Bond Finance, Bank Finance, and Bank Regulation

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Abstract

A dynamic general equilibrium model of bank regulation that omits bond financing is imprecise because such a model prevents firms from raising credit via alternative channels, and thus artificially lowers the price elasticity of demand for bank loans. In this paper, I build a continuous-time macro-finance model in which firms can use both bond credit and bank credit. Risky firms appreciate bank credit because banks are efficient at liquidating assets for troubled firms. However, risky firms must pay a risk premium for banks' exposure to aggregate risks. This paper shows that a model that does not allow for bond financing overestimates both the welfare benefits of tightening bank capital requirements and the rate at which the banking sector recovers after a recession. More importantly, the calibrated model highlights that the optimal level of capital requirement is very sensitive to the presence of bond financing. In addition, I show that the optimal bank regulation highly depends on the efficiency of the bankruptcy procedure in an economy and the risk profile of its real sector.

Keywords: bank credit, bond credit, capital requirement, and macro-prudential regulation

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Introduction

Like bank loans, bond finance is an important source of external credit for firms. For instance, during the 2007-2009 financial crisis when the supply of bank loans declined substantially, firms, especially those with relatively high credit ratings, largely substituted bank credit with bond credit (Adrian et al., 2012). Nevertheless, the implication of direct bond finance for optimal bank regulation in dynamic general equilibrium frameworks has rarely been acknowledged in the literature, even though many papers have assessed the welfare-maximizing role of bank regulation in such frameworks (Van den Heuvel, 2008; Repullo and Suarez, 2012; Christiano and Ikeda, 2013; Martinez-Miera and Suarez, 2014; Nguyen, 2014; Derviz et al., 2015; Begenau, 2018; Elenev et al., 2018; Phelan, 2016; Davydiuk, 2017; Corbae and D'Erasmo, 2018; Mendicino et al., 2018; Pancost and Robatto, 2018).[1] In this paper, I will show that a general equilibrium bank regulation model that omits the bond market generate imprecise results. Moreover, I highlight that the socially optimal level of the capital ratio requirement for banks largely depends on the efficiency of the bankruptcy system and the risk profile of the real sector in an economy because both factors affect the aggregate demand for bank credit.

I propose a continuous-time macro-finance framework with a productive expert sector, a less productive household sector, and an explicit banking sector. The production sector comprises safe firms and risky firms. Both types of firms can access the bond market and the loan market. The difference between bond finance and bank finance is that banks can liquidate troubled firms' assets in a more efficient fashion (Bolton and Freixas, 2000). The net interest spread charged by banks compensate for their exposure to the aggregate risk that they assume via loan lending. Households can both hold corporate bonds directly and deposit their savings into banks.

In my framework, risky firms prefer bank credit while safe firms rely mainly on bond credit. Since banks can liquidate troubled firms' assets in a more efficient way, banks request less compensation for bankruptcy costs relative to bondholders. The liquidation efficiency of bank credit is more important for risky firms than for safe firms because safe firms are less likely to face costly liquidation. This setting is consistent with empirical findings in Rauh and Sufi (2010) and Becker and Josephson (2016). Bank credit does not always dominate bond credit for risky firms. Since risky firms must pay bank a risk premium for the aggregate risk that banks are exposed to, risky firms will replace bank finance with bond finance when the risk premium increases. The risk premium in the model is the net interest spread earned by banks.

The net interest spread depends on the leverage of the intermediary sector, the aggregate risk of the economy, and the capital requirement faced by banks. Given the same amount of aggregate risk, banks with low leverage have low risk exposure. Therefore, the risk premium required by banks tends to be low. Hence, bank credit is relatively cheap when the banking sector has adequate equity capital. The capital ratio requirement also affects the net interest spread because a tightening of the capital requirement would lower the supply of bank loans. When there is excess demand for bank loans, the loan spread increases, as does the net interest spread earned by banks.

The impacts of exogenous aggregate shocks on the economy vary over time because the effects of financial amplification depend on the balance sheets of both banks and experts (Bernanke, Gertler and Gilchrist, 1999; Kiyotaki and Moore, 1997). Suppose a series of adverse shocks hit the economy. Both bank capital's and productive experts' net worth decline disproportionately due to their use of leverage. As a result, the supply of bank loans shrinks, leading to a decrease in experts' holdings of assets, aggregate productivity, and asset

[1] See Thakor (2014) for a review of the literature on the capital ratio requirement using microeconomic models of banking.

prices. The depreciation of asset prices hurts balance sheets of both banks and experts, and further lowers the loan supply and experts' holdings of assets. I label the effect of the financial amplification as endogenous risk.

The first key result of this paper concerns economic dynamics. In a model where the real sector does not issue bonds, the predicted recovery of the banking sector after a negative shock is overly swift. Suppose the net worths of both the real sector and the banking sector deteriorate due to a negative shock. The real sector's demand for bank loans increases as it has to rely more on external funds. Given the decline in the supply of bank loans, the loan spread must increase. If loans are the only source of external finance that can be accessed by the real sector, then the demand for bank loans is not very elastic. Hence, if the real sector cannot access bond financing, bank profitability can increase substantially due to a significant increase in the loan spread. As a consequence, the banking sector recovers more quickly after adverse shocks in a model that omits bond financing than it would in a model with bond financing.

Bank regulation in my framework can improve social welfare because my model is subject to pecuniary externalities that are common in incomplete market models (Hart, 1975; Stiglitz, 1982; Geanakoplos and Polemarchakis, 1986). In particular, experts and bankers in my model do not internalize the impact of their leverage decisions on asset prices and endogenous risks. Hence, bank regulation such as the capital ratio requirement can adjust bankers' leverage, lower the loan supply, and raise the net interest spread. In this way, bank regulation can increase the profitability of banking and strengthen the banking sector to lower endogenous risks and improve social welfare.

The second key result of this paper is that a model that omits bond financing overemphasizes the benefit of bank capital requirements. The intuition is also related to the elasticity of the aggregate demand for bank loans. If capital requirement rises, there will be excess demand for bank loans. Thus, loan spread increases and loan demand declines. If the magnitude of the decline in loan demand is small enough, bank profitability could increase, and the banking sector can expand after accumulating more and more profit. A larger banking sector can provide more credit for the real sector and indirectly raise aggregate productivity. These are the ways in which tightening capital requirement improves social welfare. Consider two otherwise identical economies: one has a bond market and the other does not. Obviously, the aggregate demand for bank loans is much more elastic in the economy where firms can raise credit from the bond market. In this economy, when loan spread increases, the demand for bank loans declines more substantially, as does bank profitability. Therefore, tightening capital requirement is more likely to cause the banking sector to shrink, and social welfare to decline. Hence, the optimal capital ratio requirement should be more lenient if we consider a model that allows for bond financing.

I calibrated the model. Its quantitative implication is that the current capital ratio requirement is too stringent. This is a natural result since my model highlights an additional negative effect of raising capital adequacy ratio. If I close the bond market, the otherwise identical model suggests that it is optimal to raise the current capital requirement. Therefore, my paper shows that the optimal level of capital requirement is very sensitive to the presence of bond financing.

The previous discussion shows that the loan spread elasticity of the demand for bank loans plays a crucial role in the welfare implication of capital requirement. In light of this property, I explore two factors that affect the elasticity of bank loan demand: the efficiency of the bankruptcy system in an economy and firms' average idiosyncratic default risks. The more efficiently bankruptcy cases are processed, the smaller the advantage of banks over bondholders in terms of liquidating insolvent firms' assets. In an efficient bankruptcy system, bondholders enjoy higher recovery value ex post and request smaller premium ex ante.

From the perspective of firms, replacing bank credit with bond credit is less costly, and thus firms' demand for bank loans is more price elastic. Hence, tightening capital requirement can cause a substantial decline in bank loans, and a decrease in bank profits. Overall, the optimal capital requirement should be more lenient in an economy with a more efficient bankruptcy system.

Firms' average default risk also influences the elasticity of demand for bank loans. Since bondholders demand higher default premium for firms that are more likely to fail, riskier firms find it costly to switch from bank credit to bond credit. Hence, the demand for loans is less elastic if firms in an economy tend to be risky. Subsequently, the optimal capital requirement should be tighter.

Related Literature. My paper is related to four strands of literature. First, I use a continuous-time macro-finance framework that emphasizes the financial amplification mechanism (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2012; Di Tella, 2017). The major contribution of this paper is that I explicitly model a financial intermediary sector rather than grouping the real sector and financial intermediary sector together. With my proposed framework, I can explicitly analyze the macroeconomic implications of bank regulation. This framework highlights two layers of financial amplification — one at the firm level and the other at the intermediary level.

Second, since the 2007–2009 financial crisis, a number of papers have investigated the macro-prudential role of banking regulation in a dynamic general equilibrium framework (see, e.g., Begenau 2018; Elenev et al. 2018). Most of these papers are quantitative, and typically incorporate many ingredients, ranging from the liquidity premium of bank debt to the risk-shifting problem caused by either deposit insurance or implicit government guarantees. The framework proposed in this paper is rather simple as it is meant to highlight a feature that is currently missing in the literature, that is, the effectiveness of banking regulation highly depends on the elasticity of demand for bank loans, which in turn relies on the presence of the bond market.[2] In my model, banking regulation mitigates pecuniary externalities and improves social welfare via the distributive effects emphasized by Dávila and Korinek (2017).

Thirdly, my paper contributes to a strand of macroeconomic literature that highlights the capital structure of firms (see, e.g., De Fiore and Uhlig 2011, 2015; Crouzet 2017). These papers model the surge in the cost of bank financing as an exogenous shock. Therefore, these papers are missing the rich characterizations of the dynamics of bank financing and bond financing that are captured in my paper. In this regard, my paper is similar to Rampini and Viswanathan (forthcoming), who also endogenize the cost of financial intermediation. However, they do not address the substitution between bank credit and bond credit. My paper shows that the dynamics of both the real sector and the intermediary sector would be significantly different if bond financing is absent in an economy.

Finally, there is a large corporate finance and banking literature that investigates firms' choices of bond finance and bank finance (Chemmanur and Fulghieri, 1994; Bolton and Freixas, 2000). My paper highlights the dynamic properties of firms' capital structure, and explores the general equilibrium effects of firms' financing choices. In addition, my paper stresses that the cost of bank financing fluctuates over business cycles, and this fluctuation has important effects on financial stability and economic growth.

The structure of the rest of the paper is as follows. Section 1 describes the set-up of the model and

[2] Two recent papers, Xiang (2018) and Dempsey (2018), acknowledge the role of bond finance for bank capital requirements. Although both papers are intended to be quantitative, firms are modeled as short-lived. Hence, neither paper captures the dynamic interaction between the real sector and the banking sector, which turns out to have profound effects on the general equilibrium implication of bank capital requirements, as shown by my paper.

defines the equilibrium. In Section 2, I characterize the optimal choice of individual agents and the Markov equilibrium. I highlight that the presence of bond financing has distinctive impacts on an economy's dynamics. Section 3 shows that the optimal level of capital requirement depends heavily on the existence of a bond market, its development, and the distribution of borrowers' risk characteristics. Section 4 concludes.

