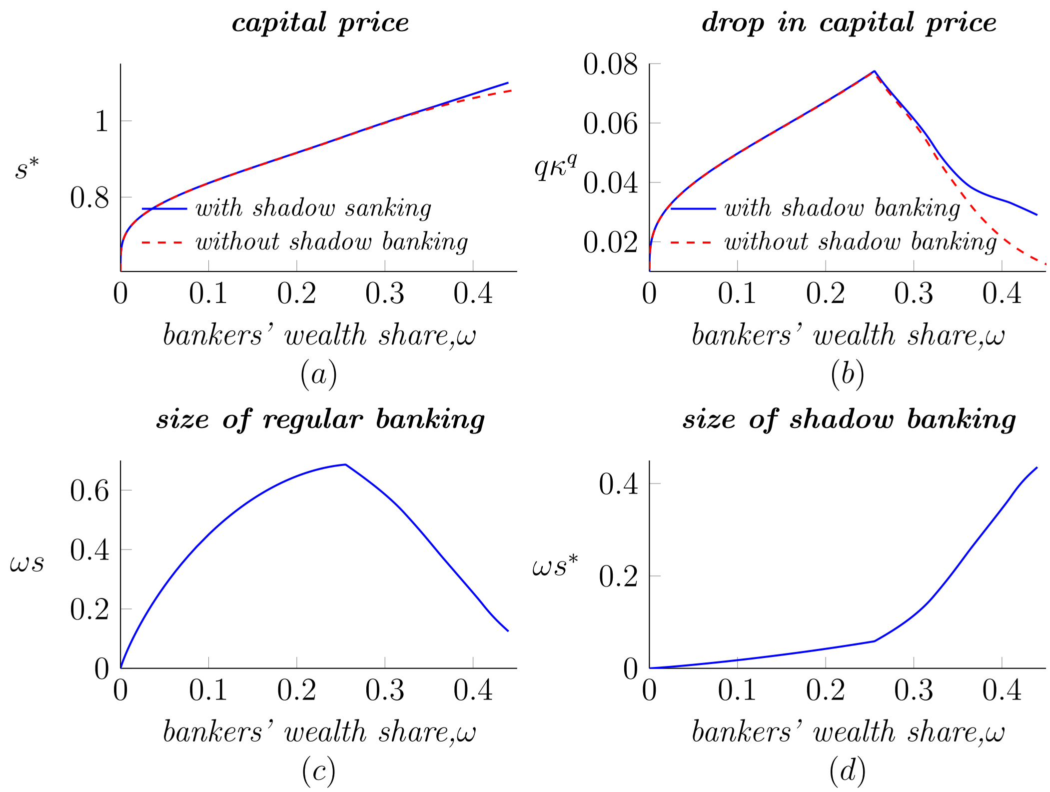

Figure 5: Reintermediation
This figure presents the price of capital (blue solid line in upper left panel), the jump in the price of capital (blue solid line in upper right panel), the size of the regular banking sector (lower left panel), and the size of the shadow banking sector (lower right panel) as functions of the state variable ω (i.e., bankers' wealth share) in the "good" equilibrium. For comparison, this figure also shows the price of capital (red dashed line in upper left panel) and the jump in the price of capital (red dashed line in upper right panel) as functions of the state variable ω in the "bad" equilibrium. For parameter values, see Section 2.3

Reintermediation. The combined effect of pro-cyclical shadow banking and reintermediation increases endogenous risk in the economy. In our model, the pro-cyclicality of shadow banking means that shadow banks purchase a large number of assets in economic upturns. The funding cost of regular banking is expensive due to bank regulation. Therefore, in economic booms, the scale of asset accumulation by shadow banks exceeds what regular banks would pursue in the absence of an accompanying shadow banking system. If an adverse shock hits the economy, shadow banks have to divest large amounts of assets as the leverage constraint tightens, and regular banks are reluctant to acquire these assets because it is very expensive to expand their balance sheets. As a result, the price of capital declines more than it would if there were no shadow banking

(Panel b in Figure 5). Finally, the decline in the price of capital raises the profitability of banking as well as the incentive to take high leverage. Thus, the tightening enforcement constraint (17) leads to the further decline in the leverage of shadow banking. Overall, our calibrated model shows that shadow banking increases the price volatility of capital $q\kappa^q$ by 28% on average.

Shadow Banking: Innocent or Not? The answer is “Yes and No” in our model. The answer is “Yes” because all shadow banks hold the same type of capital goods as regular banks do. The average quality of investments in the economy does not deteriorate because of shadow banking. Thus, the asset side of the shadow banking system is not responsible for high financial instability. Moreover, even if we move to the liability side, a single shadow bank that borrows up to the limit causes no harm, either. The answer is “No” because when shadow banks expand in economic upturns, bankers fail to take into account negative pecuniary externalities of asset fire-sales in economic downturns.

Regulatory Paradox. The conventional wisdom that tough regulation always secures financial stability may not hold when we take shadow banking into account. In economies without shadow banking, if the regulatory authority tightens regulation by raising τ banks' leverage and the price volatility of capital decline accordingly (Panel a in Figure 6). However, among economies with shadow banking one with tighter regulation experience higher market risk (Panel b in Figure 6). The intuition is that regular banks will face higher tax burdens if regulation becomes tighter. Thus, tighter regulation comes with the larger cost of default because regular banking becomes bankers' only option after default (Panel c in Figure 6). Furthermore, the larger cost of default leads to the higher leverage of shadow banking. Thus, the shadow banking sector is larger in economies with more stringent regulation. Since shadow banking adds to financial instability, tough regulation imposed on regular banks can jeopardize financial stability.

Regulatory Smile. The regulatory paradox does not mean that relaxing bank regulation always reduces financial instability. It depends on the relative size of the shadow banking system. Recall the feedback loop discussed earlier. If the regulatory authority lowers the tax rate τ, the benefit of shadow banking declines as well as the cost of default. This, in turn, lowers the maximum leverage of shadow banking and further reduces the cost of default. The feedback loop can be so significant that the shadow banking system becomes unsustainable. In our numerical

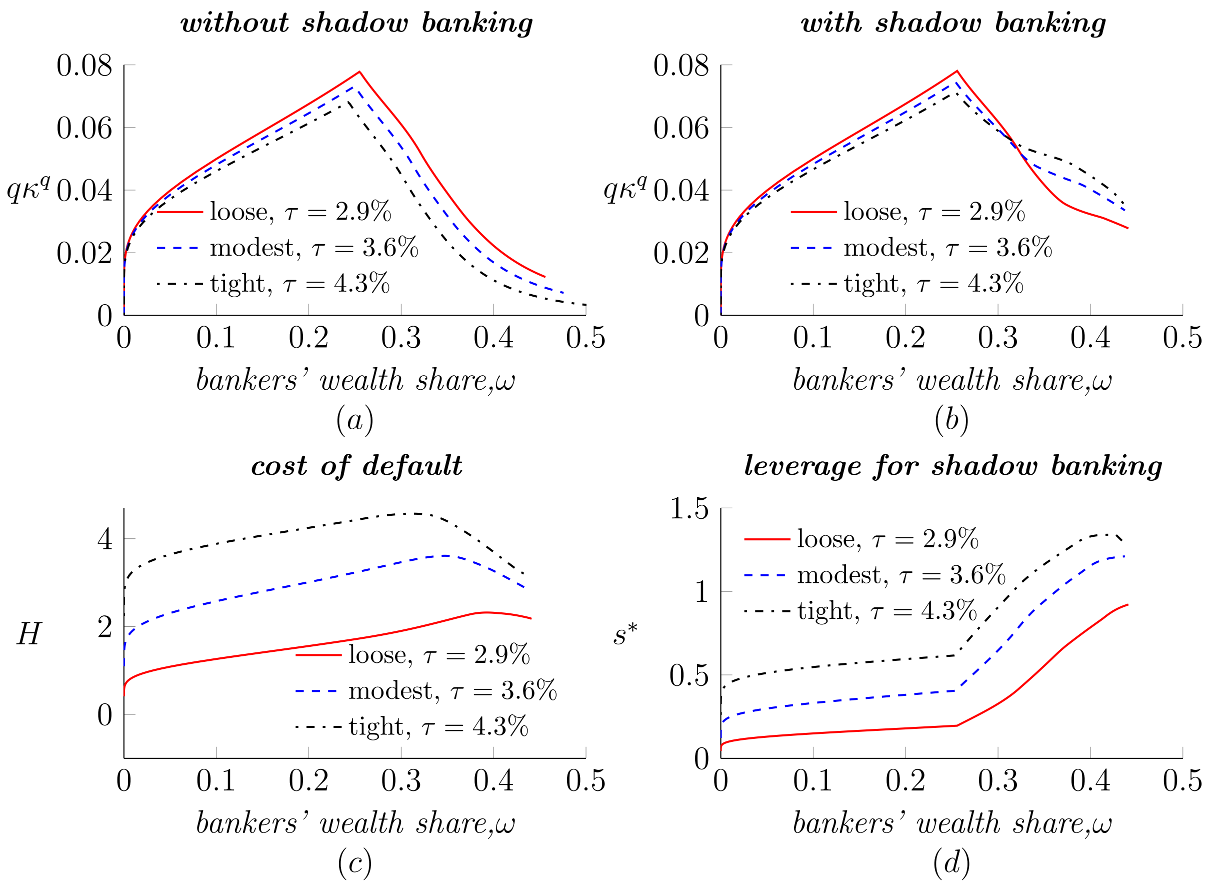

Figure 6: Regulatory Paradox
This figure shows the price of capital in the "bad" equilibrium (upper left panel), the price of capital in the "good" equilibrium (upper right panel), the cost of default (lower left panel), and the leverage of shadow banking (lower right panel). The red solid line is for the economy with loose bank regulation ($\tau = 2.9\%$); the blue dashed line for the economy with modest regulation ($\tau = 3.6\%$); the black dash-dot line for the economy with tight regulation ($\tau = 4.3\%$). For other parameter values, see Section 2.3

example, when τ declines from 3% to 2.5% (lower panel in Figure 7), the shadow banking system disappears. In the regime where the level of bank regulation is lenient enough to eliminate the shadow banking system, the conventional wisdom that tightening regulation secures financial stability still holds (upper panel in Figure 7).

