Bond Finance, Bank Finance, and Bank Regulation

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Abstract

In this paper, I build a continuous-time macro-finance model in which firms can access both bond credit and bank credit. The model captures the simple idea that the presence of bond financing lowers the price elasticity of demand for bank loans. I find that the optimal capital adequacy ratio is quantitatively sensitive to the presence of bond financing and that models would overstate the banking sector's recovery rate if they omit bond financing. Furthermore, the model highlights that an economy's optimal capital requirement highly depends on the efficiency of its bankruptcy procedure and the risk profile of its real sector.

Keywords: bank credit, bond credit, capital requirement, and macro-prudential regulation

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Introduction

Like bank loans, bond finance is an important source of external credit for firms. For instance, during the 2007-2009 financial crisis when the supply of bank loans declined substantially, firms, especially those with relatively high credit ratings, largely substituted bank credit with bond credit (Adrian, Colla and Shin, 2012). Nevertheless, bond finance is absent in many papers that quantitatively examines the role of bank capital requirement as a macro-prudential policy, e.g., Van den Heuvel (2008), Begenau (2019), Elenev, Landvoigt and Van Nieuwerburgh (2018).[1] In this paper, I show that it could raise the optimal capital requirement from 4% to 24% by closing down the bond market in a quantitative general equilibrium model. In addition, this paper highlights that the socially optimal level of the capital ratio requirement largely depends on the efficiency of the bankruptcy system and the risk profile of the real sector in an economy because both factors affect the aggregate demand for bank credit.

I propose a continuous-time macro-finance framework with a productive expert sector, an unproductive household sector, and an explicit banking sector. The production sector comprises safe firms and risky firms. Both types of firms can access the bond market and the loan market. The difference between bond finance and bank finance is that banks can liquidate troubled firms' assets in a more efficient fashion (Bolton and Freixas, 2000). The net interest spread charged by banks compensates for their exposure to the aggregate risk that they assume via loan lending. Households can both hold corporate bonds directly and deposit their savings into banks.

In my framework, risky firms prefer bank credit while safe firms rely mainly on bond credit. Since banks can liquidate troubled firms' assets in a more efficient way, banks request less compensation for bankruptcy costs relative to bondholders. The liquidation efficiency of bank credit is more important for risky firms than for safe firms because safe firms are less likely to face costly liquidation. This setting is consistent with empirical findings in Rauh and Sufi (2010) and Becker and Josephson (2016). Bank credit does not always dominate bond credit for a risky firm because the firm must pay a premium for its bank lender's exposure to aggregate risks. If the premium is too high, the risky firm will replace bank finance with bond finance. The risk premium in the model is the net interest spread earned by banks.

The net interest spread depends on the leverage of the intermediary sector, the aggregate risk of the economy, and the capital requirement faced by banks. Given the same amount of aggregate risk, banks with low leverage have low risk exposure. Therefore, the risk premium required by banks tends to be low. Hence, bank credit is relatively cheap when the banking sector has adequate equity capital. The capital ratio requirement also affects the net interest spread because a tightening of the capital requirement would lower the supply of bank loans. When there is excess demand for bank loans, the loan spread increases, as does the net interest spread earned by banks.

The impacts of exogenous aggregate shocks on the economy vary over time because the effects of

[1] See Thakor (2014) for a review of the literature on the capital ratio requirement using microeconomic models of banking.

financial amplification depend on the balance sheets of both banks and experts (Bernanke, Gertler and Gilchrist, 1999; Kiyotaki and Moore, 1997). Suppose a series of adverse shocks hit the economy. Both bank capital and productive experts' net worth decline disproportionately due to their use of leverage. As a result, the supply of bank loans shrinks, leading to a decrease in experts' holdings of assets, aggregate productivity, and asset prices. The depreciation of asset prices hurts balance sheets of both banks and experts, and further lowers the loan supply and experts' holdings of assets. I label the effect of the financial amplification as endogenous risk.

The first key result of this paper concerns economic dynamics. In a model without bond financing, the predicted recovery of the economy after a negative shock is overly swift. Suppose the net worths of both the real sector and the banking sector deteriorate due to a negative shock. Since bank capital decreases, the supply of bank loans is limited and the loan spread must increase. If loans are the only source of external finance that can be accessed by the real sector, then the demand for bank loans is not very elastic. The rise in loan spreads leads to the substantial increase in bank profitability in the absence of bond financing. Therefore, the banking sector recovers more quickly after adverse shocks in a model that omits bond financing than it would in a model with bond financing. As does the entire economy.