1 Model

In this section, I build an infinite-horizon continuous-time general equilibrium model, in which firms can issue corporate bonds as well as raise credit via financial intermediaries. The economy has two types of goods: perishable final goods (the numéraire) and durable physical capital goods. Three types of agents populate the economy: experts, bankers, and households. All agents have the same logarithmic preferences and the same time discount factor ρ. None of them accepts negative consumption. Although all three types of agents are able to hold physical capital goods and produce final goods, experts are the most productive while bankers specialize in financial intermediation. To prevent either experts or bankers take over all the wealth in the economy, I assume that experts (bankers) become households at rate χ (χ_η).

For the purpose of exposition, I present the discrete-time version of the model with the length of each period being a positive constant Δ.[3] The continuous-time model that I actually solve is the limit of the discrete-time version when Δ becomes arbitrarily small.

1.1 Technology

In period t, an expert can produce $ak_t\Delta$ units of final goods with k_t efficiency units of physical capital. Both households and bankers are unproductive. All three types of agents can convert $\iota_t k_t \Delta$ units of final goods into $k_t\Phi(\iota_t)\Delta$ units of physical capital, where

$$\Phi(\iota_t) = \frac{\log(\iota_t\phi + 1)}{\phi}.$$

Thus, there is technological illiquidity on the production side. In each period, physical capital in the possession of experts and households depreciates by $\delta\Delta$ percent and physical capital in the possession of bankers depreciates by $\delta^b\Delta$ percent.

Exogenous *aggregate* shocks are driven by an i.i.d. process $\{z_t, t = 1, 2, ...\}$, and z_t is normally distributed with mean 0 and variance Δ.[4] In the absence of any idiosyncratic shock, physical capital managed by an expert evolves according to

$$k_{t+1} = k_t + (\Phi(\iota_t) - \delta)k_t\Delta + \sigma k_t z_t, \tag{1}$$

where σ is a positive constant that captures the direct impact of the exogenous shock on physical capital. Similarly, physical capital managed by households follows

$$k_{t+1} = k_t + (\Phi(\iota_t) - \delta)k_t\Delta + \sigma k_t z_t,$$

[3] In a typical discrete-time macroeconomics model, the length of a period is one.

[4] As Δ converges to 0, the limit of $\sum_{u=1}^{t/\Delta} z_u$ is a Brownian motion.

and physical capital managed by bankers follows

$$k_{t+1} = k_t + (\Phi(\iota_t) - \delta^b)k_t\Delta + \sigma k_t z_t.$$

At the beginning of each period, an expert becomes a *safe* expert with probability α or a *risky* expert with probability $1 - \alpha$. Whether an expert becomes risky within a period is independent across the time. Within a period, an exogenous default event may occur to a risky firm (a firm managed by a risky expert) with probability λ after the firm has made its investment, production, and financing decisions. Since the default risk is independent across different firms, a risky expert establishes an infinite number of firms to diversify this idiosyncratic risk. Safe firms do not experience such adverse idiosyncratic shocks.

1.2 Corporate Bond, Bank Loan, and Liquidation

A firm can raise credit either from issuing corporate bonds or from obtaining a bank loan. In addition, assume that no firm can issue outside equity, and all firms have limited liability.

Both corporate bonds and bank loans are collateralized short-term contingent debt. Collateralized borrowing implies that if a firm raises L dollars from creditors, it must put down physical capital worth L dollars as collateral. If a risky firm defaults on the loan, the firm's creditors will seize the collateral and liquidate physical capital.[5] No liquidation is involved if a firm is self-financed.

Bondholders are assumed to be less efficient than banks in terms of liquidating physical capital. This is because it is harder and more time-consuming to achieve a collective decision for a number of bondholders during the liquidation process than it is for a single bank. In particular, assume that the depreciation rate of physical capital is $\kappa + \delta$ if banks liquidate the collateral, while the depreciation rate rises to $\kappa^d + \delta$ if bondholders seize the collateral, where $\kappa < \kappa^d$.

For simplicity, assume that there is a passive mutual fund that serves the intermediary in the corporate bond market. The fund charges its borrowers the risk-free rate plus the expected loss due to costly liquidation, and promises the risk-free rate r_t to its investors. Any loss or profit realized by the mutual fund is driven by the aggregate shock z_t. Assume that the loss or profit realized in each period is instantly shared by all agents via lump-sum transfers. Thus, the unit borrowing cost of bond-financing is $r_t + \lambda\kappa^d$ for a risky firm.

Similar to the mutual fund, banks raise funds from households, and promise the risk-free rate r_t. Unlike the passive mutual fund, banks require a risk premium because their equity capital is exposed to the aggregate risk. Overall, risky firms' unit borrowing cost of bank financing is $r_t^\lambda + \lambda\kappa$, and the net interest spread is $r_t^\lambda - r_t$.

1.3 The Expert's Problem

I conjecture that the law of motion for the equilibrium price of physical capital can be approximated by

$$q_{t+1} = q_t + \mu_t^q q_t\Delta + \sigma_t^q q_t z_t, \tag{2}$$

[5] The micro-foundation for creditors' optimal decision is as follows. We can think of the default event as a publicly-known adverse signal, which increases the information asymmetry of the quality of collateral. As a result, it becomes easier for the firm's owner to steal the collateral, leaving nothing to creditors. Therefore, given the negative signal, the optimal decision for creditors is to seize the collateral.

where both μ_t^q and σ_t^q are equilibrium objects that I will solve for. A nice property of the continuous-time approach is that I can decompose the dynamics of the stochastic process $(q_{t+1}-q_t)/q_t$ into the linear combination of a deterministic part $\mu_t^q\Delta$ and a stochastic part $\sigma_t^q z_t$. As in the macro-finance literature, I label σ_t^q the endogenous risk. An expert's rate of net return from holding physical capital is

$$\frac{q_{t+1}k_{t+1} + ak_t\Delta - \iota_t k_t\Delta - q_t k_t}{q_t k_t} = R_t\Delta + (\sigma + \sigma_t^q)z_t + o(\Delta), \text{where}$$
$$R_t \equiv \frac{a-\iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q,$$

and $o(\Delta)$ denotes terms whose order is higher than one. Hereafter, I will drop the term $o(\Delta)$ when it is involved because it will vanish in the limit as Δ converges to zero. The derivation above uses the fact that $E[z_t^2] = \Delta$.[6] Since costly liquidation does not happen to a safe expert, he or she raises external funds only through bond financing, and thus his/her dynamic budget constraint is

$$w_{t+1} = w_t + w_t\big(R_t\Delta + (\sigma+\sigma_t^q)z_t\big) + w_t b_t^0\big((R_t - r_t)\Delta + (\sigma+\sigma_t^q)z_t\big) + w_t m_t(\sigma+\sigma_t^q)z_t - c_t\Delta, \quad (3)$$

where b_t^0 is the bond-to-equity ratio and $m_t(\sigma+\sigma_t^q)z_t$ denotes the lump-sum transfer from the bond mutual fund per unit of net worth.

A risky expert will choose among corporate debt, bank loans, and self-financing. Since all of the expert's firms are identical prior to the realization of the liquidity shock, the financing decisions of all firms managed by the expert are the same. Thus, the debt-to-equity ratios of these firms are also the same, which is exactly the expert's debt-to-net-worth ratio. The law of motion for the risky expert's net worth is

$$\begin{aligned} w_{t+1} = w_t &+ w_t\big(R_t\Delta + (\sigma+\sigma_t^q)z_t\big) + w_t b_t^\lambda\Big(\big(R_t - \lambda\kappa^d - r_t\big)\Delta + \big(1-\lambda\big)(\sigma+\sigma_t^q)z_t\Big) \\ &+ w_t l_t\Big(\big(R_t - \lambda\kappa - r_t^\lambda\big)\Delta + \big(1-\lambda\big)(\sigma+\sigma_t^q)z_t\Big) + w_t m_t(\sigma+\sigma_t^q)z_t - c_t\Delta, \end{aligned} \quad (4)$$

where b_t^λ is the firms' bond-to-equity ratio, and l_t is the firm's loan-to-equity ratio. By the Law of Large Numbers, creditors seizes a proportion λ of the expert's physical capital due to default. As a result, the risky expert partially unloads his/her exposure to the aggregate risk, $\lambda(\sigma+\sigma_t^q)z_t$.

Taking $\{q_t, r_t, r_t^\lambda, m_t, t \geq 0\}$ as given, an expert chooses $\{c_t, b_t^0, b_t^\lambda, l_t, t \geq 0\}$ to maximize his/her life-time expected utility

$$E_0\left[\sum_{t=0}^{T} e^{-\rho\Delta t}\ln(c_t)\,\Delta + e^{-\rho T}J^h(W_T)\right],$$

given that his/her net worth evolves in each period according to either equation (3) or (4) depending on his/her type, where T is the time when the expert turns into a household and $J^h(W_T)$ denotes the life-time expected utility of the household.

1.4 The Banker's Problem

The instant rate of return from holding physical capital for a banker is

$$R_t^b\Delta + (\sigma+\sigma_t^q)z_t, \text{ where } R_t^b \equiv -\frac{\iota_t}{q_t} + \Phi(\iota_t) - \delta^b + \mu_t^q + \sigma\sigma_t^q.$$

[6] I use Ito's Lemma in the continuous-time setting.

Therefore, a banker's net worth n_t evolves according to

$$n_{t+1} = n_t + n_t x_t^j \left(R_t^b \Delta + (\sigma + \sigma_t^q) z_t\right) + n_t x_t \left(r_t^\lambda \Delta + \lambda(\sigma + \sigma_t^q) z_t\right) + n_t (1 - x_t^j - x_t) r_t \Delta + n_t m_t (\sigma + \sigma_t^q) z_t - c_t \Delta, \quad (5)$$

where x_t^j denotes the physical-capital-to-equity ratio and x_t the loan-to-equity ratio for the bank. When $x_t > 1$, the bank absorbs deposits, and transfers funds from households to experts. When $x_t \leq 1$, the bank puts some of its equity capital in the mutual fund. The banker is exposed to the aggregate risk $n_t x_t^\lambda \lambda(\sigma + \sigma_t^q) z_t$ because he or she takes over and resell the physical capital that backs her lending. I consider the time-invariant capital ratio requirement, which imposes an upper bound on banks' loan-to-equity ratio, that is, $x_t \leq \bar{x}$.[7] Taking $\{q_t, r_t, r_t^\lambda, m_t, t \geq 0\}$ as given, a banker chooses $\{c_t, x_t^j, x_t^\lambda, t \geq 0\}$ to maximize her life-time expected utility

$$E_0 \left[\sum_{t=0}^{\infty} e^{-\rho \Delta t} \ln(c_t) \, \Delta + e^{-T\rho} J^h(W_T) \right],$$

subject to the dynamic budget constraint (5) and the capital ratio requirement, where T is the time when the expert turns into a household and $J^h(W_T)$ denotes the life-time expected utility of the household.

1.5 The Household's Problem

The rate of return from holding physical capital for a household is

$$R_t^h \Delta + (\sigma + \sigma_t^q) z_t, \text{ where } R_t^h \equiv -\frac{\iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma \sigma_t^q.$$

The law of motion for a household's net worth w_t^h is

$$w_{t+1}^h = w_t^h + w_t^h x_t^h (R_t^h \Delta + (\sigma + \sigma_t^q) z_t) + w_t^h (1 - x_t^h) r_t \Delta + w_t^h m_t (\sigma + \sigma_t^q) z_t - c_t, \quad (6)$$

where x_t^h is the portfolio weight of physical capital. Taking $\{q_t, r_t, m_t, t \geq 0\}$ as given, a household maximizes life-time expected utility

$$J^h(w_0^h) \equiv E_0 \left[\sum_{t=0}^{\infty} e^{-\rho \Delta t} \ln(c_t) \, \Delta \right],$$

by choosing $\{c_t, x_t^h, t \geq 0\}$ that satisfy the dynamic budget constraint (6).