2.4.3 Breaking down the Feedback Loop

The feedback loop between the maximum leverage of shadow banking and the opportunity cost of default is essential for the "regulatory paradox" result. To demonstrate this, we characterize a variant of the baseline model, in which the endogenous cost of default $\{H_t\}_{t=0}^{\infty}$ is replaced by a constant $\bar{H}$.

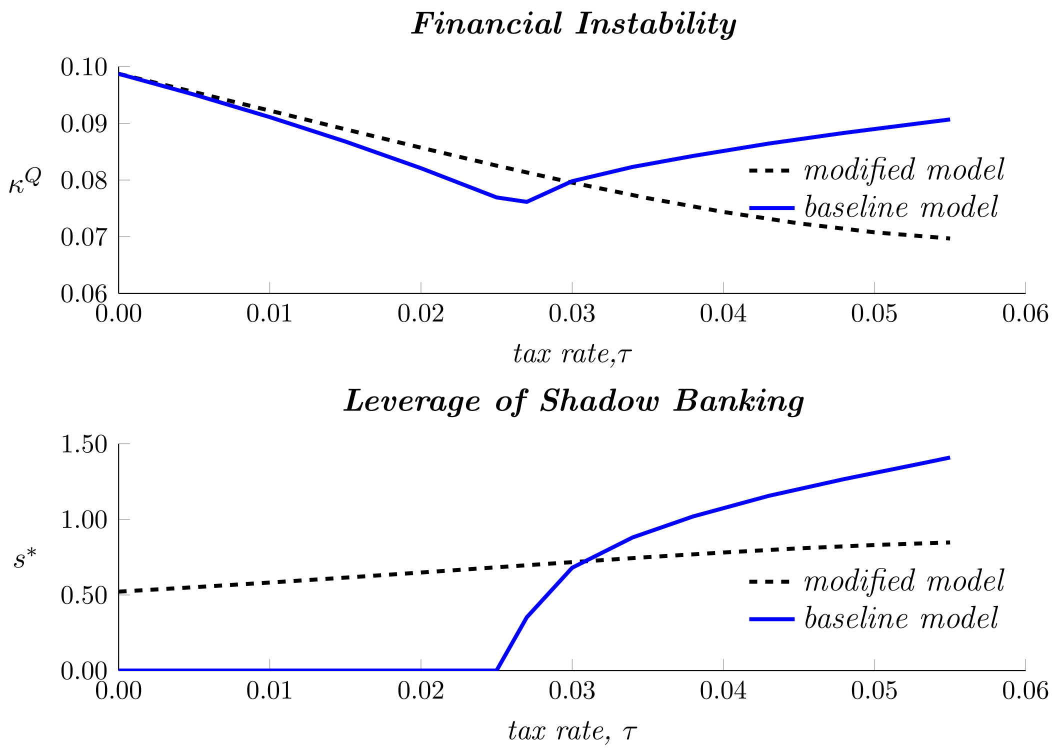

Figure 7: Financial Instability and Bank Regulation.
This figure shows how the change in the tax rate influences the volatility (the upper panel) and the leverage of shadow banking (the lower panel) at the stochastic steady states in the baseline model (blue solid lines) and in the modified model with an exogenous leverage constraint for shadow banking (black dashed lines). The stochastic steady state is the state where $\omega\mu^{\omega} - \lambda\omega\kappa^{\omega} = 0$. The exogenous cost of default H equals 2.3973, which is the average cost of default in the calibrated model in Section 2.3. For other parameter values, see Section 2.3.

Figure 14 in Appendix C shows that most basic results of our baseline model are preserved in the modified model. However, the "regulatory paradox" result does not hold (upper panel of Figure 7). The primary reason is that in the absence of the feedback loop the size of the shadow banking sector does not change much as the tightness of bank regulation varies (lower panel of Figure 7). When the regulatory authority raises the tax on regular banking, not many banking activities migrate to the shadow banking sector. As a result, the magnitude of the reintermediation does not change as significantly as it does in the baseline model. Therefore, financial instability simply declines as the tax rate increases in the modified model, which is the same as what we observe in an economy without shadow banking.

Our results have two major implications. First, we demonstrate that models, which do not endogenize the borrowing capacity of shadow banking and ignore the feedback loop between the borrowing capacity of shadow banking and the opportunity cost of default, cannot capture the non-monotonic relationship between financial instability and bank regulation. Second, our results imply that if the borrowing capacity of shadow banking is unrelated to the regulation of regular banking in an economy, tightening regulation may not necessarily squeeze a large amount of banking activities into the shadow banking sector because the credit market itself can prevent the shadow banking sector from growing.

3 Welfare Analysis

In this section, we highlight the welfare-improving feature of bank regulation in our framework, and particularly illustrate novel channels through which the regulation of regular banking influences social welfare in the presence of shadow banking. To have a clear understanding of the trade-off between growth and stability, we first focus on bankers' welfare in the baseline model. Second, we consider a variant of the baseline model, in which households have Epstein-Zin preferences.

3.1 Risk-Neutral Household

To avoid the problem of welfare aggregation, we reinterpret the baseline model as one that consists of a representative banker and a representative household.[11] Without loss of generality, we assume that the total capital stock in period 0 equals one. Note that the banker's wealth share ω_0 is exactly the fraction of capital goods that she owns. Thus, her wealth in period 0 is $\omega_0 q_0$, and the wealth of the representative household is $\big(1-\omega_0\big)q_0$. Hence, the welfare pair of the representative household and banker is $\big((1-\omega_0)q_0, \ln(\omega_0 q_0)/\rho + h_0\big)$. It is straightforward to see that the household's welfare only depends on the price of capital goods since he is risk-neutral.

Conditional on that shadow banking does not emerge, tightening bank regulation improves the

[11] According, we modify Assumption 5 such that the regulatory authority redistributes χ proportion of the banker's wealth to the household per unit of time.

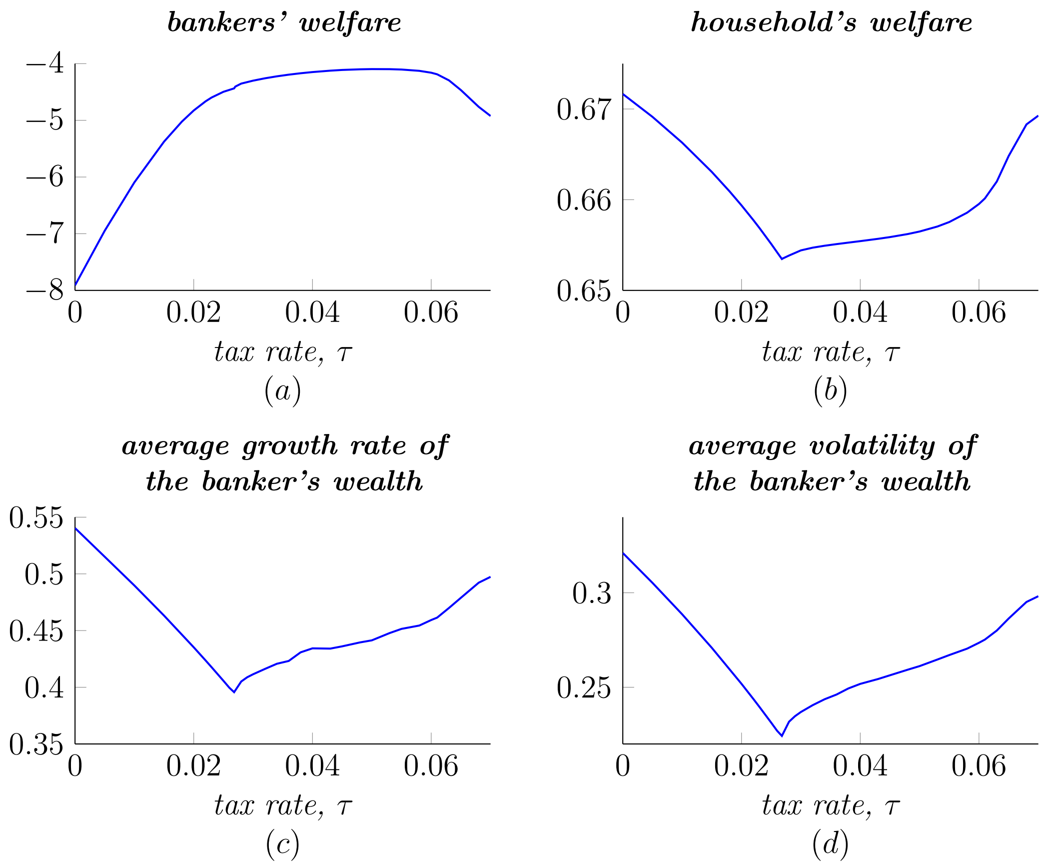

Figure 8: Welfare
This figure shows how the change in tax rate affects the representative banker's welfare (panel a), the representative household's welfare (panel b), the average growth rate of the banker's wealth (panel c), and the average volatility of the banker's wealth (panel d). For agents' welfare, we focus on state $\omega_0 = 0.38$. We use the stationary distribution to calculate moments. For parameter values other than τ, see Section 2.3.

banker's welfare (Panel a in Figure 8) but lowers the household's welfare (Panel b in Figure 8). As the tax rate τ increases from zero, the volatility of the banker's wealth increases and the growth rate of her wealth declines (Panel c and d in Figure 8). Overall, the benefit of the low wealth volatility dominates. From the banker's perspective, the unregulated competitive equilibrium ($\tau = 0$) is sub-optimal because of the pecuniary externality that she does not internalize the negative impact of her leverage choice on endogenous risk κ_t^q. Tightening bank regulation deteriorates the household's welfare because it prevents the productive banker from raising credit to hold capital goods and thus lowers the price of capital goods.