Bank regulation in my framework can improve social welfare because my model is subject to pecuniary externalities that are common in incomplete market models (Hart, 1975; Stiglitz, 1982; Geanakoplos and Polemarchakis, 1986). In particular, experts and bankers in my model do not internalize the impact of their leverage decisions on asset prices and endogenous risks. Hence, bank regulation such as the capital ratio requirement can adjust bankers' leverage, lower the loan supply, and raise the net interest spread. In this way, bank regulation can increase the profitability of banking and strengthen the banking sector to lower endogenous risks and improve social welfare.

The second key result of this paper is that a model that omits bond financing overemphasizes the benefit of the bank capital requirement. The intuition is also related to the elasticity of the aggregate demand for bank loans. If the capital requirement tightens, there will be excess demand for bank loans. Thus, loan spread increases and loan demand declines. If the magnitude of the decline in loan demand is small enough, bank profitability could increase, and the banking sector can expand after accumulating more and more profits. A larger banking sector can provide more credit for the real sector and raise aggregate productivity. These are the ways in which tightening capital requirement improves social welfare. Consider two otherwise identical economies: one has a bond market and the other does not. Obviously, the aggregate demand for bank loans is much more elastic in the economy where firms can issue bonds. In this economy, when loan spread increases, the demand for bank loans declines more substantially, as does bank profitability. Therefore, tightening capital requirement is more likely to cause the banking sector to shrink, and social welfare to decline. Hence, the optimal capital ratio requirement should be more lenient if I consider a model that allows for bond financing.

The key quantitative result of the calibrated model is that the optimal capital ratio requirement is not robust to whether the real sector can obtain bond credit. The calibrated model with bond financing suggests that the current capital ratio requirement is too stringent. This is a natural result since my model highlights an additional negative effect of raising capital adequacy ratio. If I close the bond market, however, the otherwise identical model suggests that it is optimal to raise the current capital requirement by more than three times.

The previous discussion shows that the loan spread elasticity of the demand for bank loans plays a crucial role in the welfare implication of capital requirement. In light of this property, I explore two factors that affect the elasticity of bank loan demand: the efficiency of the bankruptcy system in an economy and firms' idiosyncratic default risks. The more efficiently bankruptcy cases are processed, the smaller the advantage that banks have over bondholders in terms of liquidating insolvent firms' assets. In an efficient bankruptcy system, bondholders enjoy higher recovery value ex post and request smaller premium ex ante. From the perspective of firms, replacing bank credit with bond credit is less costly, and thus firms' demand for bank loans is more price elastic. Hence, tightening capital requirement can cause a substantial decline in bank loans, and a decrease in bank profits. Overall, the optimal capital requirement should be more lenient in an economy with a more efficient bankruptcy system.

Firms' average default risk also influences the elasticity of demand for bank loans. Since bondholders demand higher default premium for firms that are more likely to fail, riskier firms find it more costly to switch from bank credit to bond credit. Hence, the demand for loans is less elastic if firms in an economy tend to be risky. Subsequently, the optimal capital requirement should be more stringent.

Related Literature. My paper is related to four strands of literature. First, my paper contributes to the literature that investigate the optimal level of capital requirement in a dynamic general equilibrium framework (Repullo and Suarez, 2012; Christiano and Ikeda, 2013; Martinez-Miera and Suarez, 2014; Nguyen, 2014; Derviz et al., 2015; Phelan, 2016; Davydiuk, 2017; Corbae and D'Erasmo, 2018; Mendicino et al., 2018; Pancost and Robatto, 2018). In this literature, capital requirement could mitigate banks' excessive risk-taking induced by government guarantees such as the deposit insurance scheme (e.g., Van den Heuvel 2008, Begenau 2019, and Dempsey 2018), or their excessive leverage-taking that comes with pecuniary externalities in incomplete markets (e.g., Lorenzoni 2008, Bianchi 2011, and Phelan 2016). My model falls to the second category.

Different from most papers in the literature, I explicitly compare two otherwise identical economies with and without bond financing to demonstrate the effects of opening up the bond market. In this regard, my paper is similar to Pancost and Robatto (2018) that investigate the impact of firms' bank deposits on optimal capital requirement. Two recent papers, Dempsey (2018) and Xiang (2018), also acknowledge the role of bond finance for bank capital requirements. Different from the two papers, experts' wealth share is an important state variable in my model, which shows that the models without bond financing tend to overstate the recovery of the real sector.