1.6 Equilibrium

The aggregate shock $\{z_t\}_{t+0}^{\infty}$ drives the evolution of the economy. $\mathbf{I} = [0, 1)$ denotes the set of experts, $\mathbf{J} = [1, 2)$ the set of bankers, and $\mathbf{H} = [2, 3]$ the set of households. Given the idiosyncratic shock in period t, $\mathbf{I}_t^s$ is the set of safe experts in period t and $\mathbf{I}_t^r$ the set of risky experts.

Definition 1 *Given the initial endowments of physical capital* $\left\{k_0^i, k_0^j, k_0^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\right\}$ *possessed by*

[7] Bankers are much less productive than experts. Hence, bankers hold physical capital only when their wealth share is close to one, and they take on no leverage. Therefore, it is with no loss of generality to assume that the capital ratio requirement only imposes restriction on banks' loan portfolio.

experts, bankers, and households such that

$$\int_0^1 k_0^i \mathrm{d}i + \int_1^2 k_0^j \mathrm{d}j + \int_2^3 k_0^h \mathrm{d}h = K_0,$$

an equilibrium is defined by a set of stochastic processes adapted to the filtration generated by $\{z_t\}_{t=0}^{\infty}$*: the price of physical capital* $\{q_t\}_{t=0}^{\infty}$*, the risk-free rate* $\{r_t\}_{t=0}^{\infty}$*, the interest rate of bank loans* $\{r_t^{\lambda}\}_{t=0}^{\infty}$*, wealth* $\left\{W_t^i, N_t^j, W_t^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\right\}_{t=0}^{\infty}$*, investment decisions* $\left\{\iota_t^i, \iota_t^j, \iota_t^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\right\}_{t=0}^{\infty}$*, asset holding decisions* $\left\{x_t^j, x_t^h, j \in \mathbf{J}, h \in \mathbf{I}_t^h\right\}_{t=0}^{\infty}$ *of bankers and households, corporate debt financing decisions* $\left\{b_t^{i,0}, b_t^{i,\lambda}, i \in \mathbf{I}_t\right\}_{t=0}^{\infty}$ *of experts, bank financing decisions* $\{l_t^i, i \in \mathbf{I}_t^r\}_{t=0}^{\infty}$ *of risky experts, bank lending,* $\{x_t^{\lambda,j}, j \in \mathbf{J}\}_{t=0}^{\infty}$ *and consumption* $\left\{c_t^i, c_t^j, c_t^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\right\}_{t=0}^{\infty}$*; such that*

1. $W_0^i = k_0^i q_0$*,* $N_0^j = k_0^j q_0$*, and* $W_0^h = k_0^h q_0$ *for* $i \in \mathbf{I}$*,* $j \in \mathbf{J}$*, and* $h \in \mathbf{H}$*;*
2. *Each expert, banker, and household solves for his/her problem given prices;*
3. *Markets for final goods and physical capital clear, that is,*

$$\begin{aligned}\int_0^3 c_t^i \mathrm{d}i &= \frac{1}{q_t}\int_1^2 (a^b - \iota_t^j) n_t^j x_t^j \mathrm{d}j + \frac{1}{q_t}\int_2^3 (a^h - \iota_t^h) w_t^h x_t^h \mathrm{d}h + \\ &\quad \frac{1}{q_t}\int_{i\in \mathbf{I}_t^s} \left(a - \iota_t^i\right) w_t^i (1 + b_t^{i,0}) \mathrm{d}i + \frac{1}{q_t}\int_{i\in \mathbf{I}_t^r} \left(a - \iota_t^i\right) w_t^i (1 + b_t^{i,\lambda} + l_t^i) \mathrm{d}i\end{aligned}$$

for the market of final goods, and

$$\frac{1}{q_t}\int_{i\in \mathbf{I}_t^s} w_t^i (1 + b_t^{i,0}) \mathrm{d}i + \frac{1}{q_t}\int_{i\in \mathbf{I}_t^r} w_t^i (1 + b_t^{i,\lambda} + l_t^i) \mathrm{d}i + \frac{1}{q_t}\int_1^2 n_t^j x_t^j \mathrm{d}j + \frac{1}{q_t}\int_2^3 w_t^h x_t^h \mathrm{d}h = K_t$$

for the market of physical capital goods, where K_t *evolves according to*

$$\begin{aligned}\frac{K_{t+1} - K_t}{\Delta} &= \frac{1}{q_t}\int_1^2 \left(\Phi(\iota_t^j) - \delta^b\right) n_t^j x_t^j \mathrm{d}j + \frac{1}{q_t}\int_2^3 \left(\Phi(\iota_t^h) - \delta\right) w_t^h x_t^h \mathrm{d}h \\ &+ \frac{1}{q_t}\int_{i\in \mathbf{I}_t^s} \left(\Phi(\iota_t^i) - \delta\right) w_t^i (1 + b_t^{i,0})\, \mathrm{d}i \\ &+ \frac{1}{q_t}\int_{i\in \mathbf{I}_t^r} \left(\Phi(\iota_t^i) - \delta\right) w_t^i (1 + b_t^{i,\lambda} + l_t^i) - \lambda\kappa^d w_t^i b_t^i - \lambda\kappa w_t^i l_t^i \,\mathrm{d}i.\end{aligned}$$

4. *The bank loan market clears:*

$$\int_{i\in \mathbf{I}_t^r} w_t^i l_t^i \mathrm{d}i = \int_1^2 n_t^j x_t^{\lambda,j} \mathrm{d}j.$$

5. *The bond mutual fund assumes no gains or losses, i.e., the lump-sum transfer between the mutual fund and all agents perfectly hedges the fund's risk exposure to the aggregate risk*

$$\int_0^1 w_t^i m_t \mathrm{d}i + \int_1^2 n_t^j m_t \mathrm{d}j + \int_2^3 w_t^h m_t \mathrm{d}h = \int_{i\in \mathbf{I}_t^r} \lambda b^{\lambda} w^i \mathrm{d}i.$$

The credit market for corporate bonds clears automatically by Walras' Law.

2 Solving for the Equilibrium

Both experts' net worth and bank capital are crucial for the allocation of physical capital and financial resources in the equilibrium. We expect the price of physical capital to decline as experts' net worth and bank capital shrink due to adverse exogenous shocks.

To solve for the equilibrium, I first derive first-order conditions with respect to the optimal decisions of experts, bankers, and households. Next, I solve for the law of motion for endogenous state variables, wealth shares of different types of agents based on market clearing conditions and first-order conditions. Lastly, I use first-order conditions and state variables' law of motion to define partial differential equations that are satisfied by endogenous variables such as the price of physical capital. At the end of this section, I will characterize the dynamics of the economy and show that economic dynamics would be significantly different if the bond market is shut down.

2.1 Households' Optimal Choices

Households have logarithmic preferences. In the following discussion, I will take advantage of two well-known properties with respect to logarithmic preferences in the continuous-time setting: (i) a household's consumption c_t is ρ proportion of her wealth w_t^h in the same period, i.e.,

$$c_t = \rho w_t^h; \tag{7}$$

(ii) a household's portfolio weight on a risky investment is such that the Sharpe ratio of the risky investment equals the percentage volatility of the household's wealth.

A household's investment rate ι_t always maximizes $\Phi(\iota_t) - \iota_t/q_t$. The first-order condition implies that

$$\Phi'(\iota_t) = \frac{1}{q_t}, \tag{8}$$

which defines the optimal investment as a function of the price of physical capital $\iota\,(q_i)$.

Given the second property, it is straightforward to derive a household's optimal portfolio weight on physical capital x_t^h, which satisfies [8]

$$x_t^h + m_t \geq \frac{R_t^h - r_t}{(\sigma + \sigma_t^q)^2} \text{ with equality if } x_t^h > 0. \tag{9}$$

2.2 Experts' Portfolio Choices

According to the second property highlighted above, it is straightforward to characterize a safe expert's optimal bond-to-equity ratio[9]

$$1 + b_t^0 + m_t \geq \frac{R_t - r_t}{(\sigma + \sigma_t^q)^2} \text{ with equality if } b_t^0 > 0. \tag{10}$$

For a risky expert, both bond-to-equity ratio b_t^λ and loan-to-equity ratio l_t affect the percentage volatility of her wealth $(1 + (1-\lambda)b_t^\lambda + (1-\lambda)l_t + m_t)(\sigma + \sigma_t^q)$. Hence, optimal b_t^λ and l_t must satisfy

[8] Sharpe ratio is $(R_t^h - r_t)/(\sigma+\sigma_t^q)$. The percentage volatility of the household's wealth is $(x_t^h + m_t)(\sigma + \sigma_t^q)$.

[9] In this case, the Sharpe ratio is $(R_t - r_t)/(\sigma+\sigma_t^q)$. The percentage volatility of the safe expert's wealth is $(1 + b_t^0)(\sigma + \sigma_t^q)$.

$$1 + (1-\lambda)b_t^{\lambda} + (1-\lambda)l_t + m_t \geq \frac{R - \lambda\kappa^d - r_t}{(1-\lambda)(\sigma + \sigma_t^q)^2} \text{ with equality if } b_t^{\lambda} > 0; \tag{11}$$

$$1 + (1-\lambda)b_t^{\lambda} + (1-\lambda)l_t + m_t \geq \frac{R - \lambda\kappa - r_t^{\lambda}}{(1-\lambda)(\sigma + \sigma_t^q)^2} \text{ with equality if } l_t > 0. \tag{12}$$

When the cost of bond financing equals the cost of bank financing, i.e., $\lambda\kappa^d + r_t = \lambda\kappa + r_t^{\lambda}$, individual risky experts are indifferent between bond financing and bank financing, and their portfolio choices are indeterminate. Without loss of generality, I assume that portfolio weights of both bond-financing and bank-financing, b_t^{λ} and l_t, are the same across all risky experts.

2.3 Banker's Optimal Choices

A banker's optimal portfolio weights on holdings of physical capital and loans satisfy

$$x_t^j + \lambda x_t + m_t \geq \frac{R_t^b - r_t}{(\sigma + \sigma_t^q)^2}, \text{with equality if } x_t^j > 0 \tag{13}$$

and

$$x_t^j + \lambda x_t + m_t \leq (>) \frac{r_t^{\lambda} - r_t}{\lambda(\sigma + \sigma_t^q)^2}, \text{with equality if } 0 < x_t < \bar{x} \text{ (if } x_t = 0). \tag{14}$$

The loan rate r_t^{λ} depends on banks' exposure to aggregate risk $\lambda(\sigma + \sigma_t^q)$, banks' leverage x_t and x_t^j and also whether the capital requirement constraint is binding or not. If the constraint is binding, i.e., $x_t = \bar{x}$, then the positive Lagrange multiplier of the constraint implies that the loan r_t^{λ} is larger or equal to the level it would be if the constraint is not binding. The financing cost of bank loans for firms fluctuates endogenously for two reasons: the price volatility of physical capital changes over time, and banks' leverage varies across business cycles.