Once the shadow banking sector emerges, the rise in tax rate τ increases both the growth rate and the volatility of the banker's wealth (Panel c and d in Figure 8) because *i*) the shadow banking sector expands as regulation tightens, and *ii*) the growth of shadow banking more than offsets the negative effect of regulation in terms of economic growth. The growth of shadow banking benefits the banker's welfare (Panel a in Figure 8) because, as the shadow banking sector begins to expand, the growth benefit dominates the cost of increased endogenous risk. However, if regulation is too stringent, the negative effect of high volatility dominates the benefit of high growth, and thus tightening regulation hurts the banker's welfare.

As the shadow banking sector expands, the productive banker can easily raise cheap credit to fund her holding of capital goods. And, the price of capital goods appreciates accordingly. Therefore, tightening bank regulation benefits the household (Panel b in Figure 8).

3.2 Household with Epstein-Zin Preference

To further explore welfare implications of bank regulation in an economy with shadow banking, we modify our baseline model such that the representative household has the Epstein-Zin preference with the time discount rate ρ, the relative risk-aversion coefficient γ, and the elasticity of intertemporal substitution b^{-1}.

In the modified model, the household chooses $\{c_t^h, S_t^h, n_t, t \geq 0\}$ to maximize

$$U_0^h = E_0\left[\int_0^\infty f\left(c_s^h, U_s^h\right) ds\right],$$

where

$$f\left(c^h, U^h\right) = \frac{1}{1-b}\left\{\frac{\rho\left(c^h\right)^{1-b}}{\left(\left(1-\gamma\right)U^h\right)^{\frac{\gamma-b}{1-\gamma}}} - \rho\left(1-\gamma\right)U^h\right\}$$

and

$$U_s^h = E_s\left[\int_s^\infty f\left(c_v^h, U_v^h\right) dv\right], \text{ for } s > 0,$$

subject to the dynamic budget constraint (5).

In the interest of space, we skip the standard analysis of the household's optimal choice and only emphasize two market clearing conditions that differ from their counterparts in the baseline model. First, the market for consumption goods does not clear automatically in the modified model since households are risk-averse. Second, the risk-free rate is jointly determined by the portfolio choices of both the banker and the household and the dynamics of the household's continuation value.

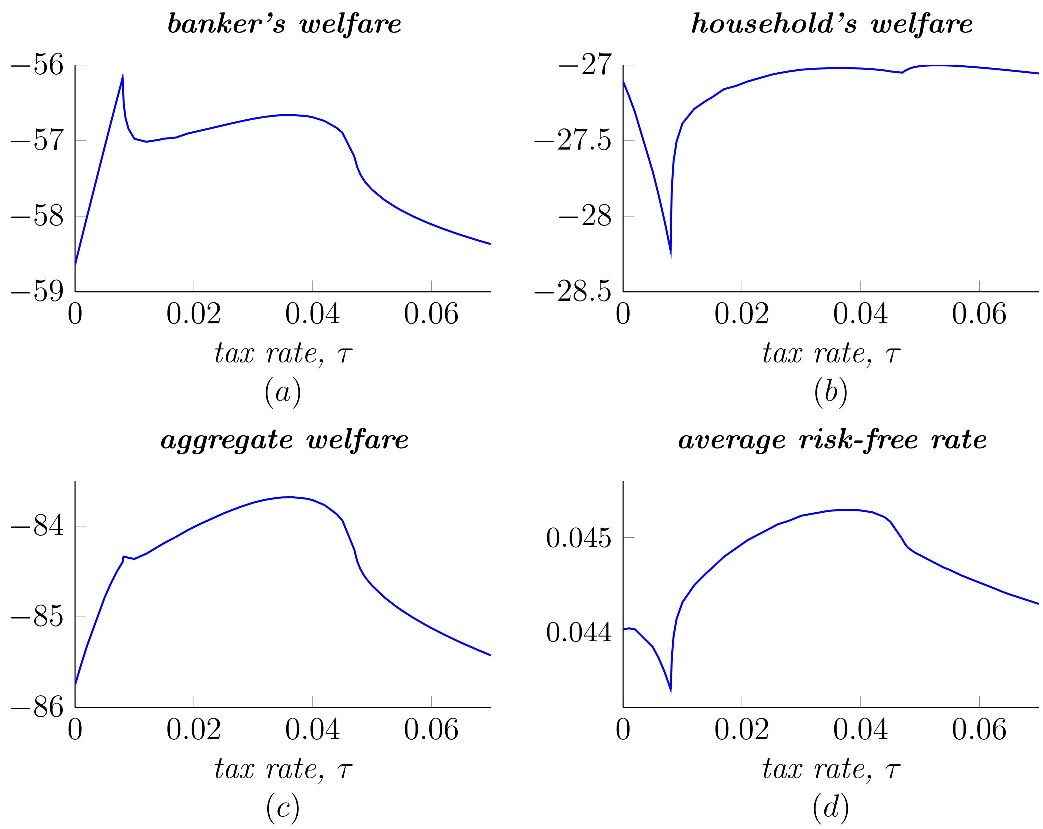

Figure 9: Welfare
This figure shows how the representative banker's welfare (panel a), the representative household's welfare (panel b), the sum of the two agents' welfare (panel c), and the average risk-free rate (panel d) change with the tax rate. For agents' welfare, we focus on state $\omega_0 = 0.38$. For parameter values other than τ, see Section 3.

As in our previous analyses, we focus on the Markov equilibrium of the modified model where shadow banking exists. The choice of parameter values is that $\rho = 4\%$, $\gamma = 2$, $b = 0.5$, $\chi = 0.1$, $a = 0.225$, $a^h = 0.1$, $\delta = 10\%$, $\phi = 3$, $\lambda = 1$, $\kappa = 4\%$, $\tau = 3\%$, and $\xi = 5\%$. Figure 15 and 16 in Appendix C show that results found in the baseline model survive in the modified model.

Given that the total capital stock in period 0 equals one, the welfare pair of the representative household and banker is

$$\left(\frac{(\zeta_0(1-\omega_0)q_0)^{1-\gamma}}{1-\gamma}, \frac{\ln(\omega_0 q_0)}{\rho} + h_0\right),$$

where ζ_0 is the continuation multiplier of the representation household in period 0.

Different from the baseline model, the equilibrium risk-free rate in the modified model depends on the credit demand and supply. If the regulatory authority starts regulating regular banking and shadow banking is still unsustainable, the decrease in the banker's credit demand lowers the risk-free rate (Panel d in Figure 9), which hurts the household (Panel b in Figure 9). Naturally, the banker's welfare improves because of the low volatility of her wealth and the low borrowing cost. Overall, imposing tax on regular banking still improves the sum of two representative agents' welfare when shadow banking does not emerge (Panel c in Figure 9).

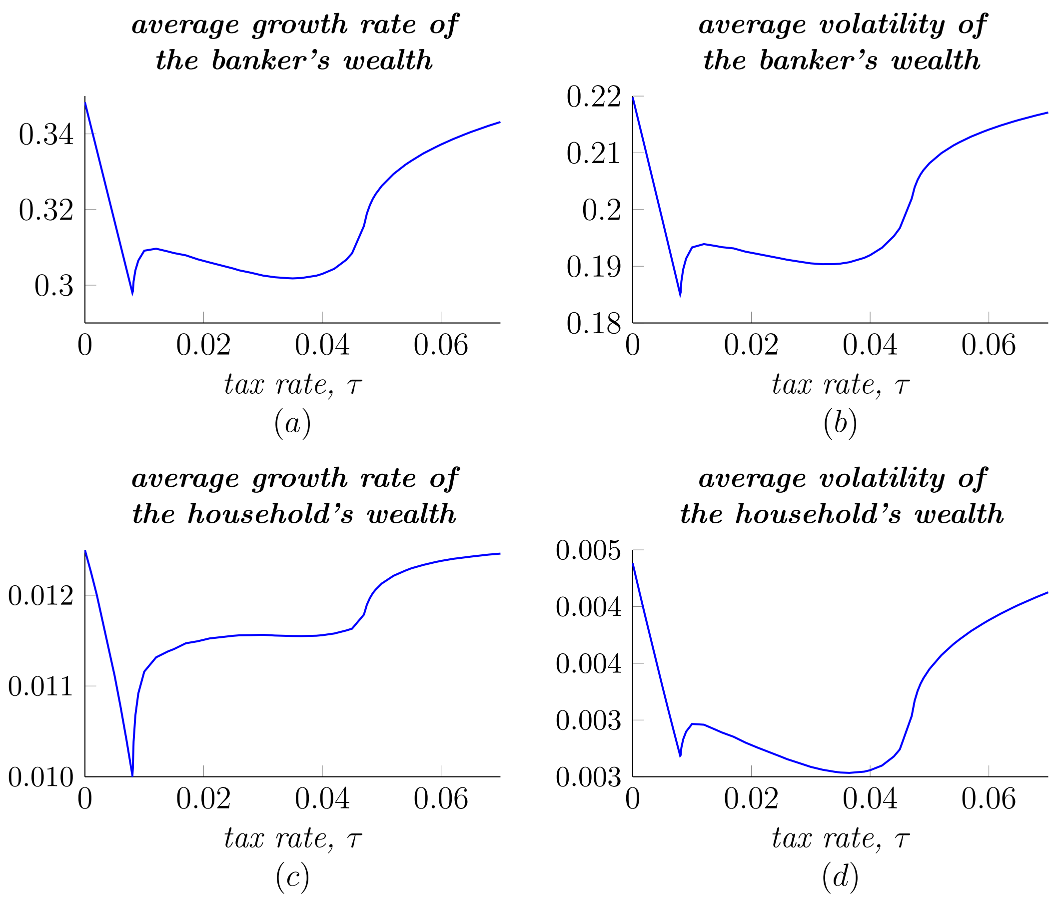

Figure 10: Welfare
This figure shows how tax rate influences the average growth rate (left panels) and the average volatility (right panels) of both the representative banker's wealth (upper panels) and the representative household's wealth (lower panels). We use the stationary distribution to calculate moments. For parameter values other than τ, see Section 3.