Second, the spirit of my paper is close to the shadow banking literature, which emphasizes that banking activities could migrate to the shadow banking sector if regulation tightens (Plantin, 2014;

Begenau and Landvoigt, 2018; Huang, 2018).

Third, I use a continuous-time macro-finance framework that emphasizes the financial amplification mechanism (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2012; Di Tella, 2017). The major contribution of this paper is that I explicitly model a financial intermediary sector rather than grouping the real sector and financial intermediary sector together. With my proposed framework, I can explicitly analyze the macroeconomic implications of bank regulation. This framework highlights two layers of financial amplification — one at the firm level and the other at the intermediary level.

Finally, my paper contributes to a strand of macroeconomic literature that highlights the capital structure of firms (see, e.g., De Fiore and Uhlig 2011, 2015; Crouzet 2017). These papers treat the surge in the cost of bank financing as an exogenous shock. In my paper, the cost of intermediated bank credit fluctuates endogenously and bank finance and bond finance interact in a dynamic fashion. In this regard, my paper is related to Rampini and Viswanathan (2018), who also endogenize the cost of financial intermediation. However, their focus is not on the substitution between bank credit and bond credit. My paper shows that the dynamics of both the real sector and the intermediary sector would be significantly different if bond financing is absent in an economy.

The structure of the rest of the paper is as follows. Section 1 describes the set-up of the model and defines the equilibrium. In Section 2, I characterize the optimal choice of individual agents and the Markov equilibrium. The calibrated model highlights that the presence of bond financing has distinctive impacts on an economy's dynamics. Section 3 shows that the optimal level of capital requirement depends heavily on the existence of a bond market, its development, and the distribution of borrowers' risk characteristics. Section 4 concludes.

1 Model

In this section, I build an infinite-horizon continuous-time general equilibrium model, in which firms can issue corporate bonds as well as raise credit via financial intermediaries. The economy has two types of goods: perishable final goods (the numéraire) and durable physical capital goods. Three types of agents populate the economy: experts, bankers, and households. All agents have the same logarithmic preferences and the same time discount factor ρ. None of them accepts negative consumption. Although all three types of agents are able to hold physical capital goods, only experts are productive while bankers specialize in financial intermediation. Like typical models with financial frictions, I assume that experts (bankers) become households at rate χ (χ_η) so that they would not take over all the wealth in the economy.[2]

[2] In addition, I need an assumption that households turn into experts and bankers at the relevant rates so that the measure of each group does not change over time. However, I dropped this additional assumption from the main text, since it is the wealth shares of each group rather than the absolute measure that serve as the endogenous state variables.

1.1 Technology

In period t, an expert can produce ak_t units of final goods with k_t efficiency units of physical capital over a short period of time. Both households and bankers are unproductive. All three types of agents can convert $\iota_t k_t$ units of final goods into $k_t \Phi(\iota_t)$ units of physical capital, where

$$\Phi(\iota_t) = \frac{\log(\iota_t \phi + 1)}{\phi}.$$

Thus, there is technological illiquidity on the production side. In each period, physical capital in the possession of experts and households depreciates at rate δ and physical capital in the possession of bankers at δ^b.[3]

Exogenous *aggregate* shock is driven by a Brownian motion $\{Z_t, t \geq 0\}$. In the absence of any idiosyncratic shock, physical capital managed by an expert or a household evolves according to

$$\frac{\mathrm{d}k_t}{k_t} = \mu_t^k \mathrm{d}t + \sigma \mathrm{d}Z_t, \tag{1}$$

where $\mu_t^k \equiv \Phi(\iota_t) - \delta$ and σ is a positive constant that captures the direct impact of the exogenous shock on physical capital. Physical capital managed by bankers follows the same law of motion except that the capital depreciation rate is replaced by δ^b. The market for physical capital is frictionless. I conjecture that the law of motion for the equilibrium price of physical capital is

$$\frac{\mathrm{d}q_t}{q_t} = \mu_t^q \mathrm{d}t + \sigma_t^q \mathrm{d}Z_t, \tag{2}$$

where both μ_t^q and σ_t^q are equilibrium objects that will be solved.