2.4 Market Clearing

Let W_t denote the total wealth that experts have in period t and N_t the total bank capital. Hence, the total bank loans issued in equilibrium denoted by $x_t N_t$ satisfies

$$x_t N_t = (1-\alpha) W_t l_t. \tag{15}$$

The demand for final goods comprises consumption and investment. The aggregate consumption of households is $\rho q_t K_t$. Therefore, the market clearing condition with respect to final goods is

$$\begin{aligned} \rho q_t K_t = {} & \alpha \frac{W_t}{q_t}(a - \iota_t)(1 + b_t^0) + (1-\alpha)\frac{W_t}{q_t}(a - \iota_t)(1 + b_t^{\lambda} + l_t) \\ & - \frac{N_t}{q_t}\iota_t x_t^j - \frac{q_t K_t - W_t - N_t}{q_t}\iota_t x_t^h \end{aligned} \tag{16}$$

The market for physical capital clears if

$$\alpha\frac{W_t}{q_t}(1+b_t^0)+(1-\alpha)\frac{W_t}{q_t}(1+b_t^\lambda+l_t)+\frac{N_t}{q_t}x_t^j+\frac{q_tK_t-W_t-N_t}{q_t}x_t^h=K_t. \tag{17}$$

Finally, the bond mutual fund's exposure to the aggregate risk must be shared by all agents $m_tq_tK_t = (1-\alpha)\lambda b_t^\lambda W_t$.

2.5 Wealth Distribution

Two endogenous state variables that characterize the dynamics of the economy are experts' wealth share $\omega_t = W_t/(q_tK_t)$ and bankers' wealth share $\eta_t = N_t/(q_tK_t)$. The decline of experts' wealth share naturally leads to a fall in average productivity since financial markets are incomplete and households are less productive. If bankers' wealth share declines, then the supply of bank loans shrinks, and the interest rate on bank loans rises, which in turn lowers the aggregate productivity of the economy due to the increased financing cost for experts.

Given dynamic budget constraints of individual experts and bankers, it is straightforward to derive laws of motion for both W_t and N_t

$$\begin{aligned}W_{t+1}=W_t+W_t\Big(R_t+\alpha b_t^0(R_t-r_t)+(1-\alpha)b_t^\lambda(R_t-\lambda\kappa^d-r_t)+(1-\alpha)l_t(R_t-\lambda\kappa-r_t^\lambda)\Big)\Delta\\ -c_t\Delta-\chi W_t\Delta+W_t\Big(1+\alpha b_t^0+(1-\alpha)(b_t^\lambda+l_t)(1-\lambda)+m_t\Big)(\sigma+\sigma_t^q)z_t\end{aligned} \tag{18}$$

$$N_{t+1}=N_t+N_t\Big(x_t^jR_t^b+x_tr_t^\lambda+(1-x_t^j-x_t)r_t-\frac{c_t}{N_t}\Big)\Delta-\chi_\eta W_t\Delta+N_t(x_t^j+x_t\lambda+m_t)(\sigma+\sigma_t^q)z_t. \tag{19}$$

Dynamics of state variables in equilibrium also depend on the law of motion of the aggregate physical capital, which is

$$\begin{aligned}K_{t+1}&=K_t+K_t\mu_t^K\Delta+K_t\sigma z_t, \text{where}\\ \mu_t^K&\equiv\Phi(\iota_t)-\delta-\eta_tx_t^j(\delta-\delta^b)-(1-\alpha)\omega_t\lambda(b_t^\lambda\kappa^d+l_t\kappa).\end{aligned} \tag{20}$$

Given laws of motion of W_t, N_t, q_t, and K_t, we can derive laws of motion for ω_t and η_t in equilibrium, which are summarized in the following lemma.[10]

Lemma 1 *In equilibrium, experts' wealth share ω_t evolves according to*

$$\omega_{t+1}=\omega_t+\omega_t\mu_t^\omega\Delta+\sigma_t^\omega z_t, \tag{21}$$

where

$$\begin{aligned}\mu_t^\omega&=R_t-\mu_t^q-\mu_t^K-\sigma\sigma_t^q+\alpha b_t^0(R_t-r_t)+(1-\alpha)b_t^\lambda(R_t-\lambda\kappa^d-r_t^\lambda)\\ &\quad+(1-\alpha)l_t(R_t-\lambda\kappa-r_t^\lambda)-\big(\alpha b_t^0+(1-\alpha)b_t^\lambda(1-\lambda)+(1-\alpha)l_t(1-\lambda)+m_t\big)(\sigma+\sigma_t^q)^2-\rho-\chi\\ \sigma_t^\omega&=\big(\alpha b_t^0+(1-\alpha)b_t^\lambda(1-\lambda)+(1-\alpha)l_t(1-\lambda)+m_t\big)(\sigma+\sigma_t^q).\end{aligned}$$

[10] I apply Ito's Lemma for this derivation in the continuous-time setting.

The state variable η_t evolves according to

$$\eta_{t+1} = \eta_t + \eta_t \mu_t^\eta \Delta + \sigma_t^\eta z_t, \tag{22}$$

where

$$\mu_t^\eta = (x_t^j + \lambda x_t + m_t)(x_t^j - 1)(\sigma + \sigma_t^q)^2 + x_t(r_t^\lambda - r_t) + r_t - \mu_t^q - \mu_t^K - \sigma\sigma_t^q + (\sigma + \sigma^q)^2 - \rho - \chi_\eta$$
$$\sigma_t^\eta = (x_t^j + \lambda x_t + m_t - 1)(\sigma + \sigma_t^q)$$

The proof of Lemma 1 is in the appendix.

2.6 Markov Equilibrium

Like other continuous-time macro-finance models (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2012), my framework also has the property of scale-invariance with respect to total physical capital K_t. I focus on the equilibrium that is Markov in state variables ω_t and η_t. In the Markov equilibrium, dynamics of endogenous variables such as q_t can be characterized by laws of motion of ω_t and η_t and functions $q(\omega, \eta)$.

To solve for the full dynamics of the economy, I derive a partial differential equations with respect to $q(\omega, \eta)$. The partial differential equation as well as its boundary conditions originate from equilibrium conditions and Ito's formula with $q(\omega, \eta)$. Ito's lemma with respect to the volatility of the price of physical capital implies that

$$q_t \sigma_t^q = q_\omega(\omega_t, \eta_t)\omega_t \sigma_t^\omega + q_\eta(\omega_t, \eta_t)\eta_t \sigma_t^\eta. \tag{23}$$

Given (q, ω, η), we can solve the equilibrium and derive all endogenous choice variables $(c, b^0, b^\lambda, l, x, x^h)$ and endogenous price variables $(r, r^\lambda, \mu^q, \sigma^q)$ as well as the lump-sum transfer related to the bond mutual fund m.[11] Therefore, volatility terms of two state variables $(\sigma^\eta, \sigma^\omega)$ are also known. Hence, equation (23) is a well-defined partial differential equation with respect to $q(\omega, \eta)$.

In addition to the differential equation, I need boundary conditions to solve for $q(\omega, \eta)$. There are three boundary conditions that correspond to three boundaries for the domain of $q(\omega, \eta)$: $\{(\omega, \eta) : \omega = 0, 0 \leq \eta \leq 1\}$, $\{(\omega, \eta) : 0 \leq \omega \leq 1, \eta = 0\}$, and $\{(\omega, \eta) : 0 \leq \omega \leq 1, 0 \leq \eta \leq 1, \omega + \eta = 1\}$. For any of the three boundaries, one of the three agents has zero net worth and the economy now has only two types of agents. Accordingly, differential equation (23) on boundaries reduces to an ordinary differential equation, which is straightforward to characterize.

2.7 Calibration

The calibration of the model mainly follows He and Krishnamurthy (forthcoming) because the two models share the same continuous-time framework and both emphasize the role of the financial intermediary. I choose standard values from the real business cycle literature for time discount rate ρ, the depreciation of physical capital held by experts and households δ, and capital adjustment cost parameter ϕ (see Table 1). I choose the minimum capital requirement $1/\bar{x}$ as 6% to be consistent with Elenev, Landvoigt and Van Nieuwerburgh (2018). The reason I use the capital adequancy ratio calculated by Elenev, Landvoigt and Van Nieuwerburgh (2018) is that both models are composed of a real sector and an explicit banking

[11] At this stage given (q, ω, η), I can only solve for $r - \mu^q$. However, it is straightforward to solve for r_t and μ^q after I derive the entire $q(\omega, \eta)$.

Table 1: Parameters

	Parameter	Choice	Reference
Technology Parameters			
a	experts' productivity	0.16	calibration (average Sharpe ratio)
δ	capital depreciation	10%	literature
δ_b	capital depreciation (bankers)	55%	calibration (highest to average Sharpe ratio)
ϕ	capital adjustment cost	3	literature
σ	capital quality shock	2.7%	calibration (volatility)
λ	idiosyncratic default likelihood	5%	calibration (risk premium)
α	the fraction of safe experts	16%	calibration
Finance Parameters			
$1/\bar{x}$	capital requirement	6%	Basel accord (see Elenev et al. 2018)
κ	bankruptcy cost (loan)	15%	calibration (loan risk premium)
κ_d	bankruptcy cost (bond)	60%	calibration (bond risk premium)
Preference Parameters			
ρ	time discount rate	2%	literature
χ	experts' retirement rate	25%	calibration
χ_η	bankers' retirement rate	17%	calibration

sector and thus the real world counterparts of bank assets in both models are corporate loans. For the rest nine parameters, I target nine key moments (see table 2), six of which are standard moments that He and Krishnamurthy (forthcoming) and three of which are related to bond and bank finance (i.e., the ratio of bank credit to total credit, risk premiums of bank loans and corporate bonds).

Table 2 reports the sample moments of the model simulation. I simulate the economy for 1500 years and keep the results of the last 1000 years. The moments generated by the model are based on the sample of 50,000 simulations. Like He and Krishnamurthy (forthcoming), I solve for the global solution of the equilibrium and characterize the economy in both financially distressed states and non-distressed states. Since risky firms are the most important type of agents in the economy, we use the Sharpe ratio of their investments to measure to what extent the economy is financially constrained. As in He and Krishnamurthy (forthcoming), we label states with the lowest 67% Sharpe ratio as the non-distressed states. The experts' productivity a is chosen to target the average Sharpe ratio and the investment-to-capital ratio. My model generates the average Sharpe ratio 45% and the investment-to-capital ratio 11%, which are not far from their real world counterparts (see He and Krishnamurthy forthcoming). Notice that the turnover rates χ and χ_η provide additional freedom of targeting two moments for the single parameter, which also applies to following cases. The depreciation rate of physical capital held by bankers is the key crisis parameter because bankers are the least efficient owners of physical capital. Bankers' depreciation rate δ_b affects the price of physical capital in crises and thus the ratio of the highest Sharpe ratio to the average one. He and Krishnamurthy (forthcoming) notice that this ratio is 15 based on the observation of 2008-09 financial crisis. The ratio generated by my model is 11.5, which is still in the reasonable range.

Second moments of the model simulated data tend to be larger than their real data counterparts. One reason is that the model has only one exogenous shock and its volatility is governed by the single parameter σ. Moreover, the fixed parameter capital adjustment cost ϕ has a significant impact on endogenous risk σ^q and the Sharpe ratio itself is also influenced by the aggregate risk $\sigma + \sigma^q$. Therefore, I sacrifice the performance of my model regarding second moments to some extent.