When shadow banking is sustainable, tightening bank regulation starts to improve the household's welfare. There are two underlying forces. First, the increased credit demand from shadow banks pushes up the risk-free rate, which benefits the household. Panel d in Figure 9 shows that this effect is so large that the average risk-free rate in an economy with a positive tax rate could be higher than it is in an economy without any tax. The underlying intuition is that if the banker has to pay a high tax rate for regular banking her willingness to pay a relatively high interest rate for shadow banking must be high as well.

The second force is that as the shadow banking sector expands the banker holds more fraction of capital goods in the economy and thus the household's exposure to the aggregate risk declines. Panel d in Figure 10 indicates that the volatility of the household's wealth declines as the tax rate increases until it hits 4%.

Overall, if we consider the sum of the two agent's welfare, we observe that strengthening bank regulation can raise the total welfare of the economy. Nevertheless, when regulation is too tight, tightening regulation makes social welfare worse off. This is primarily the consequence of the increased financial instability that the expansion of the shadow banking sector causes.

In summary, this section highlights novel channels, through which tightening regulation of regular banking improves the household's welfare. In particular, strengthening bank regulation helps the shadow banking sector expand, and the consequential increase in credit demand raises the return for the household to supply funds and lowers the household's exposure to the aggregate risk.

4 Policy Implications

In this section, we analyze the policy implications of our framework. First, we find that countercyclical regulation can generally improve financial stability because it alleviates the risk of the asset fire-sales between shadow banks and regular banks. Second, we show that our regulatory smile result still holds when the price control analyzed in the baseline model is replaced by a quantity control (e.g., capital requirement constraint).

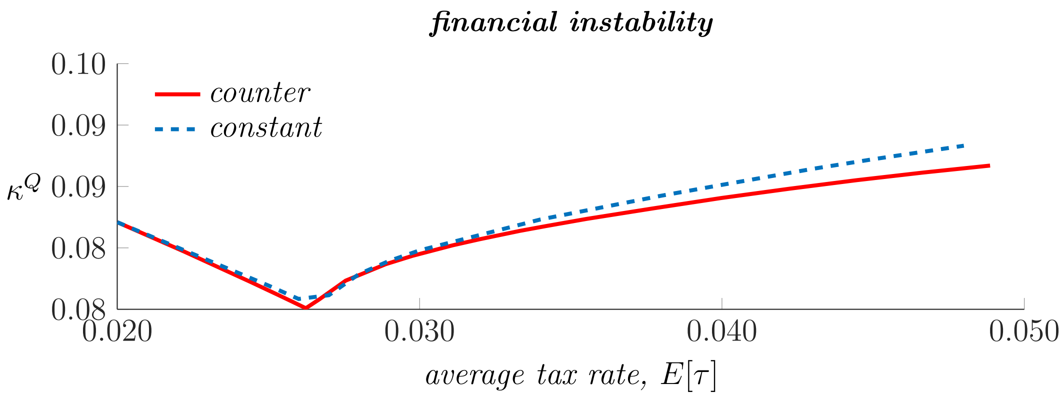

Figure 11: Financial Instability and Counter-Cyclical Regulation
This figure shows how the average volatility changes with the average tax rate in the baseline model (blue dashed line) and in modified model with a counter-cyclical tax rate policy (red solid line). All moments are based on the stationary distribution. For parameter values, see Section 2.3.

4.1 Counter-Cyclical Regulation

In this section, we substitute the constant tax rate regulation (Assumption $2'$) with a counter-cyclical regulation specified by the following assumption.

Assumption $2''$ *In period t, the tax rate τ_t equals* $\min\{\tau(\omega_t), \tau(\omega_t)s_t\}$. *$\tau(\omega)$ is defined by* $\big(\underline{\tau}(\tilde{\omega}-\omega)+\bar{\tau}\omega\big)/\tilde{\omega}$, *where $\underline{\tau}$, $\bar{\tau}$, and $\tilde{\omega}$ are constants, $\underline{\tau}<\bar{\tau}$, and $\tilde{\omega}$ is larger than $\bar{\omega}$.*

The interpretation of Assumption $2''$ is that the regulatory authority alleviates the tax burden on the regular banking sector in recessions and discourages bankers' use of leverage in economic booms.

The "regulatory paradox" result still holds in the model with the counter-cyclical policy, although the financial market is more stable. Figure 11 shows that counter-cyclical regulation can enhance financial stability when the average tax is high. This is because regular banks face declining borrowing costs when they need to raise funds to acquire assets dumped by shadow banks in economic downturns. Thus, the counter-cyclical regulation mitigates the magnitude of asset fire-sales and lowers the financial instability of an economy.

4.2 Quantity Control

In this section, we investigate a modified model, in which the regular banking sector is subject to a quantity control instead of the price control in the baseline model. In particular, we consider the capital-requirement constraint, which imposes an upper bound $\bar{s}$ for a regular bank's debt-to-equity ratio.

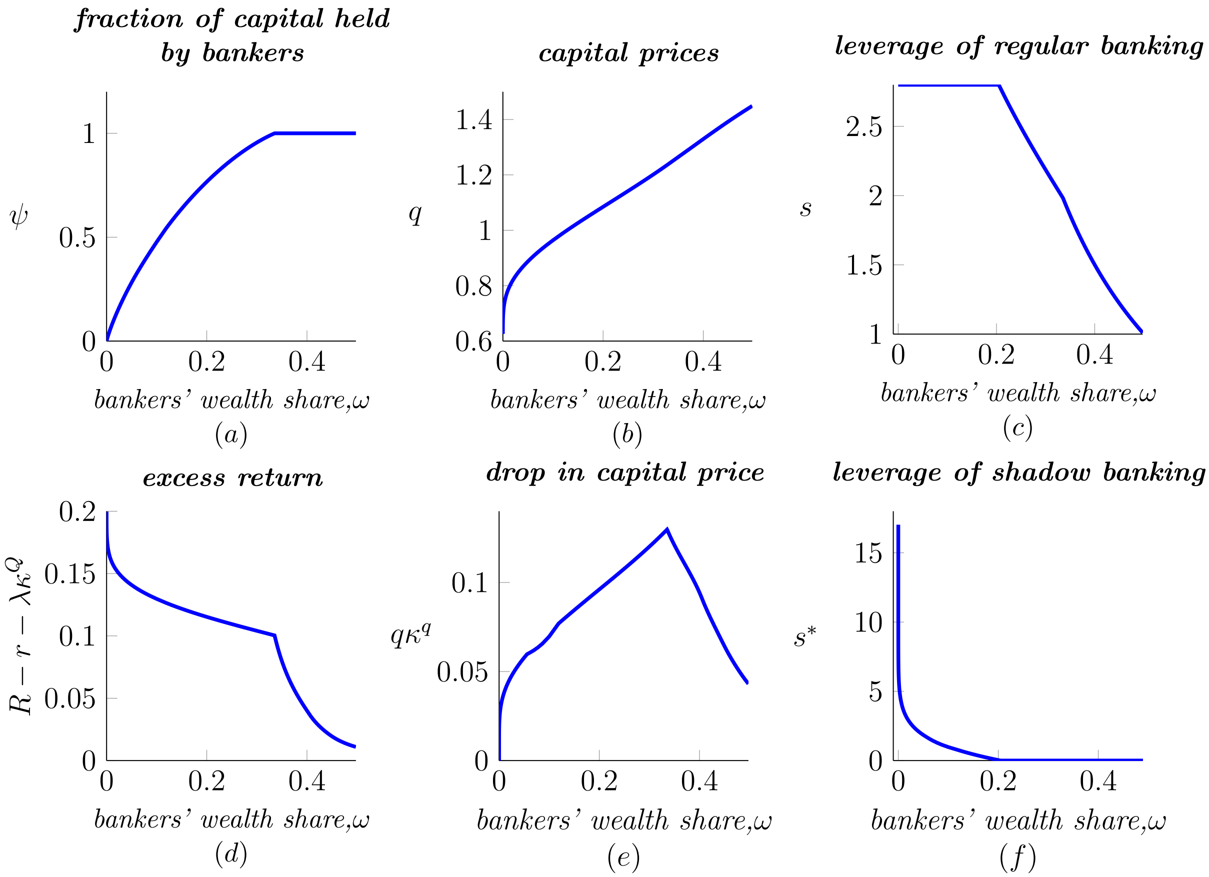

Figure 12: $\psi, q, s + s^*, R - r - \tau, q\kappa^q$, and $\left(1 + s + s^*\right)\kappa^Q$ as functions of the state variable ω, i.e., bankers' wealth share, in the modified model with the capital requirement constraint. The choice of parameter values follows: $\rho = 3\%$, $\chi = 1\%$, $a = 0.225$, $a^h = 0.1$, $\delta = 10\%$, $\phi = 3$, $\lambda = 1$, $\kappa = 4\%$, and $\bar{s} = 2.8$.

With the price control replaced by the quantity control, a banker's dynamic budget constraint becomes

$$dW_t = \left(W_{t-}R_{t-} + \left(S_{t-} + S^*_{t-}\right)\left(R_{t-} - r_{t-}\right) - c_{t-}\right)dt - \left(W_{t-} + S_{t-} + S^*_{t-}\right)\kappa_t^Q dN_t.$$

In addition to the leverage constraint for shadow banking (3), the banker in the modified model faces the capital-requirement constraint $S_t \leq \bar{s}W_t$. Similar changes apply to bankers who cannot access shadow banking due to default.