To capture firm heterogeneity without raising the number of state variables, I assume that an expert becomes a *safe* expert with probability α or a *risky* expert with probability $1 - \alpha$ at the beginning of each period. Whether an expert becomes risky in a period is independent across the time. Given that a risky firm (i.e., a firm managed by a risky expert) has raised external credit, it is subject to a firm-level default shock that occurs with probability λ. If the default shock hits the firm, its production stalls and its owner could potentially steal physical capital under the cover of the adverse shock. If the shock does not occur, the firm's productivity is assumed to be $\tilde{a}$ such that $(1 - \lambda)\tilde{a} = a$, i.e., risky firms' average productivity is the same as safe firms. To diversify the firm-specific adverse shock, a risky expert establishes an infinite number of firms. Safe firms do not experience such adverse idiosyncratic shocks.

1.2 Corporate Bond, Bank Loan, and Liquidation

A firm can raise credit by issuing corporate bonds and obtaining a bank loan. In addition, assume that no firm can issue outside equity, and all firms have limited liability.

[3] Bankers' capital depreciation rate is different to match certain moments. See Section 2.7 for details of the model calibration.

Both corporate bonds and bank loans are *short-term* collateralized debt. Collateralized borrowing means that if a firm raises 1 dollar, it must put down physical capital worth 1 dollar as collateral. Since the firm owner could steal physical capital when default occurs, creditors seize the collateral in the event of default and resell the asset immediately at the market price. Nevertheless, the liquidation process accelerates the depreciation of physical capital. If banks liquidate the collateral, the depreciation rate increases by κ; if it is bondholders, the rate rises by κ^d, where $\kappa < \kappa^d$. Bondholders are assumed to be less efficient than banks in managing liquidation because it is harder and more time-consuming to achieve a collective decision for a number of bondholders than for a single bank (Bolton and Freixas, 2000).

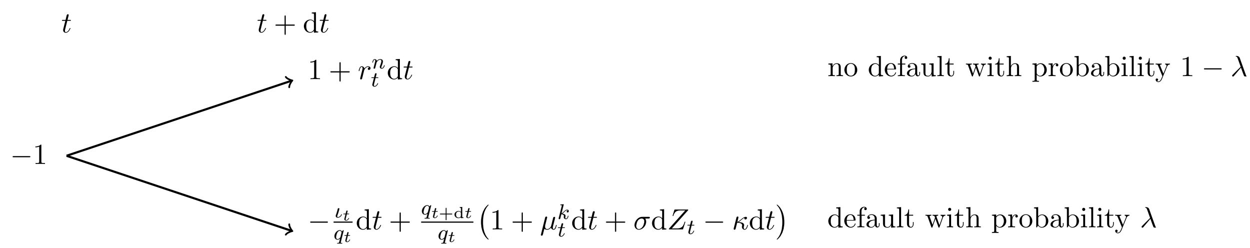

Figure 1: Debt payoff

Next, I will use Figure 1 illustrate the payment structure of 1 dollar lent to a risky firm by a bank, where r_t^n denotes the nominal interest rate. The collateral of 1 dollar lending is $1/q_t$ units of capital at time t. If default occurs, the collateral available becomes $\big(1+\mu_t^k \mathrm{d}t+\sigma \mathrm{d}Z_t-\kappa \mathrm{d}t\big)/q_t$ given the capital production with expense $(\iota_t \mathrm{d}t)/q_t$, the law of motion for physical capital (1) and the liquidation cost κ.

Given that a bank can lend to a large number of risky firms, the average payment of such 1 dollar lending is

$$1 + r_t^\lambda \mathrm{d}t + \lambda(\sigma + \sigma_t^q)\mathrm{d}Z_t, \text{where } r_t^\lambda \equiv (1-\lambda)r_t^n + \lambda\bigg(-\frac{\iota_t}{q_t} + \mu_t^q + \mu_t^k + \sigma\sigma_t^q\bigg) - \lambda\kappa$$

where I use Ito's Lemma for the derivation. r_t^λ is the expected rate of return for bank loans, which is supposed to compensate banks' risk exposure to the aggregate risk $\lambda(\sigma + \sigma_t^q)\mathrm{d}Z_t$.