The key moments of my model are the three related to bond finance and bank finance: the ratio of bank credit to total credit, loans' risk premium, and corporate bonds' risk premium. The targets of the three moments are from De Fiore and Uhlig (2011). I choose relevant parameters (the fraction of safe firms α, idiosyncratic default likelihood λ, bankruptcy costs of bond and bank finance κ and κ^d) to exactly match the ratio of bank credit to total credit as well as the other two moments.

Table 2: Moments[1]

Moment	Model	Target	Source
Sharpe ratio (mean)	45%	48%	He and Krishnamurthy (forthcoming)
investment-to-capital ratio[2] (mean)	11%	9%	He and Krishnamurthy (forthcoming)
$\frac{\text{highest Sharpe ratio}}{\text{average Sharpe ratio}}$	11.5	15	He and Krishnamurthy (forthcoming)
investment growth[2] (volatility)	12.8%	5.8%	He and Krishnamurthy (forthcoming)
consumption growth[2] (volatility)	5.36%	1.24%	He and Krishnamurthy (forthcoming)
Sharpe ratio[2] (volatility)	26.5%	16.56%	He and Krishnamurthy (forthcoming)
bank credit to total credit ratio (mean)	0.406	0.401	De Fiore and Uhlig (2011)
risk premium on loans (mean)	2.07%	1.70%	De Fiore and Uhlig (2011)
risk premium on bonds (mean)	0.72%	1.43%	De Fiore and Uhlig (2011)

[1] We use the density of the stationary distribution to calculate all moments.
[2] These are moments conditional on non-distressed states, which are defined as states with lowest 67% Sharpe ratio.

2.8 Equilibrium Characterization

In this subsection, I highlight that the equilibrium of an economy highly depends on whether risky firms can directly issue bonds. To illustrate the role of bond financing in the aggregate economy, I compare the equilibrium of the benchmark economy with bond financing with the equilibrium of a second economy without the bond market. In the second economy, safe firms obtain risk-free loans from banks. To ensure that the two economies are comparable, capital requirement imposes an upper bound on the risky loan-to-equity ratio.[12]

First-Moment Comparison. The economy with bond financing tends to perform less than the one without bond financing according to several metrics (see Table 3). I run 50,000 simulations for both economies and each round of simulation continues for 1,500 years. Table 3 reports the average of nine key endogenous variables based on the sample of 50,000 draws. The table highlights one significant difference between the two economies, that is, bankers hold more wealth in the economy without a bond market. This is an intuitive result since bank finance becomes the only source of external finance for firms in the absence of bond financing, which generates more profits for the banking sector. The two rows in the bottom of Table 3 show that risky firms raise much more bank loans if they cannot issue bonds and the loan spread also increases slightly. The improvement of the banking sector also helps the real sector raise more external credit that it could in the economy with bond financing (see row 6 of table 3). On the real side, the economy without bond financing also outperforms: higher TFP, higher consumption ratio, and higher investment ratio (see rows 3-5 of Table 3). Notice that since the model is scale-invariant I focus on the consumption and investment to physical capital ratios.

The above comparison indicates that the framework that omits bond financing tends to *overstates* the benefiting role of the banking sector for the real sector. If we take into account the effect of the bond market,

[12] Safe firms obtain risk-free bank loans in the second economy.

Table 3: First-Moment Comparison[1]

		with bond	without bond
1	experts' wealth share ω	0.1545	0.1547
2	bankers' wealth share η	0.0156	0.0240
3	TFP	0.1372	0.1385
4	consumption to capital ratio	0.0266	0.0267
5	investment to capital ratio	0.1106	0.1117
6	the real sector's liability	0.6915	0.7002
7	outstanding bonds[2]	0.4361	0.3599
8	outstanding loans[3]	0.2554	0.3403
9	loan spread	0.0132	0.0134

[1] We use the density of the stationary distribution to calculate all moments.
[2] Outstanding bonds in the economy without bond financing refer to safe firms' borrowing.
[3] Outstanding bonds in the economy without bond financing refer to risky firms' borrowing.

banks enjoy less profits and the banking sector shrinks. The general equilibrium consequence is that the overall external credit that the real sector can raise actually decrease when the alternative financing channel, bond issuance, becomes available (see row 6 of Table 3).

Dynamics. To illustrate the economic dynamics, Figure 1 shows a set of nonlinear impulse response functions in the continuous-time Brownian environment [13] In particular, I set the initial state of the economy at the highest density in the long-run stationary distribution. The expectation is approximated by the sample average of 50,000 simulated economies.

Before discussing economic dynamics in detail, let us review the *transmission mechanism* of the model. When a negative shock hits the economy, experts' dynamic budget constraints (3) and (4) imply that their net worth will decline disproportionately due to the leverage effect. On top of the exogenous shock, the decline in the price of physical capital causes additional losses to experts' net worth, as indicated by equations (3) and (4). The exogenous shock also affects bankers' net worth, which is the other state variable. Bankers' exposure to the aggregate risk comes from the collateral that backs their loans. When banks liquidate risky firms' physical capital, the exogenous shock affects the (efficient) units of physical capital seized by banks, and also the price at which they can sell the physical capital in the secondary market. Note that banks also take on high leverage and thus have high risk exposure to the exogenous shock as shown by equation (5). The decline in the net worth of both productive experts and financial intermediaries has persistent effects on the productivity, investment, asset prices, and external financing in the economy.

The key message of Figure 1 is that the banking sector will recover more quickly after a negative shock if risky firms cannot access the bond market (see the top middle panel). Given the initial adverse shock, the wealth of experts and bankers decline by a similar magnitude in both types of economies. However, the two types of economies experience quite different dynamics in the aftermath of the shock. The top left and middle panels in Figure 1 show that both experts' wealth and bankers' wealth tend to recover more quickly in the economy without bond financing. The difference is more significant for bankers' wealth. Since experts are the most productive type of agents in the economy, and bankers provide relatively cheap credit for productive agents, the economy without bond financing would also experience much faster recovery in

[13] Effectively, I show the numerical approximation of the shock-exposure elasticities. See Borovička, Hansen and Scheinkman (2014) and Borovička and Hansen (2016) for details of shock elasticities.

its average productivity, consumption, and investment (see the top right, middle left, and center panels in Figure 1). This result emphasizes a crucial point that models that do not permit direct bond financing cannot capture the exact dynamics of an economy with an active bond market.

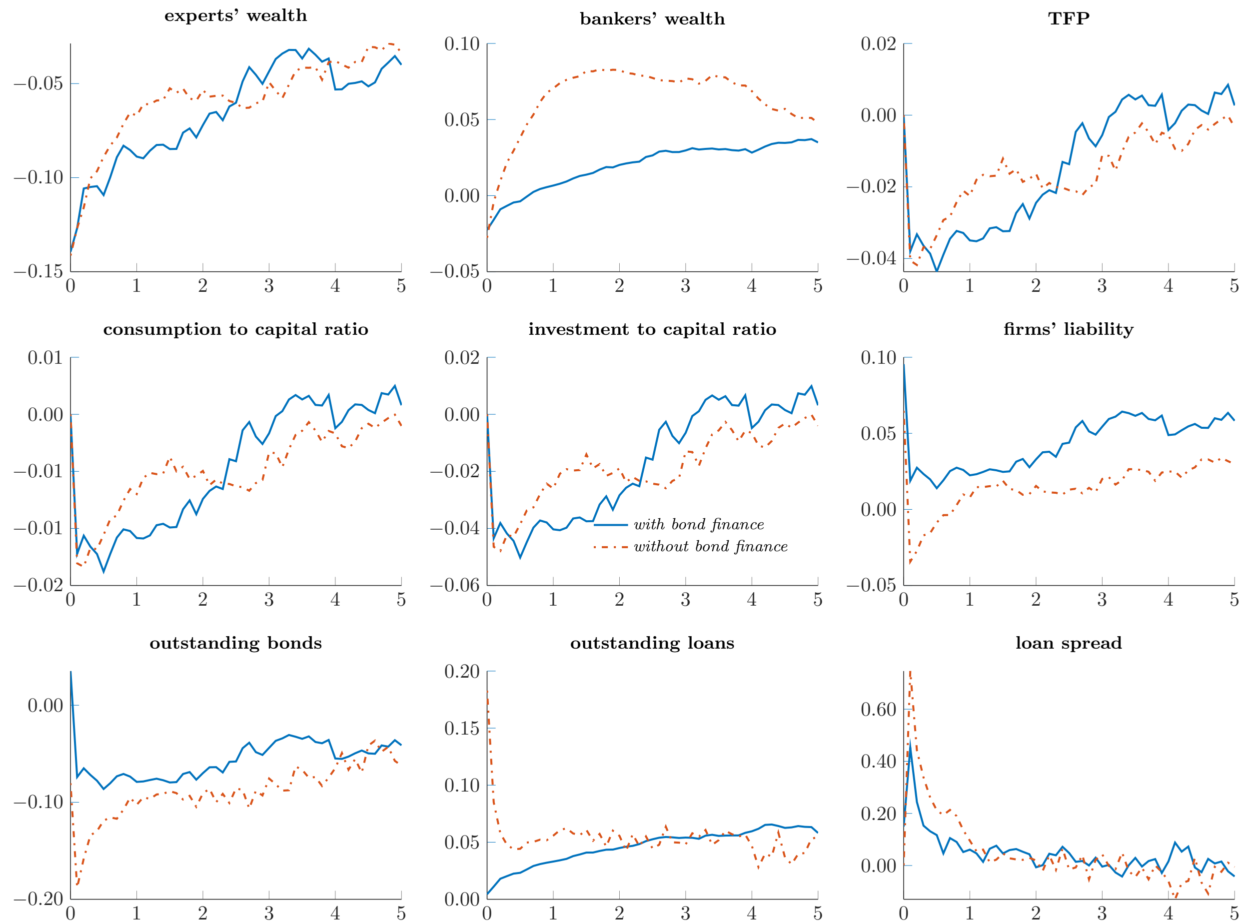

Figure 1: Dynamics
This figure shows the dynamics of the mean of nine key aggregate variables in two economies after being hit by an aggregate capital quality shock with a magnitude of 1.58 times the standard deviation: experts' wealth share (top left), bankers' wealth share (top middle), TFP (top right), consumption-to-physical capital ratio (middle left), investment-to-capital ratio (center), risky firms' liability (middle right), outstanding bonds (bottom left), outstanding loans (bottom middle), loan spread (bottom right). The horizontal axis depicts the number of calendar quarters following the shock. Each line indicates the percentage change relative to the initial state of the variable over time. The solid lines refer to an economy with bond financing, and the dashed lines refer to an economy without bond financing. The initial states of the two economies prior to the aggregate shock are at the highest probability density of the long-run stationary distribution.

I now explain why the banking sector can recover faster when the bond market is shut down in an economy. In the aftermath of an adverse shock, both the firm sector and the banking sector shrink. Experts' wealth declines much more significantly that bankers' wealth (see the upper left and middle panels of Figure 1). It turns out that bankers are relatively better capitalized than firms. In the absence of bond financing, the dashed line in the lower middle panel of Figure 1 shows that risky firms raise more loans after the shock and the loan spread also increases substantially (see the lower right panel of Figure 1). Intuitively, the rise in both loan volumes and loan spreads raises bank profitability and thus enhances the quick growth of the banking sector.