We first focus on the dynamics of endogenous variables and then move to the regulatory smile result of the quantity-control model. A number of endogenous variables have dynamics similar to the baseline model (Panels a-e in Figure 12). However, the leverage dynamics of shadow banking change drastically. This is the consequence of the fact that when bankers' share of wealth is small the excess return is high and the incentives to build up leverage are strong. In these states, it is extremely costly to default to shadow bank debt because regular banking only allows for considerably low leverage. Therefore, when bankers' share of wealth is small, the enforcement problem is not severe and the leverage of shadow banking is high. This property is absent in the baseline model because bankers do not face a binding leverage constraint for regular banking.

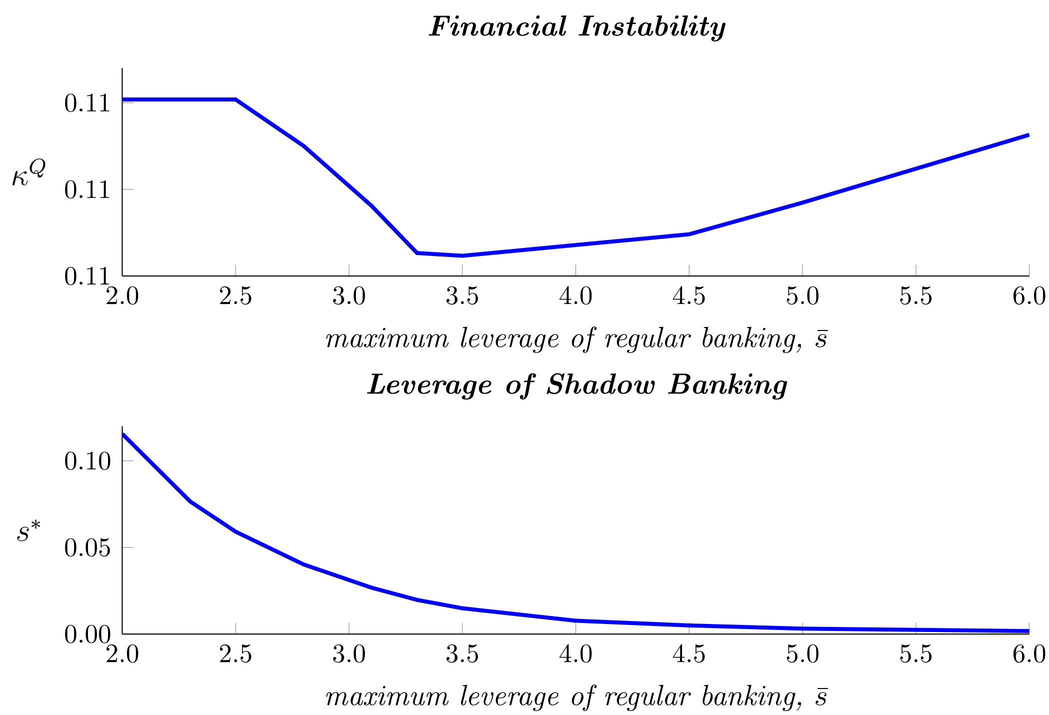

Figure 13: This figure shows the investment risk κ^Q (upper panel) and the leverage of shadow banking (lower panel) at the stochastic steady state of different economies with different capital-requirement constraints in the modified model. The stochastic steady state is the state where $\omega\mu^{\omega} - \lambda\omega\kappa^{\omega} = 0$. The maximum leverage of regular banking $\bar{s}$ is 2.8. For the choice of other parameter values, see Figure 12.

The constant quantity control is less appealing than the price control because it prevents productive bankers from leveraging when the entire economy needs them to do so. When bankers' wealth share is small, the aggregate productivity could be as low as households' productivity if the capital-requirement constraint is binding for regular banks and shadow banking is not available. We can see that micro-prudential policies such as the capital-requirement constraint may not benefit the overall economy, even though it contains the credit risk of individual banks.

Figure 13 shows that the regulatory smile result continues to hold in the modified model with quantity control. Very lenient bank regulation comes with the low leverage of shadow banking and high financial instability. As bank regulation tightens (i.e., the maximum leverage of regular banking $\bar{s}$ declines), financial instability initially diminishes. However, if the regulation is so tight that the shadow banking sector becomes sizeable, tighter regulation causes higher financial instability.

5 Conclusions

In this paper, we emphasize the enforcement problem with shadow banking that the use of implicit guarantees causes. By modeling this friction in a dynamic general equilibrium setting, our paper captures dynamics of shadow banking and uncovers the general equilibrium mechanism through which shadow banking adds to financial instability. Since the borrowing capacity of shadow banking is endogenous in our framework, our paper clearly highlights that tightening regulation of regular banking actually helps shadow banks increase their debt capacity. The general equilibrium framework that we use allows for welfare and policy discussions in the modern economy where the unregulated shadow banking sector plays a critical role.

The framework proposed in this paper could be extended in several directions. The most straightforward follow-up work is to characterize the social planner's constrained efficient regulatory rule by, say, a process of tax rate $\{\tau_t, t \geq 0\}$. However, this exercise requires a completely new methodology that can characterize the set of *all* competitive equilibria under *all* sorts of regulations. Secondly, one could investigate the collapse of the shadow banking system by exploring the stability property of the "good" equilibrium and endogenizing the regime switch between the "good" equilibrium and the "bad" one.

Appendix

A Derivation of Bankers' Optimal Choices

A banker's optimal overall leverage has an upper bound. This is because the second part of the utility specification (2) effectively imposes a nonnegative wealth constraint for bankers.

Proposition 4 *A banker's overall leverage has an upper bound in any period; in particular,*

$$(W_{t-} + S_{t-})\,\kappa_t^Q + S^*_{t-}\kappa_t^Q \mathbf{1}_{(d_t>0)} < W_{t-}$$

always holds.

Proof. See Appendix B. ■

Next, we spell out the Hamilton-Jacobi-Bellman (HJB) equation for the banker's optimal control problem

$$0 = \max_{c_{t-},S_{t-},S^*_{t-},\mathcal{D}_{t-}} \left\{(1-\mathcal{D}_{t-})\mathbf{HJB}_{\mathcal{N}} + \mathcal{D}_{t-}\mathbf{HJB}_{\mathcal{D}}\right\}, \text{ where} \tag{28}$$

$$\begin{aligned}
\mathbf{HJB}_{\mathcal{N}} &\equiv \max_{c_{t-},S_{t-},S^*_{t-}} \left\{ \begin{array}{c} \ln(c_{t-}) - \rho J_{t-} + \frac{\mu_W}{\rho} + h_{t-}\mu^h_{t-} + \chi\left(J^r_t(W_t) - J_{t-}\right) \\ +\lambda\left(\frac{1}{\rho}\ln\left(W_{t-} - \left(W_{t-} + S_{t-} + S^*_{t-}\right)\kappa_t^Q\right) + h_{t-}(1-\kappa_t^h) - J_{t-}\right) \end{array} \right\}, \\
\mathbf{HJB}_{\mathcal{D}} &\equiv \max_{c_{t-},S_{t-},S^*_{t-}} \left\{ \begin{array}{c} \ln(c_{t-}) - \rho J_{t-} + \frac{\mu_W}{\rho} + h_{t-}\mu^h_{t-} + \chi\left(J^r_t(W_t) - J_{t-}\right) \\ +\lambda\left(\frac{1}{\rho}\ln\left(W_{t-} - \left(W_{t-} + S_{t-}\right)\kappa_t^Q\right) + \hat{h}_{t-}(1-\kappa_t^{\hat{h}}) - J_{t-}\right) \end{array} \right\}, \\
\mu_W &\equiv \frac{1}{W_{t-}}\left(W_{t-}(R_{t-} + \pi_{t-}) + S_{t-}\left(R_{t-} - r_{t-} - \tau_{t-}\right) + S^*_{t-}\left(R_{t-} - \tilde{r}_{t-}\right) - c_{t-}\right),
\end{aligned}$$

and $J^r_t(\cdot)$ is the banker's continuation value function if she retires in period t. While choosing her portfolio and consumption in period t, the banker also decides whether she would default to her shadow bank obligations ($\mathbf{HJB}_{\mathcal{D}}$) in the event of an adverse shock or not ($\mathbf{HJB}_{\mathcal{N}}$). Because of the time-consistency problem, a banker's portfolio choice (S_{t-}, S^*_{t-}) with respect to both $\mathbf{HJB}_{\mathcal{N}}$ and $\mathbf{HJB}_{\mathcal{D}}$ must satisfy their corresponding incentive-compatible constraints:

$$\frac{\ln\left(W_{t-} - \left(W_{t-} + S_{t-} + S^*_{t-}\right)\kappa^Q_t\right)}{\rho} + h_{t-}(1-\kappa^h_t) \geq \frac{\ln\left(W_{t-} - \left(W_{t-} + S_{t-}\right)\kappa^Q_t\right)}{\rho} + \hat{h}_{t-}(1-\kappa^{\hat{h}}_t)$$

for $\mathbf{HJB}_{\mathcal{N}}$ and

$$\frac{\ln\left(W_{t-} - \left(W_{t-} + S_{t-} + S^*_{t-}\right)\kappa^Q_t\right)}{\rho} + h_{t-}(1-\kappa^h_t) \leq \frac{\ln\left(W_{t-} - \left(W_{t-} + S_{t-}\right)\kappa^Q_t\right)}{\rho} + \hat{h}_{t-}(1-\kappa^{\hat{h}}_t)$$

for $\mathbf{HJB}_{\mathcal{D}}$. First-order conditions with respect to portfolio choices are given by (13) and (14) for $\mathbf{HJB}_{\mathcal{N}}$, and

$$R_{t-} - r_{t-} - \tau_{t-} \leq \frac{\lambda\kappa^Q_t}{1-\left(1+\tilde{s}_{t-}\right)\kappa^Q_t}, \quad = \text{ if } \tilde{s}_{t-} > 0,$$

$$R_{t-} - \tilde{r}_{t-} \geq 0, \quad = \text{ if } \tilde{s}^*_{t-} < \bar{s}^*_{t-}.$$

for $\mathbf{HJB}_{\mathcal{D}}$. The banker find it optimal to honor her shadow bank debt if $\mathbf{HJB}_{\mathcal{N}} \geq \mathbf{HJB}_{\mathcal{D}}$.