Assume that a passive mutual fund serves as the intermediary in the bond market. Given the nominal interest rate of bonds $\tilde{r}_t^n$, similar derivation yields the average rate of return the fund earns

$$1 + \Bigg((1-\lambda)\tilde{r}_t^n + \lambda\bigg(-\frac{\iota_t}{q_t}\mu_t^q + \mu_t^k + \sigma\sigma_t^q\bigg) - \lambda\kappa^d\Bigg)\mathrm{d}t + \lambda(\sigma + \sigma_t^q)\mathrm{d}Z_t$$

For simplicity, assume that the bond mutual fund sets the nominal interest $\tilde{r}_t^n$ such that the expected rate of return it earns equals the risk-free rate r_t it pays to its investors, i.e.,

$$r_t = (1-\lambda)\tilde{r}_t^n + \lambda\bigg(-\frac{\iota_t}{q_t} + \mu_t^q + \mu_t^k + \sigma\sigma_t^q\bigg) - \lambda\kappa^d.$$

Any profit or loss realized by the mutual fund is driven by the aggregate shock Z_t. Assume that the profit or loss realized in each period is instantly shared by all agents via lump-sum transfers.

1.3 The Expert's Problem

The instant return of holding physical capital for an expert R_t consists of final and capital good productions and capital gains, i.e.,

$$R_t \mathrm{d}t + (\sigma + \sigma_t^q)\mathrm{d}Z_t, \text{ where } R_t \equiv \frac{a - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q,$$

Since costly liquidation does not happen to a **safe expert**, without loss of generality we assume that the safe expert raises external funds only through bond financing, and thus his/her net worth w_t^s evolves according to

$$\frac{\mathrm{d}w_t^s}{w_t^s} = R_t \mathrm{d}t + (\sigma + \sigma_t^q)\mathrm{d}Z_t + b_t^0\big((R_t - r_t)\mathrm{d}t + (\sigma + \sigma_t^q)\mathrm{d}Z_t\big) + m_t(\sigma + \sigma_t^q)\mathrm{d}Z_t - \frac{c_t \mathrm{d}t}{w_t^s}. \tag{3}$$

where b_t^0 is the bond-to-net-worth ratio and $m_t(\sigma + \sigma_t^q)\mathrm{d}Z_t$ denotes the lump-sum transfer from the bond mutual fund per dollar. The first block of the above equation's right hand side represents the return of capital holding funded with inside equity and the second block funded with bond credit.

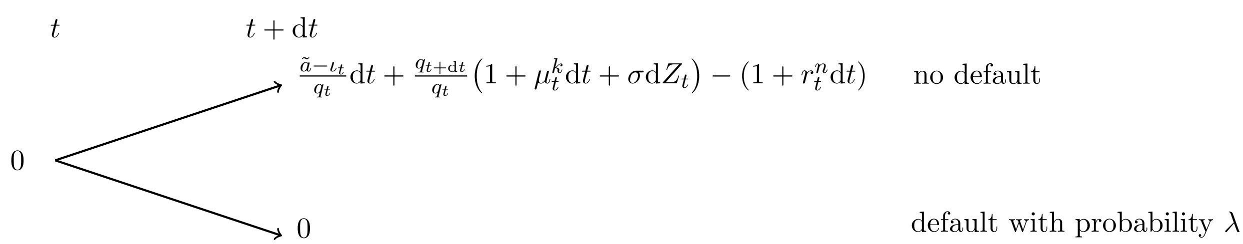

Figure 2: Risky firm's payoff

A **risky expert** will choose among corporate debt, bank loans, and self-financing. Since all of the expert's firms are identical prior to the realization of the default shock, the financing decisions of all firms managed by the expert are the same. Thus, the debt-to-equity ratios of these firms are also the same, which is exactly the expert's debt-to-net-worth ratio.

Next, I will derive the average excess return of 1 dollar raised from bank financing as shown by Figure 2. Given the nominal interest rate derived in the previous section, the average excess return of 1 dollar investment financed by bank financing is

$$\begin{aligned}&\frac{(1-\lambda)\tilde{a}}{q_t}\mathrm{d}t + (1-\lambda)\bigg(-\frac{\iota_t}{q_t}\mathrm{d}t + \frac{q_{t+\mathrm{d}t}}{q_t}\big(1 + \mu_t^k \mathrm{d}t + \sigma \mathrm{d}Z_t\big) - (1 + r_t^n \mathrm{d}t)\bigg)\\ =&\big(R_t - \lambda\kappa - r_t^\lambda\big)\mathrm{d}t + \big(1-\lambda\big)(\sigma + \sigma_t^q)\mathrm{d}Z_t.\end{aligned}$$