However, the same effect that triggers the fast recovery of the banking sector could be dampened by the presence of bond financing. As bank loans become gradually more expensive, the lower left panel of Figure 1 shows that risky firms switch to bond credit if it is available. Notice that the dashed line in the lower left panel represents safe firms' borrowing in the economy without a bond market. Therefore, banks' profitability might not increase much as risky firms switch to bond issuance and the aggregate demand for bank loans tends to be sluggish. Consequently, bankers' wealth grows at a much slower rate in an economy with a bond market than it does in an economy without a bond market. Similarly, risky firms' outstanding loans are restored more slowly in an economy with a bond market relative to an economy without a bond market (see the bottom middle panels in Figure 1).

3 Optimal Capital Requirement

In this section, I emphasize that the socially optimal level of capital ratio requirement highly depends on (*i*) whether bond financing is present in a model, (*ii*) the efficiency of an economy's bankruptcy procedure, and (*iii*) the distribution of borrowing firms' idiosyncratic default risk. All these results are connected in the sense that the elasticity of aggregate demand for bank loans is the key factor that determines the general equilibrium costs and benefits of bank capital requirements.

The welfare of an individual agent is the weighted sum of the agent's lifetime expected utility over all possible states of the economy. The weight of each state is the density of the long-run stationary distribution at that state. The social welfare is the equal-weighted sum of the welfare of all agents.

3.1 The Consequences of Omitting Bond Financing

An economic model that omits bond financing overstates the benefit of capital ratio requirements, and thus prescribes an optimal requirement that is overly tight. We compare the social welfare of two economies — one with bond financing and the other without bond financing — under different degrees of capital requirement. Figure 2 clearly shows that the capital adequacy ratio $1/\bar{x}$ that maximizes social welfare is lower in the economy with a bond market than in the economy without a bond market. In other words, the socially optimal capital requirement should be more lenient in the presence of bond financing. This statement holds regardless of whether we focus on the welfare of experts, bankers, or households (see the second, third, and fourth panels from the left in Figure 2). Before expounding why this difference exists, I explain the channel through which capital requirement influences social welfare.

Elenev, Landvoigt and Van Nieuwerburgh (2018) highlights that tightening the capital requirement shifts wealth from savers to borrowers. Here, I emphasize that part of the wealth is actually diverted to financial intermediaries. To illustrate this effect more clearly, first consider an economy without bond financing. The dashed lines in the two left panels in Figure 3 show that the wealth share of both experts and bankers rises as the capital ratio requirement tightens. Dashed lines in the top middle and upper right panels in Figure 3 clearly show why bankers' wealth share increases. Tightening the capital requirement lowers the supply of bank loans. Therefore, the loan spread that banks can charge increases accordingly. To some extent, the overall effect leads to the increase in bank profitability as shown by the dashed line in the bottom right panel in Figure 3. The cumulative effect of high bank profits naturally leads to an increasingly stronger banking sector, which translates to an improvement in bankers' welfare.

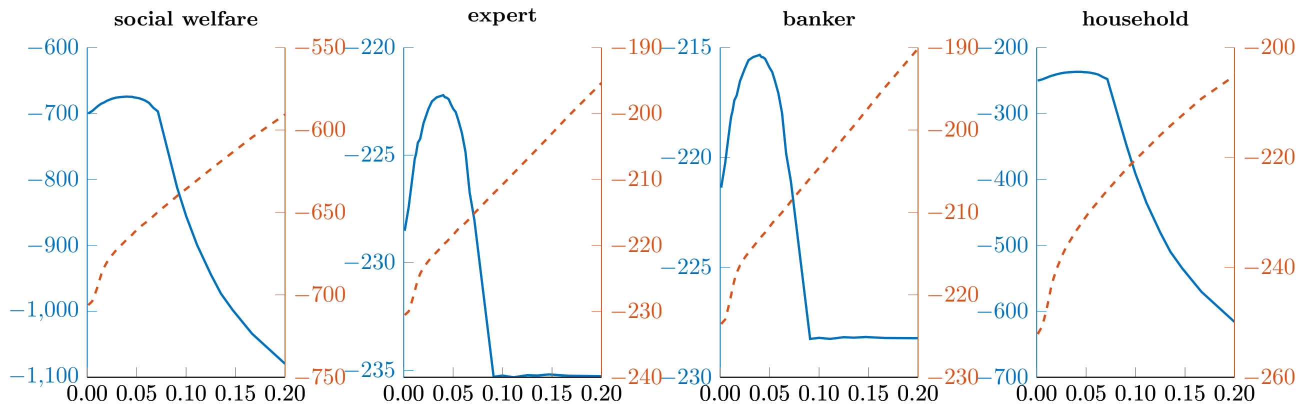

Figure 2: Welfare

This figure shows the relationship between the capital adequacy ratio $1/\bar{x}$ (horizontal axis) and the welfare of different types of agents in economies with and without bond financing. Solid lines refer to an economy with bond financing and dashed lines refer to an economy without bond financing. The aggregate welfare shown in the left panel is the sum of the welfare of the three types of agents. For the values of parameters other than $\bar{x}$, see Section 2.7.

Tightening the capital requirement increases experts' wealth share as well as the welfare of both experts and households. Lowering the maximum leverage of bankers limits the supply of overall credit. Given the excessive credit supply from less productive households, the overall borrowing costs decrease, and thus experts' wealth share increases. In sum, the strengthening of both the firm sector and the banking sector increases the average productivity of the economy as highlighted in the lower middle panel in Figure 3. The rise in the average TFP results in the improvement of households' welfare (see the right panel in figure 2). Furthermore, I should highlight that tightening the capital requirement does not always lead to positive effects although I did not present the results for cases where capital requirement constraint is further tightened. The logic is that if the capital ratio requirement is too tight bank profitability ultimately declines due to the substantial decrease in loans that banks can originate. This reasoning also applies to social welfare.

The presence of bond financing, however, can significantly dampen the wealth transfer effect of the capital ratio requirement. We now turn to an economy with a bond market. The solid line in the upper middle panel in Figure 3 displays that bankers' wealth share increases as the capital ratio requirement increases up to around 5%. This increase is similar to their reactions to the regulatory change in an economy without bond financing. Nevertheless, if the capital adequacy ratio continues rising, the wealth share of the intermediary sector shrinks drastically until it completely vanishes. The shrinkage of the banking sector has adverse effects on the the real sector's borrowing and the average productivity, although experts' wealth share becomes larger (see the upper left, lower left, and lower middle panels of Figure 3).

Why does the financial intermediary sector react so differently in the two economies? The key underlying reason is that firms have an alternative way of raising external credit in an economy with bond financing. Thanks to the alternative channel, firms can resort to bond financing when loan spreads rise. Hence, when bond financing is feasible, the decline in loan demand would be more substantial than in an economy where loan financing is the only option for the real sector. Therefore, bank profits are more likely to decline in an economy where firms have a second option for raising external credit. The decline in bank profitability, in turn, hurts bankers' wealth share and loan supply, which ultimately lowers experts' wealth share and the economy's average productivity. In sum, tightening capital requirement in the presence of bond financing is more inclined to hurt the financial intermediary sector and the entire economy.

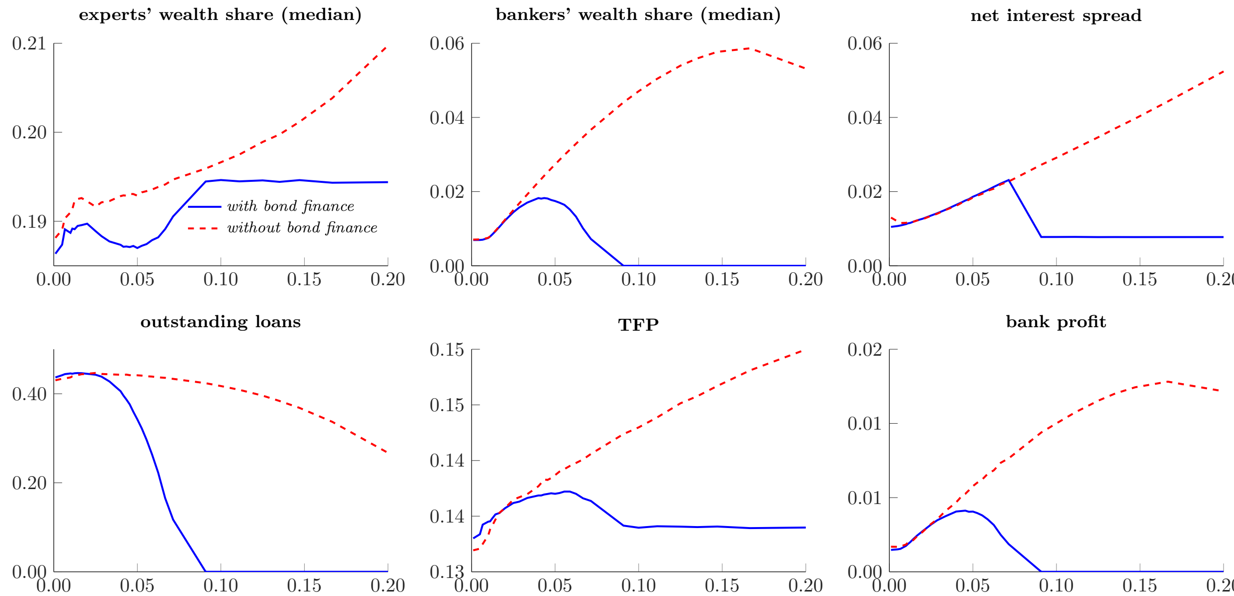

Figure 3: Wealth Distribution
This figure shows the relationship between the capital adequacy ratio $1/\bar{x}$ (horizontal axis) and the moments of six financial variables in the long-run stationary distribution: median experts' wealth share (upper left), average net interest spread (upper middle), average outstanding loans (upper right), median bankers' wealth share (lower left), average total factor productivity (lower middle), and average bank profit (lower right). For the values of parameters other than $\bar{x}$, see Section 2.7.

Quantitative Implications. The calibrated model indicates that the socially optimal capital adequacy ratio is 4%, which is much more lenient than the benchmark 6%. This is not a surprising result since, unlike Elenev, Landvoigt and Van Nieuwerburgh (2018) and others, my model takes into account an additional channel that dampens the positive effects of the capital requirement constraint. If I drop the channel, then the framework will have the similar prediction as in Elenev, Landvoigt and Van Nieuwerburgh (2018) that raising capital requirement from 6% would increase the social welfare. One caveat regarding the model's quantitative prediction is that the idiosyncratic default risk is exogenous in my model and thus the capital adequacy ratio has no effects from the micro-prudential perspective. My model only focuses on the macro-prudential implications of capital requirement.

The most crucial insight of the quantitative finding is that the optimal level of capital ratio requirement could be very sensitive to the presence of the bond market. If I deprive the real sector of the bond financing option, the social welfare maximizing capital requirement would point to more than 20%, which is significantly different from 4% that the benchmark model with bond financing suggests. This result raises an important regarding the *robustness* of optimal bank regulation against different market forces.