B Proofs

Proof of Proposition 4. Now suppose in period t, the banker's wealth W_t is negative. The law of motion for the banker's wealth is

$$dW_t = \left(W_{t-}R_{t-} + S_{t-}\left(R_{t-} - r_{t-}\right) + S^*_{t-}\left(R_{t-} - r_{t-}\right) - c_{t-}\right)dt - \left(W_{t-}\kappa^Q_t + S_{t-}\kappa^Q_t + S^*_{t-}\kappa^Q_t\right)dN_t$$

Given a fixed time T, we can construct a new measure under which

$$R_{s-} - r_{s-} = \tilde{\lambda}_{t-}\kappa^Q_s$$

for each time s between t and T. Under this new measure,

$$\tilde{E}_t\left[W_T \exp\left(-\int_t^T r_u du\right) + \int_t^T \left(c_s \exp\left(-\int_t^s r_u du\right)\right) ds\right] \leq W_t < 0$$

Suppose the banker retires at the stopping time S with positive wealth W_S. After the banker retires, her wealth evolves as

$$dW_{S+u} = W_{S+u}\left(r_{S+u} - \rho\right) du.$$

Her wealth in period $S+s$ is $W_S \exp\left(\int_S^{S+s} r_u du - \rho s\right)$. It is easy to see that

$$\lim_{s\to\infty} E_S\left[\exp\left(-\int_S^{S+s} r_u du\right) W_{S+s}\right] = 0.$$

Thus, if T is large enough, $E_t\left[W_T \exp\left(-\int_t^T r_u du\right)\right]$ could be arbitrarily small. Since $W_t < 0$, the consumption of the banker must be negative at some point between t and T with a strictly positive probability. Since the banker has logarithm preference, the banker's expected lifetime discounted utility in period t must be negative infinity. Therefore, we show that it is never optimal for the banker to have negative wealth and that a banker's overall leverage must have an upper bound. ■

Proof of Proposition 1. Without loss of generality, we focus a banker with wealth W_t in period t and explicitly express her continuation value in different cases.

We start with the case that the banker is retired. Since logarithmic agents only consumer ρ fraction of their wealth, the growth rate of her wealth is $r_{t+v} - \rho$ in period $t+v$. Hence, the banker's wealth in period $t+u$ will be $W_t \exp\left(\int_0^u r_{t+v} - \rho\ dv\right)$. The banker's continuation value in period t is

$$\begin{aligned}
& \int_0^\infty \exp\left(-\rho u\right)\left(\ln\left(\rho W_t\right) + \int_0^u r_{t+v} - \rho\ dv\right) du \\
= & \frac{\ln\left(W_t\right)}{\rho} + \frac{\ln\left(\rho\right)}{\rho} + \int_0^\infty \exp\left(-\rho u\right)\int_0^u r_{t+v} - \rho\ dv\ du \\
= & \frac{\ln\left(W_t\right)}{\rho} + \frac{\ln\left(\rho\right)}{\rho} + \frac{1}{\rho}\int_0^\infty \exp(-\rho v)\left(r_{t+v} - \rho\right)\ dv,
\end{aligned}$$

which is denoted by $\ln\left(W_t\right)/\,\rho + h_t^r$.

We use the same idea to express the continuation value of a banker who can access shadow banking. Given the banker's optimal portfolio choices $\left(s_{t+u}, s_{t+u}^*\right)$, if she does not retire in period

$t+u$, her wealth is

$$W_t \exp\left(\begin{array}{c} \int_0^u \left(R_{t+v} + s_{t+v}\left(R_{t+v} - r_{t+v}\right) + s^*_{t+v}\left(R_{t+v} - r_{t+v}\right) - \rho\right) dv \\ + \int_0^u \ln\left(1 - \left(1 + s_{t+v} + s^*_{t+v}\right)\kappa^Q_{t+v}\right) dN_{t+v} \end{array}\right)$$

Let $t+T$ denote the stopping time that the banker retires. Her continuation value in period t is

$$\begin{aligned} & E_t\left[\begin{array}{c} \int_0^T \exp\left(-\rho u\right)\left(\ln\left(\rho W_t\right) + \int_0^u R_{t+v} + s_{t+v}\left(R_{t+v} - r_{t+v}\right) + s^*_{t+v}\left(R_{t+v} - r_{t+v}\right) - \rho \; dv\right) du \\ + \int_0^T \exp\left(-\rho u\right)\int_0^u \ln\left(1 - \left(1 + s_{t+v} + s^*_{t+v}\right)\kappa^Q_{t+v}\right) dN_{t+v} \; du + \exp\left(-T\rho\right)\left(\frac{\ln(W_{t+T})}{\rho} + h^r_{t+T}\right). \end{array}\right] \\ = \; & \frac{\ln\left(W_t\right)}{\rho} + \frac{\rho \ln\left(\rho\right)}{\rho + \chi} \\ & + E_t\left[\int_0^\infty \exp\left(-\left(\rho + \chi\right)v\right)\left(\begin{array}{c} R_{t+v} + s_{t+v}\left(R_{t+v} - r_{t+v}\right) + s^*_{t+v}\left(R_{t+v} - r_{t+v}\right) - \rho \\ + \lambda \ln\left(1 - \left(1 + s_{t+v} + s^*_{t+v}\right)\kappa^Q_{t+v}\right) \end{array}\right) dv\right] \\ & + E_t\left[\int_0^\infty \chi \exp\left(-\left(\rho + \chi\right)v\right) h^r_{t+v} \; dv\right], \end{aligned}$$

which we denote as $\ln\left(W_t\right)/\rho + h_t$.

Finally, we consider the case that the banker who cannot use shadow banking but obtain such opportunity at intensity ξ. Let $\hat{s}_{t+u}$ denote her optimal portfolio choices and T_ξ the stopping when the banker obtain the access to shadow banking. Her continuation value in period t is

$$E_t\left[\begin{array}{c} \int_0^{\min\left(T,T_\xi\right)} \exp\left(-\rho u\right)\left(\ln\left(\rho W_t\right) + \int_0^u R_{t+v} + \hat{s}_{t+v}\left(R_{t+v} - r_{t+v} - \tau_{t+v}\right) + s_{t+v}\tau_{t+v} - \rho \; dv\right) du \\ + \int_0^{\min\left(T,T_\xi\right)} \exp\left(-\rho u\right)\int_0^u \ln\left(1 - \left(1 + \hat{s}_{t+v}\right)\kappa^Q_{t+v}\right) dN_{t+v} \; du \\ + \exp\left(-T_\xi \rho\right)\left(\frac{\ln\left(W_{t+T_\xi}\right)}{\rho} + h_t\right)\mathbf{1}_{\left(\min\left(T_\xi,T\right)=T_\xi\right)} + \exp\left(-T\rho\right)\left(\frac{\ln(W_{t+T})}{\rho} + h^r_{t+T}\right)\mathbf{1}_{\left(\min\left(T_\xi,T\right)=T\right)} \end{array}\right]$$

$$\begin{aligned} = \; & E_t\left[\begin{array}{c} \int_0^T \exp\left(-\rho u\right)\left(\ln\left(\rho W_t\right) + \int_0^u R_{t+v} + s_{t+v}\left(R_{t+v} - r_{t+v}\right) + s^*_{t+v}\left(R_{t+v} - r_{t+v}\right) - \rho \; dv\right) du \\ + \int_0^T \exp\left(-\rho u\right)\int_0^u \ln\left(1 - \left(1 + s_{t+v} + s^*_{t+v}\right)\kappa^Q_{t+v}\right) dN_{t+v} \; du \exp\left(-T\rho\right)\left(\frac{\ln(W_{t+T})}{\rho} + h^r_{t+T}\right) \end{array}\right] - \\ & E_t\left[\int_0^{\min\left(T,T_\xi\right)} \exp(-\rho u)\int_0^u \left(\begin{array}{c} \left(\left(s_{t+v} + s^*_{t+v} - \hat{s}_{t+v}\right)\left(R_{t+v} - r_{t+v} - \tau_{t+v}\right) + s^*_{t+v}\tau_{t+v}\right) dv \\ + \left(\ln\left(1 - \left(1 + s_{t+v} + s^*_{t+v}\right)\kappa^Q_{t+v}\right) - \ln\left(1 - \left(1 + \hat{s}_{t+v}\right)\kappa^Q_{t+v}\right)\right) dN_{t+v} \end{array}\right) du\right] \\ = \; & \ln\left(W_t\right)/\rho + h_t - H_t, \end{aligned}$$

where

$$H_t = E_t \left(\int_0^\infty \exp\left(-\left(\rho+\chi+\xi\right)v\right) \left(\begin{array}{c} \left(s_{t+v}+s^*_{t+v}-\hat{s}_{t+v}\right)\left(R_{t+v}-r_{t+v}-\tau_{t+v}\right)+s^*_{t+v}\tau_{t+v} \\ +\lambda\left(\ln\left(1-\left(1+s_{t+v}+s^*_{t+v}\right)\kappa^Q_{t+v}\right)-\ln\left(1-\left(1+\hat{s}_{t+v}\right)\kappa^Q_{t+v}\right)\right) \end{array} \right) dv \right]$$

■

Proof of Lemma 1. W_t denotes $\int_0^1 W_t^i di$. In a Markov equilibrium, bankers' dynamic budget constraint, the optimal choice of bankers, and the market-clearing for notes, and the balanced budget of the regulatory authority imply that

$$\begin{aligned} dW_t &= W_{t-}\left(\left(R_{t-}+s_{t-}\left(R_{t-}-r_{t-}\right)+s^*_{t-}\left(R_{t-}-r_{t-}\right)-\rho-\chi\right)dt-\left(s_{t-}+s^*_{t-}\right)\kappa^Q_t dN_t\right) \\ &= W_{t-}\left(\left(R_{t-}+s_{t-}\left(R_{t-}-r_{t-}\right)+s^*_{t-}\left(R_{t-}-r_{t-}\right)-\rho-\chi\right)dt-\left(s_{t-}+s^*_{t-}\right)\kappa^Q_t dN_t\right). \end{aligned}$$