By the Law of Large Numbers, creditors seize a proportion λ of the expert's physical capital due to default. As a result, the risky expert partially unloads his/her exposure to the aggregate risk,

$\lambda(\sigma+\sigma_t^q)\mathrm{d}Z_t$. Similarly, the average excess return of investment funded by bond credit is

$$\big(R_t-\lambda\kappa^d-r_t\big)\mathrm{d}t+\big(1-\lambda\big)(\sigma+\sigma_t^q)\mathrm{d}Z_t.$$

The law of motion for the risky expert's net worth w_t^r is

$$\begin{aligned}\frac{\mathrm{d}w_t^r}{w_t^r}&=R_t\mathrm{d}t+(\sigma+\sigma_t^q)\mathrm{d}Z_t+b_t^\lambda\Big(\big(R_t-\lambda\kappa^d-r_t\big)\mathrm{d}t+\big(1-\lambda\big)(\sigma+\sigma_t^q)\mathrm{d}Z_t\Big)\\&\quad+l_t\Big(\big(R_t-\lambda\kappa-r_t^\lambda\big)\mathrm{d}t+\big(1-\lambda\big)(\sigma+\sigma_t^q)\mathrm{d}Z_t\Big)+m_t(\sigma+\sigma_t^q)\mathrm{d}Z_t-\frac{c_t\mathrm{d}t}{w_t^r},\end{aligned}\tag{4}$$

where b_t^λ is the firms' bond-to-equity ratio, and l_t is the firm's loan-to-equity ratio. Similar to equation 3, the second and third parts of equation 4 are returns of capital holdings financed by bond credit and bank credit, respectively.

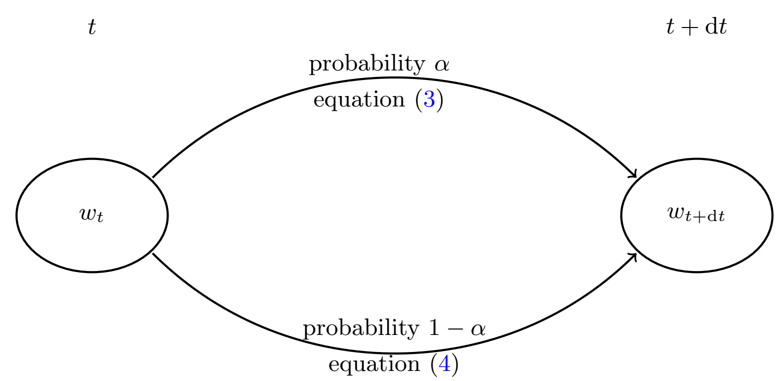

Figure 3: dynamics of an expert's net worth

Taking $\{q_t, r_t, r_t^\lambda, m_t, t\geq 0\}$ as given, an expert chooses $\{c_t, b_t^0, b_t^\lambda, l_t, t\geq 0\}$ to maximize his/her life-time expected utility

$$E_0\left[\int_0^T e^{-\rho t}\ln(c_t)\,\mathrm{d}t+e^{-\rho T}J^h(W_T)\right],$$

given that his/her net worth evolves in each period according to either equation (3) or (4) depending on his/her type realized when entering the period (see Figure 3), where T is the stopping time when the expert turns into a household and $J^h(W_T)$ denotes the life-time expected utility of the household given the net worth W_T.

1.4 The Banker's Problem

The instant rate of return from holding physical capital for a banker is

$$R_t^b\mathrm{d}t+(\sigma+\sigma_t^q)\mathrm{d}Z_t,\ \text{ where } R_t^b\equiv-\frac{\iota_t}{q_t}+\Phi(\iota_t)-\delta^b+\mu_t^q+\sigma\sigma_t^q.$$