3.2 Policy Experiments

The previous section shows that the discussion on the optimal capital requirement could be misleading if bond financing is omitted from the model. In this subsection, I conduct three policy experiments, and discuss whether and how the optimal capital requirement depends on the structure of the bond market. In the first experiment, I vary the liquidation cost of bondholders κ^d, and characterize the relationship between the optimal capital requirement and the development of the bond market (Djankov, Hart, McLiesh and Shleifer, 2008; Becker and Josephson, 2016). In the second experiments, I investigate the policy implication of the risk profile of borrowing companies in the bond market.

3.2.1 Development of the Bond Market

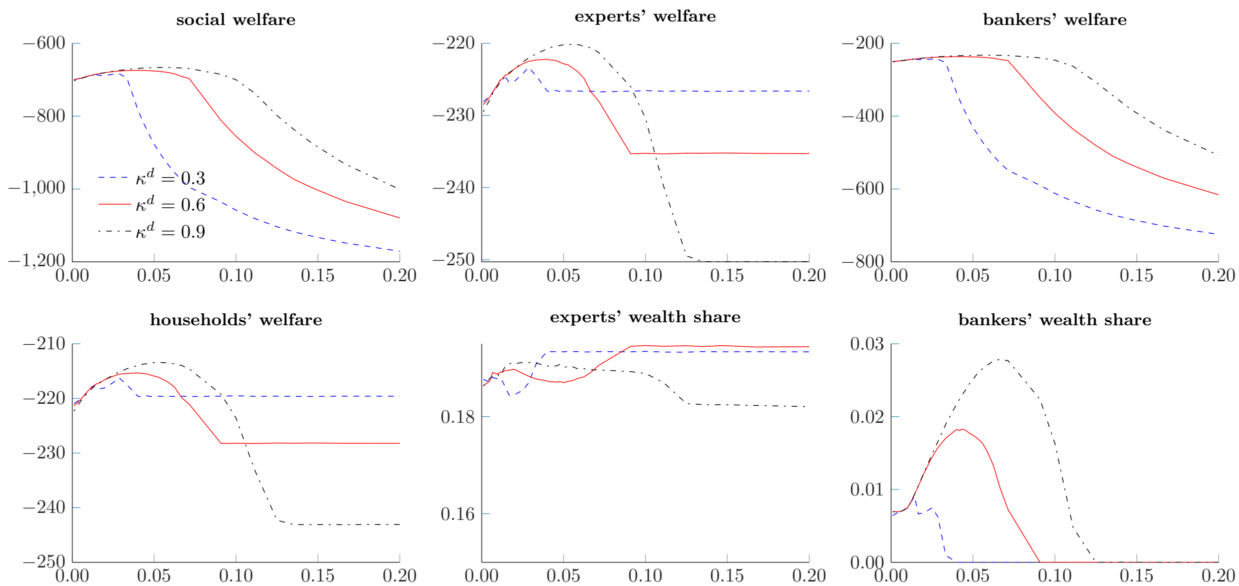

Figure 4: Development of bond market
This figure shows the welfare implications of a change in the capital adequacy ratio $1/\bar{x}$ (horizontal axis) for economies with different degrees of bond market development: more developed bond market ($\kappa^d = 0.3$), benchmark ($\kappa^d = 0.6$), less developed bond market ($\kappa^d = 0.9$). The bottom middle and right panels display effects of a change in $\bar{x}$ on the median wealth shares of experts and bankers. For the values of parameters other than $\bar{x}$ and κ^d, see Section 2.7.

Becker and Josephson (2016) emphasize that the efficiency differences in the processing of insolvency and bankruptcy cases (e.g., bankruptcy recoveries) can explain the cross-firm and also cross-country heterogeneity regarding the use of bond financing and bank financing. Their empirical evidence as well as theoretical results show that inefficient bankruptcy procedures in an economy is associated with less bond financing by risky firms. The efficiency of bankruptcy procedures, in turn, can be traced back to the legal origin and income per capita according to Djankov et al. (2008). Here, I treat bondholders' liquidation cost κ^d as an exogenous parameter that captures the efficiency of bankruptcy procedures in an economy. A lower liquidation cost κ^d signifies a more efficient bankruptcy system and a more developed bond market. Based on this assumption, I investigate how the optimal capital ratio requirement in a country depends on how developed its bond market is.

The top left panel in Figure 4 shows that the socially optimal capital requirement ought to be more stringent in an economy with a less developed bond market (i.e., higher κ^d). The intuition is the same as that in the previous analysis on the absence of bond financing. If the cap on bank leverage decreases, the loan spread increases; at the same time, the amount of loans originated by banks also declines. In an economy with a less developed bond market, (i.e., higher liquidation cost κ^d), risky firms find it more costly to switch from bank financing to bond financing. Hence, the decrease in the amount of bank loans is not so significant; in fact, banks' overall profitability may actually increase when the loan spread increases. The capital adequacy ratio is relatively low, raising this ratio actually increases bankers' wealth share in the economy. However, the bottom right panel in Figure 4 shows that if the capital requirement is too tight, the banking sector is more likely to vanish in an economy with a developed bond market ($\kappa^d = 0.3$). The reason is that when risky firms switch to bond financing, there is a substantial decline in the quantity of bank loans and also a sizable drop in bank profitability. When the banking sector vanishes, the borrowing cost of the firm sector increases and its borrowing capacity declines substantially. Since the average productivity of the economy depends on to what extent firms can raise external funds, the capital ratio requirement affects social welfare through its impact on experts' borrowing.

3.2.2 Average Firm Riskiness

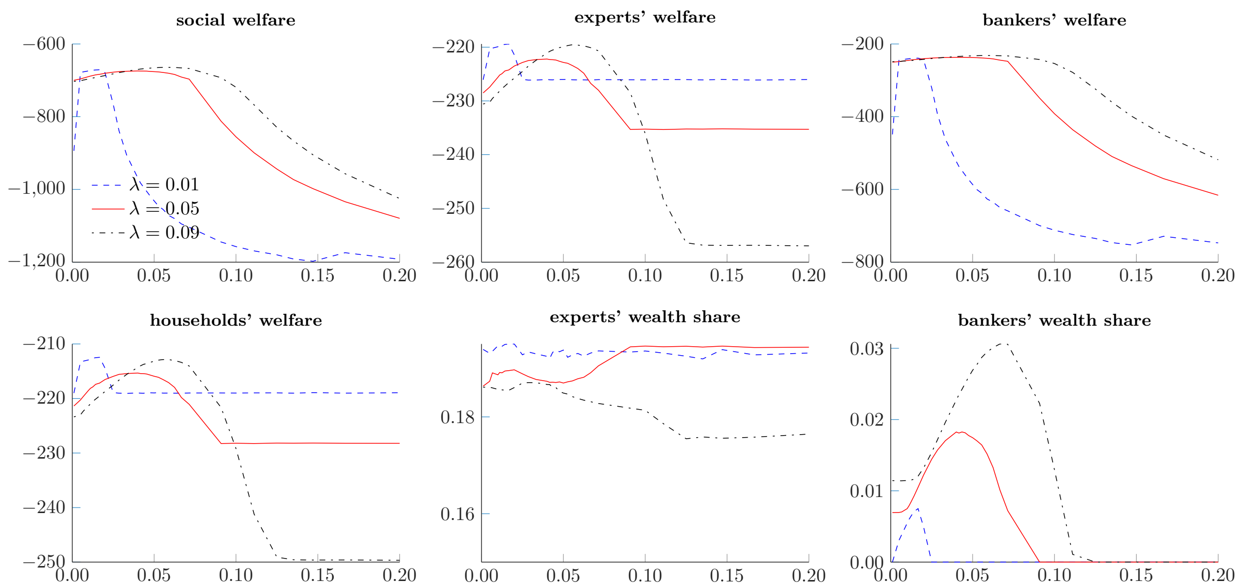

Figure 5: Riskier firms
This figure shows the welfare implications of a change in banks' maximum leverage $\bar{x}$ (horizontal axis) for economies in which firms have different degrees of riskiness: less risky ($\lambda = 0.25$), benchmark ($\lambda = 0.3$), and more risky ($\lambda = 0.35$). The bottom middle and right panels display effects of a change in $\bar{x}$ on the median wealth shares of experts and bankers. For the values of parameters other than $\bar{x}$ and λ, see Section 2.7.

Other than the efficiency of the bankruptcy process in an economy, the risk profile of its ultimate borrowers also affects the use of bond financing and bank financing. I consider two experiments by varying the distribution of experts' idiosyncratic riskiness. First, I investigate how the average riskiness of firms affects the optimal capital requirement. In particular, I keep all parameters unchanged and only adjust the value of individual firms' bankruptcy probability λ. Notice that a change in λ does not change the skewness of firms' idiosyncratic default risk distribution. In light of this property, I vary the fraction of safe firms (i.e., α) to adjust the skewness in the second experiment.

The top right panel in Figure 5 shows that the socially optimal capital requirement is tighter in an economy where firms are riskier on average. The same conclusion holds regardless of whether we focus on the welfare of experts, bankers, or households (the top middle, top right, and bottom left panels in Figure 5). When a risky firm switches from bank financing to bond financing, it has to pay an additional premium $\lambda(\kappa^d - \kappa)$ to compensate creditors for their loss in the event of a firm liquidation. This switch cost is increasing in the likelihood of firm failure, i.e., λ. Hence, relative to safe firms, risky firms find it more costly to replace bank loans with bonds. When the capital requirement tightens, the decrease in the amount of bank loans is less significant in an economy with riskier firms. In such an economy, bank profitability is less likely to decline given the rise in the loan spread. Consequently, the banking sector is less likely to shrink (see the bottom right panels in Figure 5). Hence, the optimal capital requirement ought to be tighter in an economy with riskier firms.

4 Conclusion

In this paper, I point out that bond financing is a critical feature in a dynamic general equilibrium framework analyzing the welfare implications of bank capital regulations. A model that omits the bond market overemphasizes the benefit of capital requirements. In addition, I highlight three factors that affects the optimal level of bank capital requirements via their influences on the demand elasticity of bank loans: the efficiency of the bankruptcy system in an economy, as well as the mean and skewness of the distribution of firms' idiosyncratic default risks.

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Appendix

A Proofs

Proof of Lemma 1.