Note bankers retire at the intensity χ. Next, consider the scaling factor $1/\left(q_t K_t\right)$.

$$d\left(q_t K_t\right) = q_{t-}K_{t-}\left(\left(\mu^q_{t-}+\mu^K_{t-}\right)dt-\kappa^Q_t dN_t\right),$$

and

$$d\left(\frac{1}{q_t K_t}\right) = \left(\frac{1}{q_{t-}K_{t-}}\right)\left(-\left(\mu^q_{t-}+\mu^K_{t-}\right)dt+\frac{\kappa^Q_t}{1-\kappa^Q_t}dN_t\right).$$

Then,

$$\begin{aligned} d\omega_t &= \omega_{t-}\left(\mu^\omega_{t-}dt-\kappa^\omega_t dN_t\right) \\ \text{where } \mu^\omega_t &= R_{t-}+s_{t-}\left(R_{t-}-r_{t-}\right)+s^*_{t-}\left(R_{t-}-r_{t-}\right)-\mu^q_{t-}-\mu^K_{t-}-\rho-\chi \\ \text{and } \kappa^\omega_t &= \frac{\left(s_{t-}+s^*_{t-}\right)\kappa^Q_t}{1-\kappa^Q_t}. \end{aligned}$$

■

Proof of Proposition 3. To justify the algorithm proposed by Proposition 6, we need to show that if the leverage constraint for shadow banking is satisfied bankers' HJB equation can reduce to

$$0 = \mathbf{HJB}_{\mathcal{N}}$$

without considering the incentive-compatible constraint. First, we know that the optimal choice of $\mathbf{HJB}_{\mathcal{D}}$ is dominated by that of $\mathbf{HJB}_{\mathcal{N}}$ by the definition of the maximum leverage of shadow banking. Thus, what we need to show next is that if the portfolio choice (s_{t-}, s^*_{t-}) satisfies the leverage constraint for shadow banking it automatically meets the incentive-compatible constraint. Now, suppose (s_{t-}, s^*_{t-}) is such that

$$s^*_{t-} \leq \frac{\rho\lambda\big(h_{t-}(1-\kappa_t^h) - \hat{h}_{t-}(1-\kappa_t^{\hat{h}})\big)}{R_{t-} - r_{t-} - \tau_{t-}}.$$

Thus,

$$s^*_{t-} \leq \rho\big(h_{t-}(1-\kappa_t^h) - \hat{h}_{t-}(1-\kappa_t^{\hat{h}})\big)\frac{1-(1+s_{t-}+s^*_{t-})\kappa_t^Q}{\kappa_t^Q},$$

which comes from the first-order condition with respect to (s_{t-}, s^*_{t-}). Since $x > \ln(1+x)$ for $x > 0$,

$$\ln\left(1 + \frac{s^*_{t-}\kappa_t^Q}{1-(1+s_{t-}+s^*_{t-})\kappa_t^Q}\right) < \frac{s^*_{t-}\kappa_t^Q}{1-(1+s_{t-}+s^*_{t-})\kappa_t^Q} \leq \rho\big(h_{t-}(1-\kappa_t^h) - \hat{h}_{t-}(1-\kappa_t^{\hat{h}})\big).$$

Hence, we show that the incentive-compatible constraint is satisfied. ■

Proof of Theorem 1. We will apply the contraction mapping theorem to show the uniqueness of the solution $H(\omega) = 0$. First, we define a complete metric space. Since the state variable ω is between 0 and $\bar{\omega}$, we focus on the space $B((0, \bar{\omega}])$ of bounded continuous functions $h : (0, \bar{\omega}] \to R$ under sup norm. Theorem 3.1 in Stokey et al. (1989) implies that $B((0, \bar{\omega}])$ is a complete metric space.

We will use Blackwell's sufficient conditions to show Γ is a contraction mapping. Suppose both h, $\tilde{h} \in B((0, \bar{\omega}])$ and $h(\omega) \geq \tilde{h}(\omega)$, for all $\omega \in (0, \bar{\omega}]$, since

$$\bar{s}^* = \frac{\rho\lambda H}{R - r - \tau}$$

all portfolio choices permitted under $\tilde{h}(\omega)$ are feasible under $h(\omega)$. Hence, $\Gamma h \geq \Gamma\tilde{h}$, for all $\omega \in (0, \bar{\omega}]$. Next, we need to show that there exists a positive constant $\beta < 1$ such that $\Gamma(h + v) \leq$

$\Gamma h + \beta v$, for all $h \in B\left((0,\bar{\omega}]\right)$, $v \geq 0$, $\omega \in (0,\bar{\omega}]$. Consider

$$\Gamma\left(h+v\right)[\omega] = E_0\left[\int_0^\infty \exp\left(-\left(\rho+\xi+\chi\right)u\right) f\left(\omega_u\right)du \middle| \omega_0 = \omega\right],$$

where

$$f\left(\omega\right) = \frac{1}{\rho}\left(\begin{array}{c} \left(s+s^*\right)\left(R\left(\omega\right)-r-\tau(\omega)\right)+s^*\tau\left(\omega\right)-\hat{s}\left(R\left(\omega\right)-r-\tau\right) \\ +\lambda\left(\ln\left(1-\left(1+s+s^*\right)\kappa^Q\left(\omega\right)\right)-\ln\left(1-\left(1+\hat{s}\right)\kappa^Q\left(\omega\right)\right)\right) \end{array}\right)$$

and

$$s^* \leq \frac{\rho\lambda H}{R(\omega)-r-\tau(\omega)},$$

where (s, s^*) is the optimal portfolio choice of a banker who has the access to shadow banking given $\{q(\omega), \tau(\omega), \pi(\omega), \mu^\omega(\omega), \kappa^\omega(\omega)\}$ and $\tilde{s}$ is the portfolio choice of a banker who does not have. Since the lower bound of h is zero, then

$$\begin{aligned}
\bar{s}^* &= \frac{\rho\lambda(h+v)}{R-r-\tau} = \frac{\rho\lambda h}{R-r-\tau} + \frac{\rho\lambda v}{R-r-\tau} \\
&= \frac{\rho\lambda h}{R-r-\tau} + \frac{\rho v(1-(1+\tilde{s})\kappa^Q)}{\kappa^Q} \\
&= \frac{\rho\lambda h}{R-r-\tau} + \rho v\left(\frac{1}{\kappa^Q} - (1+\tilde{s})\right) \\
&\leq \frac{\rho\lambda h}{R-r-\tau} + \frac{\rho v}{\kappa}
\end{aligned}$$

With the assistance of above inequality, we derive that

$$\begin{aligned}
\Gamma\left(h+v\right)[\omega] &\leq \Gamma h + E_0\left[\int_0^\infty \exp\left(-\left(\rho+\xi+\chi\right)u\right)\frac{v}{\kappa}\tau du \middle| \omega_0 = \omega\right] \\
&\leq \Gamma h + \frac{\tau}{\left(\rho+\xi+\chi\right)\kappa}v.
\end{aligned}$$

If $\tau < \left(\rho+\xi+\chi\right)\kappa$, Γ is a contraction mapping. ■

C Calibration, Data, Tables, and Figures

We set the time discount factor ρ to 3% to match the real interest rate estimated by Campbell and Cochrane (1999). Bankers' retirement rate χ is set at 16% to target the average Sharpe ratio. We set bankers' productivity at 22.5% so that the average investment-to-capital ratio is close to 11% (He and Krishnamurthy, 2012a). The productivity of less-productive households is chosen at 10% to match the fact that the Sharpe ratio during the 2007-09 financial crisis was approximately 15 times the average level (He and Krishnamurthy, 2012a). Choices of the depreciation rate and the capital adjustment cost ϕ are standard in the macroeconomic literature (Christiano, Eichenbaum and Evans, 2005). We set the Poisson shock parameters to target the conditional volatility of the growth rate of bankers' wealth in distressed periods and in non-distressed periods. The distressed periods are defined as periods with the lowest 33% Sharpe ratios. The regulation parameter, tax rate τ, is set at 3% to target the average leverage of the entire banking sector. We set the intensity with which bankers can re-access shadow banking after default at 6% to target the ratio of securitization by non-agency issuers in the third quarter of 2006.

Table 1: Moments[1]

Moment	Model	Target	Source
average Sharpe ratio	33.7%	40%	Wachter (2013)
$\frac{\text{highest Sharpe ratio}}{\text{average Sharpe ratio}}$	15.3	15	He and Krishnamurthy (2012a)
average $\frac{\text{investment}}{\text{capital}}$ ratio	11.2%	11%	He and Krishnamurthy (2012a)
average ratio of securitization	25.5%	25.1%	ratio of securitization in third quarter of 2006
bankers' overall leverage	2.9	3	He and Krishnamurthy (2012a)
volatility of bankers' wealth growth rate			
in distress periods[2]	35.1%	31.5%	He and Krishnamurthy (2012a)
in non-distress periods	18.9%	17.5%	He and Krishnamurthy (2012a)

[1] We use the density of the stationary distribution to calculate all moments.
[2] The distress periods are those with highest 33% Sharpe ratio.

Securitization. We follow Loutskina (2011) to compute the ratio of securitization. The difference is that we focus on securitization done by non-agency security issuers. All data are drawn from the "Flow of Funds Accounts of the United States". There are 5 loan categories. The details of items for each category are listed in Table 2.

Table 2: Details of Securitization Data

	Outstanding	Securitized
Home Mortgages	FL383165105	FL673065105
Multifamily Residential Mortgages	FL143165405	FL673065405
Commercial Mortgages	FL383165505	FL673065505
Commercial and Industrial Loans[1]	FL253169255 FL263168005 FL263169255	FL673069505
Consumer Credit	FL153166000	FL673066000

[1] Becuase item FL253169255 is not available now, we use the ratio of securitization calculated by Loutskina (2011) to estimate the outstanding commercial and industrial loans.