Therefore, a banker's net worth n_t evolves according to

$$\mathrm{d}n_t = n_t x_t^j \big(R_t^b \mathrm{d}t + (\sigma + \sigma_t^q)\mathrm{d}Z_t\big) + n_t x_t \big(r_t^\lambda \mathrm{d}t + \lambda(\sigma + \sigma_t^q)\mathrm{d}Z_t\big) + n_t(1 - x_t^j - x_t) r_t \mathrm{d}t \qquad (5)$$
$$+ n_t m_t (\sigma + \sigma_t^q)\mathrm{d}Z_t - c_t \mathrm{d}t,$$

where x_t^j denotes the physical-capital-to-equity ratio and x_t the loan-to-equity ratio for the bank. When $x_t > 1$, the bank absorbs deposits, and transfers funds from households to experts. When $x_t \leq 1$, the bank puts some of its equity capital in the bond mutual fund. The banker is exposed to the aggregate risk $n_t x_t^\lambda \lambda(\sigma + \sigma_t^q)\mathrm{d}Z_t$ because he or she takes over and resell the physical capital that backs his/her lending. I consider the time-invariant capital ratio requirement, which imposes an upper bound on banks' loan-to-equity ratio, that is, $x_t \leq \bar{x}$.[4] Taking $\{q_t, r_t, r_t^\lambda, m_t, t \geq 0\}$ as given, a banker chooses $\{c_t, x_t^j, x_t^\lambda, t \geq 0\}$ to maximize his/her life-time expected utility

$$E_0\left[\int_0^T e^{-\rho t} \ln(c_t)\, \mathrm{d}t + e^{-T\rho} J^h(W_T)\right],$$

subject to the dynamic budget constraint (5) and the capital ratio requirement, where T is the stopping time when the banker turns into a household and $J^h(W_T)$ denotes the life-time expected utility of the household.

1.5 The Household's Problem

The rate of return from holding physical capital for a household is

$$R_t^h \mathrm{d}t + (\sigma + \sigma_t^q)\mathrm{d}Z_t, \text{ where } R_t^h \equiv -\frac{\iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q.$$

The law of motion for a household's net worth w_t^h is

$$\mathrm{d}w_t^h = w_t^h x_t^h (R_t^h \mathrm{d}t + (\sigma + \sigma_t^q)\mathrm{d}Z_t) + w_t^h (1 - x_t^h) r_t \mathrm{d}t + w_t^h m_t (\sigma + \sigma_t^q)\mathrm{d}Z_t - c_t, \qquad (6)$$

where x_t^h is the portfolio weight of physical capital. Taking $\{q_t, r_t, m_t, t \geq 0\}$ as given, a household maximizes the life-time expected utility

$$J^h(w_0^h) \equiv E_0\left[\int_0^\infty e^{-\rho t} \ln(c_t)\, \mathrm{d}t\right],$$

by choosing $\{c_t, x_t^h, t \geq 0\}$ that satisfy the dynamic budget constraint (6).

[4] Bankers are much less productive than experts. Hence, bankers hold physical capital only when their wealth share is close to one, and they take on no leverage. Therefore, it is with no loss of generality to assume that the capital ratio requirement only imposes restriction on banks' loan portfolio.

1.6 Equilibrium

The aggregate shock $\{Z_t, t \geq 0\}$ drives the evolution of the economy. $\mathbf{I} = [0, 1)$ denotes the set of experts, $\mathbf{J} = [1, 2)$ the set of bankers, and $\mathbf{H} = [2, 3]$ the set of households. Given the idiosyncratic shock in period t, $\mathbf{I}_t^s$ denotes the set of safe experts in period t and $\mathbf{I}_t^r$ the set of risky experts.

Definition 1 *Given the capital ratio requirement $1/\bar{x}$ and the initial endowments of physical capital $\left\{k_0^i, k_0^j, k_0^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\right\}$ possessed by experts, bankers, and households such that*

$$\int_0^1 k_0^i \mathrm{d}i + \int_1^2 k_0^j \mathrm{d}j + \int_2^3 k_0^h \mathrm{d}h = K_0,$$

an equilibrium is defined by a set of stochastic processes adapted to the filtration generated by $\{Z_t, t \geq 0\}$: the price of physical capital $\{q_t\}_{t=0}^{\infty}$, the risk-free rate $\{r_t\}_{t=0}^{\infty}$, the effective rate of bank loans $\{r_t^\lambda\}_{t=0}^{\infty}$, wealth $\left\{W_t^i, N_t^j, W_t^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\right\}_{t=0}^{\infty}$, investment decisions $\left\{\iota_t^i, \iota_t^j, \iota_t^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\right\}_{t=0}^{\infty}$, asset holding decisions $\left\{x_t^j, x_t^h, j \in \mathbf{J}, h \in \mathbf{I}_t^h\right\}_{t=0}^{\infty}$ of bankers and households, corporate debt financing decisions $\left\{b_t^{i,0}, b_t^{i,\lambda}, i \in \mathbf{I}_t\right\}_{t=0}^{\infty}$ of experts, bank financing decisions $\{l_t^i, i \in \mathbf{I}_t^r\}_{t=0}^{\infty}$ of risky experts, bank lending $\{x_t^{\lambda,j}, j \in \mathbf{J}\}_{t=0}^{\infty}$ and consumption $\left\{c_t^i, c_t^j, c_t^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\right\}_{t=0}^{\infty}$; such that