The laws of motion for the price of physical capital (2) and the efficiency units of physical capital (20) imply

$$\begin{aligned} q_{t+1}K_{t+1} &= \left(q_t + \mu_t^q q_t \Delta + \sigma_t^q q_t z_t\right)\left(K_t + K_t \mu_t^K \Delta + K_t \sigma z_t\right) \\ &= q_t K_t + q_t K_t(\mu_t^q + \mu_t^K + \sigma\sigma_t^q)\Delta + q_t K_t(\sigma + \sigma_t^q) z_t. \end{aligned}$$

I omit all terms of order above Δ and use the property that $E[z_t^2] = \Delta$. The equation above, together with equation (18), lead to

$$\begin{aligned}
\frac{W_{t+1}}{q_{t+1}K_{t+1}} &= \frac{1}{q_{t+1}K_{t+1}}\Big(W_t + W_t\Big(R_t + \alpha b_t^0(R_t - r_t) + (1-\alpha)b_t^\lambda(R_t - \lambda\kappa^d - r_t) + (1-\alpha)l_t(R_t - \lambda\kappa - r_t^\lambda)\Big)\Delta\Big) \\
&\quad - \frac{c_t}{q_{t+1}K_{t+1}}\Delta + \frac{W_t}{q_{t+1}K_{t+1}}\Big(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)(1-\lambda) + m_t\Big)(\sigma + \sigma_t^q)z_t \\
&= \frac{W_t}{q_tK_t} + \frac{W_t}{q_tK_t}\Big(R_t + \alpha b_t^0(R_t - r_t) + (1-\alpha)b_t^\lambda(R_t - \lambda\kappa^d - r_t) + (1-\alpha)l_t(R_t - \lambda\kappa - r_t^\lambda) - \frac{c_t}{W_t}\Big)\Delta \\
&\quad - \frac{W_t}{q_tK_t}(\mu_t^q + \mu_t^K + \sigma\sigma_t^q)\Delta - \frac{W_t}{q_tK_t}\Big(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)(1-\lambda) + m_t\Big)(\sigma + \sigma_t^q)^2\Delta \\
&\quad + \frac{W_t}{q_tK_t}(\sigma + \sigma_t^q)^2\Delta + \frac{W_t}{q_tK_t}\Big(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)(1-\lambda) + m_t\Big)(\sigma + \sigma_t^q)z_t - \frac{W_t}{q_tK_t}(\sigma + \sigma_t^q)z_t \\
\omega_{t+1} &= \omega_t + \omega_t\mu_t^\omega\Delta + \sigma_t^\omega z_t,
\end{aligned}$$

I use the approximation $\frac{a}{b+x} = \frac{a}{b} - \frac{a}{b^2}x + \frac{a}{b^3}x^2 + o(x^2)$ for x close to zero. In addition, I also omit all terms of order above Δ, and use the property that $E[z_t^2] = \Delta$.

Given one of the bankers' Euler equation (13), the law of motion for W_t can be rewritten as

$$N_{t+1} = N_t + N_t\Big(x_t^j(x_t^j + \lambda x_t + m_t)(\sigma + \sigma_t^q)^2 + x_t(r_t^\lambda - r_t) + r_t - \frac{c_t}{N_t}\Big)\Delta + N_t(x_t^j + \lambda x_t + m_t)(\sigma + \sigma_t^q)z_t.$$

Hence,

$$\begin{aligned}
\frac{N_{t+1}}{q_{t+1}K_{t+1}} &= \frac{1}{q_{t+1}K_{t+1}}\Big(N_t + N_t\Big(x_t^j(x_t^j + \lambda x_t + m_t)(\sigma + \sigma_t^q)^2 + x_t(r_t^\lambda - r_t) + r_t - \frac{c_t}{N_t}\Big)\Delta\Big) \\
&\quad + \frac{N_t}{q_{t+1}K_{t+1}}(x_t^j + \lambda x_t + m_t)(\sigma + \sigma_t^q)z_t \\
&= \frac{W_t}{q_tK_t} + \frac{N_t}{q_tK_t}\Big(x_t^j(x_t^j + \lambda x_t + m_t)(\sigma + \sigma_t^q)^2 + x_t(r_t^\lambda - r_t) + r_t - \frac{c_t}{N_t}\Big)\Delta - \frac{N_t}{q_tK_t}(\mu_t^q + \mu_t^K + \sigma\sigma_t^q)\Delta \\
&\quad - \frac{N_t}{q_tK_t}(x_t^j + \lambda x_t + m_t)(\sigma + \sigma_t^q)^2\Delta + \frac{N_t}{q_tK_t}(\sigma + \sigma_t^q)^2\Delta \\
&\quad + \frac{N_t}{q_tK_t}(x_t^j + \lambda x_t + m_t)(\sigma + \sigma_t^q)z_t - \frac{N_t}{q_tK_t}(\sigma + \sigma_t^q)z_t \\
\frac{\mathrm{d}\eta_t}{\eta_t} &= \mu_t^\eta \mathrm{d}t + \sigma_t^\eta \mathrm{d}Z_t.
\end{aligned}$$

■

B Numerical Procedure

I first simplify the final goods' market clearing condition so as to illustrate the numerical procedure more clearly. Taking the parameter choice $a^h = a^b = 0$ as given, the market clearing condition reduces to

$$a\psi_t = \rho q_t + \iota_t = \rho q_t + \frac{q_t - 1}{\phi},$$

where ψ_t denotes the fraction of physical capital held by experts and the second equality uses the result of equation (8). The connections between ψ_t and the leverage of households and experts follow

$$\begin{aligned}\psi_t &= \omega_t(1+\alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)), \text{ and} \\ x_t^h &= \frac{\omega_t + \eta_t - \psi_t}{1-\omega_t-\eta_t}.\end{aligned} \tag{24}$$

The numerical procedure of solving for the equilibrium consists of three parts: i) three boundary solutions of $q(\omega,\eta)$, where $\omega = 1$, $\eta = 1$, or $\omega+\eta = 1$, ii) the interior solutions of $q(\omega,\eta)$ where $\psi(\omega,\eta) = 1$, iii) the interior solutions of $q(\omega,\eta)$ where $\psi(\omega,\eta) < 1$. I discretize the space as $\{(\omega_{i,j},\eta_{i,j}), \text{where } i = 1,...,N, j = 1,...,I(j), \omega_{i,j} = \omega_{i,h} \text{ for any } j \text{ and } h, \eta_{i,j} = \eta_{h,j} \text{ for any } i \text{ and } h, \text{ and } \omega_{i,J(i)} + \eta_{i,J(i)} = 1\}$.

I first solve for $q(\omega,\eta)$ along the three boundaries, that is, $q(0,\eta_{i,j})$, $q(\omega_{i,j},0)$, and $q(\omega_{i,J(i)})$. In the boundary cases, equation (23) yields ordinary differential equations (ODEs). The numerical scheme of solving the three ODEs is a simplified version of the one used for the interior solutions of $q(\omega,\eta)$ where $\psi(\omega,\eta) < 1$. Hence, I do not repeat the description of the scheme for the simpler case.

I start from the boundary where $\omega_{i,J(i)} + \eta_{i,J(i)} = 1$ to solve for the interior solution of $q(\omega,\eta)$ where $\psi(\omega,\eta) = 1$. The numerical scheme begins with the conjecture $\psi(\omega,\eta) = 1$ and later verify that neither households nor bankers find it optimal to hold physical capital. The conjecture and the final goods' market clearing condition imply that $q(\omega,\eta) = \frac{a\phi+1}{\rho\phi+1}$ and thus $\sigma_q = 0$. Then, I can solve for b^0, b^λ, l, and x given the optimality conditions of experts and bankers (10), (11), (12), and (14) as well as equation (24). In the end, we check the optimality conditions of households and bankers regarding their holdings of physical capital (9) and (13) to verify the initial conjecture. For a given i, we can find a minimum j such that $\psi(\omega_{i,j},\eta_{i,j}) = 1$, which I denote as $\Psi(i)$.

Equation (23) provides me a first-order partial differential equation, which fully characterizes $q(\omega,\eta)$ along with boundary conditions mentioned above. By the nature of the first-order PDE, I essentially only need to solve for a system of ODEs that $q(\omega_i,\eta)$ satisfies for any $i = 1,...,N$. To ensure the stability of the numerical procedure, I use the implicit method that involves the root-finding of a sixth order polynomial with respect to the unknown $q(\omega_i,\eta_j)$. I rearrange equation (23) to illustrate how to formalize the polynomial

$$\begin{aligned}q\sigma^q &= q_\omega(\omega,\eta)\omega\sigma^\omega + q_\eta(\omega,\eta)\eta\sigma^\eta \\ \frac{q\sigma^q}{\sigma+\sigma_t^q} &= q_\omega\omega\big(\alpha b_t^0 + (1-\alpha)b_t^\lambda(1-\lambda) + (1-\alpha)l_t(1-\lambda) + m_t\big) + q_\eta\eta(x_t^j + \lambda x_t + m_t - 1) \\ q^2\sigma^2 &= (q - q_\omega\omega\big(\alpha b_t^0 + (1-\alpha)b_t^\lambda(1-\lambda) + (1-\alpha)l_t(1-\lambda) + m_t\big) - q_\eta\eta(x_t^j + \lambda x_t + m_t - 1))^2(\sigma+\sigma^q)^2.\end{aligned}$$

Notice that I can express b^0, b^λ, l, x^j, x, and $(\sigma+\sigma^q)^2$ as polynomial functions of q by rearranging and combining the market clearing condition and optimality conditions of different agents. I calculate $q_\omega(\omega,\eta)$ according

$$\frac{q_{i,j} - q_{i-1,j}}{\omega_{i,j} - \omega_{i-1,j}}$$

where $q_{i,j}$ denotes $q(\omega_{i,j},\eta_{i,j})$. And, the "upwind scheme" will dictate whether I use forward or backward difference to calculate $q_\eta(\omega,\eta)$. If $x_t^j + \lambda x_t + m_t \leq 1$, then I use backward difference and start the update of $q_{i,j}$ from $j = 2$ towards $\Psi(i)$. As the updating of $q_{i,j}$ proceeds, $x_t^j + \lambda x_t + m_t$ will be less than one if $\eta_{i,j}$ is large enough. Then, I will update $q_{i,j}$ starting from $\Psi(i) - 1$ towards $j = 2$.

C Global Dynamics

In this section, I briefly review the property of the global dynamics in an economy. Notice that while solving for the equilibrium object $q(\omega, \eta)$, we also obtain the value of all other endogenous variables as functions of the two state variables. Figures 1 and 2 in the online appendix of the paper show the solution of fifteen key endogenous variables including the drift and volatility terms of the two state variables. Hence, we know the exact global dynamics of the economy, i.e., the laws of motion for ω and η, which are depicted in equations (21) and (22).

The top plots in Figure 1 of the online appendix show that if the productive experts' wealth share rises, they will hold more physical capital. Consequently, the price of physical capital as well as the investment in physical capital increase. Since bankers lower the financing costs of risky experts, the price of physical capital and the other two relevant terms also increase when bankers' wealth share rises. The middle right panel in Figure 1 of the online appendix depicts the scenario when experts' wealth share is relatively small; the volatility of the price of physical capital is high when the magnitude of asset fire-sale is large. The same plot reveals another interesting fact: the increase in bankers' wealth share does not necessarily mitigate the financial amplification (see the bottom right corner of the plot). The intuition is as follows. The magnitude of asset fire-sale ultimately depends on the real sector. When experts' wealth share is low, excess supply of bank credit allows the real sector to take excess leverage, and amplifies asset fire-sale effects.

The bottom middle panel in Figure 1 of the online appendix shows that the leverage of risky firms highly depends on bankers' wealth share as risky firms depend mainly on bank credit for external financing. The bottom right panel in Figure 1 of the online appendix shows that in the case where risky firms' demand for bank loans is still high, bank leverage naturally declines when bankers' wealth share declines. The top left panel in Figure 2 of the online appendix shows that risky firms only issue bonds when the banking sector is poorly capitalized. In this scenario, experts' wealth share is low, bankers' wealth share is high, and the net interest spread is high (the top middle panel in Figure 2 of the online appendix). The price volatility of physical capital is high, which raises bankers' exposure to aggregate risks.

The density of the stationary distribution of (ω, η) is displayed by the bottom right panel in Figure 2 of the online appendix. The economy considered in our numerical example is mainly anchored the states where $\omega = 0.054$ and $\eta = 0.021$.