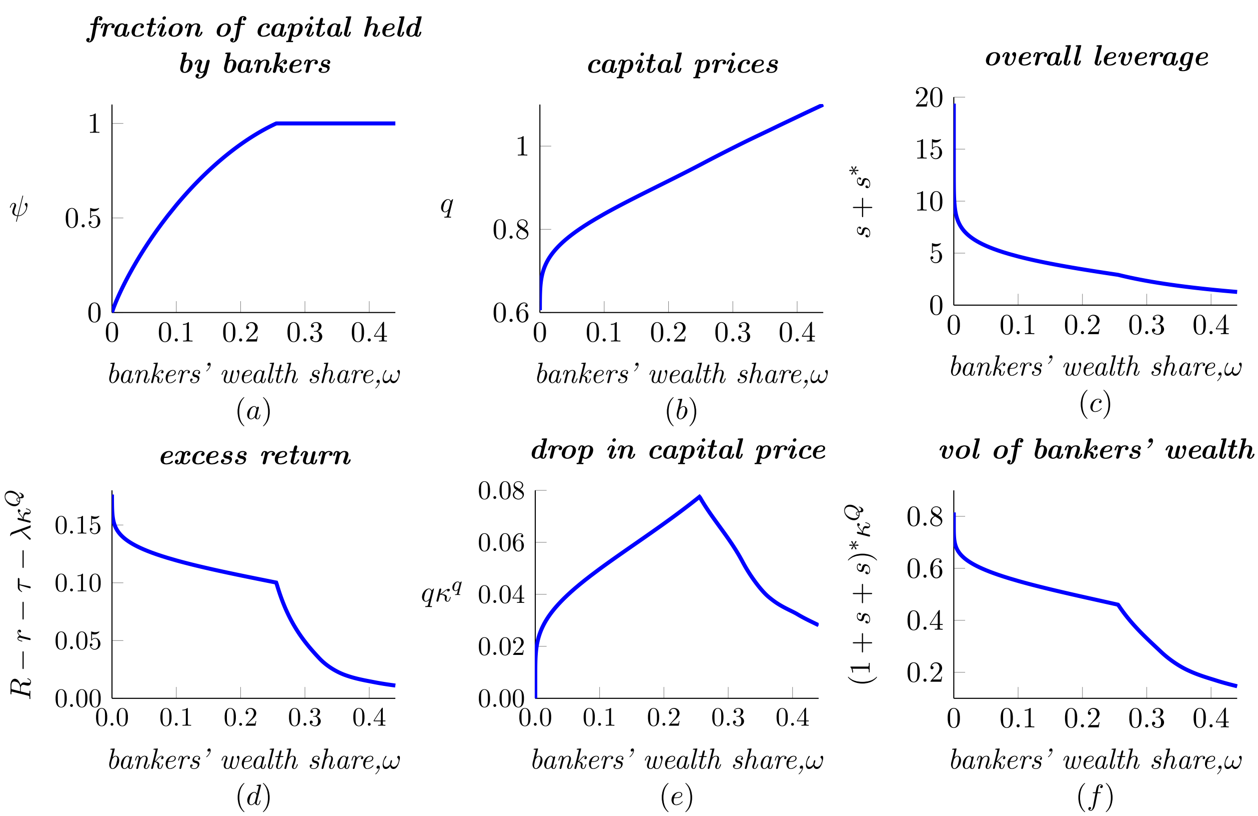

Figure 14: $\psi, q, (s+s^*), R-r-\tau-\lambda\kappa^Q, q\kappa^q$, and $\left(1+s+s^*\right)\kappa^Q$ as functions of the state variable ω in the modified model with a constant cost of default $\bar{H}=2.3973$, which equals the average cost of default in the calibrated model in Section 2.3. For other parameter values see the same section.

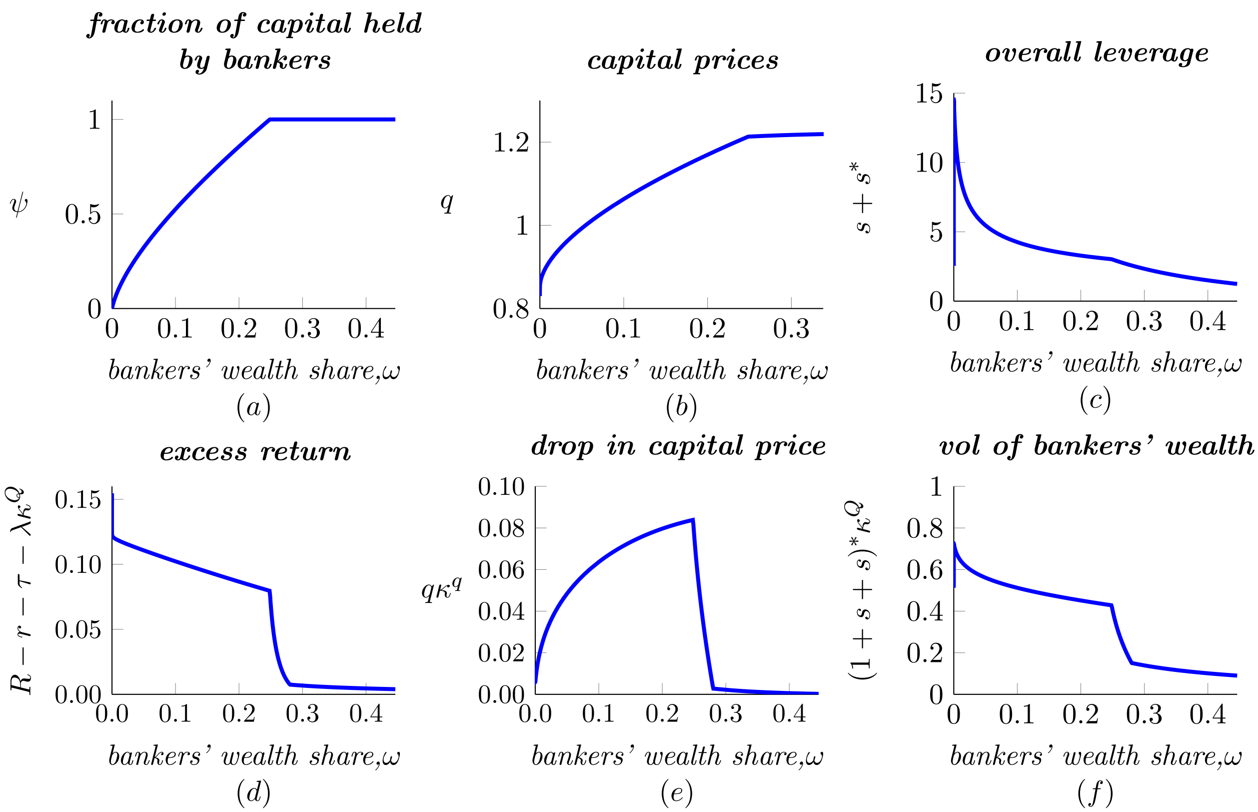

Figure 15: $\psi, q, (s+s^*), R-r-\tau-\lambda\kappa^Q, q\kappa^q$, and $\left(1+s+s^*\right)\kappa^Q$ as functions of the state variable ω, i.e., bankers' wealth share, in the "good" equilibrium of the modified model in which households have Epstein-Zin preferences. For the choice of parameter values, see Section 3.

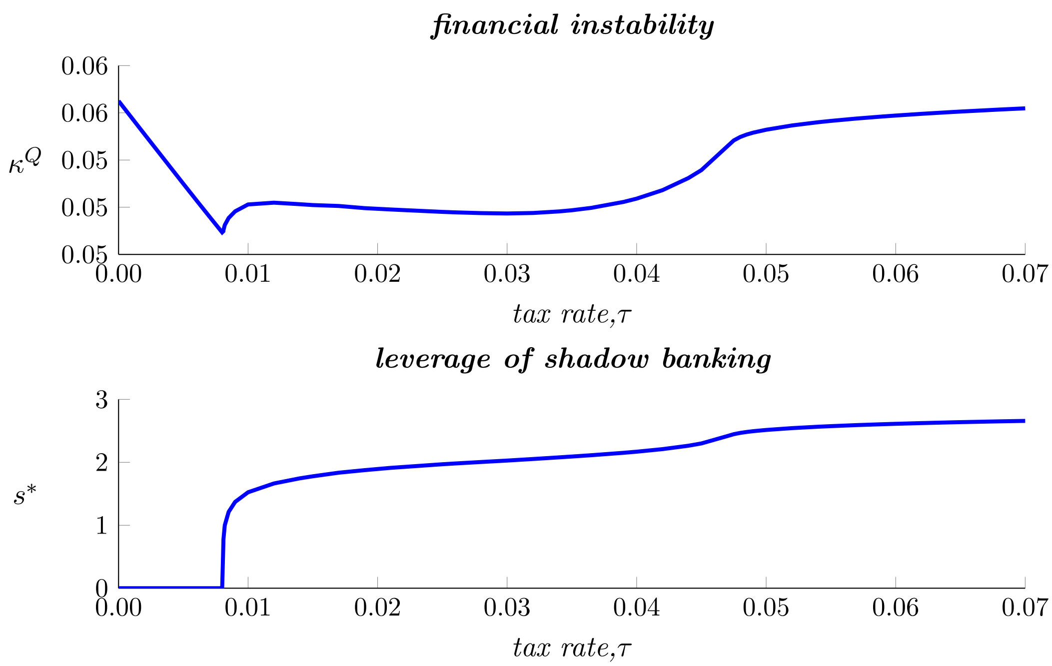

Figure 16: This figure shows the average investment risk κ^Q (upper panel) and the average leverage of shadow banking (lower panel) in different economies with different tax rates τ in the modified model with households of Epstein-Zin preference. We use the stationary distribution to calculate moments. For parameter values other than τ, see Section 3.

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