1. *$W_0^i = k_0^i q_0$, $N_0^j = k_0^j q_0$, and $W_0^h = k_0^h q_0$ for $i \in \mathbf{I}$, $j \in \mathbf{J}$, and $h \in \mathbf{H}$;*
2. *Each expert, banker, and household solves for his/her problem given prices and the capital adequacy ratio for bankers;*
3. *Markets for final goods and physical capital clear, that is,*

$$\begin{aligned}\int_0^3 c_t^i \mathrm{d}i &= \frac{1}{q_t}\int_1^2 (a^b - \iota_t^j) n_t^j x_t^j \mathrm{d}j + \frac{1}{q_t}\int_2^3 (a^h - \iota_t^h) w_t^h x_t^h \mathrm{d}h + \\ &\quad \frac{1}{q_t}\int_{i\in\mathbf{I}_t^s} \left(a - \iota_t^i\right) w_t^i (1 + b_t^{i,0}) \mathrm{d}i + \frac{1}{q_t}\int_{i\in\mathbf{I}_t^r} \left(a - \iota_t^i\right) w_t^i (1 + b_t^{i,\lambda} + l_t^i) \mathrm{d}i\end{aligned}$$

for the market of final goods, and

$$\frac{1}{q_t}\int_{i\in\mathbf{I}_t^s} w_t^i(1 + b_t^{i,0})\mathrm{d}i + \frac{1}{q_t}\int_{i\in\mathbf{I}_t^r} w_t^i(1 + b_t^{i,\lambda} + l_t^i)\mathrm{d}i + \frac{1}{q_t}\int_1^2 n_t^j x_t^j \mathrm{d}j + \frac{1}{q_t}\int_2^3 w_t^h x_t^h \mathrm{d}h = K_t$$

for the market of physical capital goods, where K_t evolves according to

$$\begin{aligned}\mathrm{d}K_t &= \frac{1}{q_t}\int_1^2 \left(\Phi(\iota_t^j) - \delta^b\right) n_t^j x_t^j \mathrm{d}j + \frac{1}{q_t}\int_2^3 \left(\Phi(\iota_t^h) - \delta\right) w_t^h x_t^h \mathrm{d}h \\ &+ \frac{1}{q_t}\int_{i\in\mathbf{I}_t^s} \left(\Phi(\iota_t^i) - \delta\right) w_t^i (1 + b_t^{i,0})\, \mathrm{d}i \\ &+ \frac{1}{q_t}\int_{i\in\mathbf{I}_t^r} \left(\Phi(\iota_t^i) - \delta\right) w_t^i (1 + b_t^{i,\lambda} + l_t^i) - \lambda\kappa^d w_t^i b_t^i - \lambda\kappa w_t^i l_t^i \,\mathrm{d}i + K_t \sigma \mathrm{d}Z_t.\end{aligned}$$

4. *The bank loan market clears:*

$$\int_{i\in\mathbf{I}_t^r} w_t^i l_t^i \mathrm{d}i = \int_1^2 n_t^j x_t^{\lambda,j} \mathrm{d}j.$$

5. *The bond mutual fund assumes no gains or losses, i.e., the lump-sum transfer between the mutual fund and all agents perfectly hedges the fund's risk exposure to the aggregate risk*

$$\int_0^1 w_t^i m_t \mathrm{d}i + \int_1^2 n_t^j m_t \mathrm{d}j + \int_2^3 w_t^h m_t \mathrm{d}h = \int_{i\in\mathbf{I}_t^r} \lambda b^\lambda w^i \mathrm{d}i.$$

The credit market for corporate bonds clears automatically by Walras' Law.

2 Solving for the Equilibrium

Both experts' net worth and bank capital are crucial for the allocation of physical capital and financial resources in the equilibrium. I expect the price of physical capital to decline as experts' net worth and bank capital shrink.

To solve for the equilibrium, I first derive first-order conditions with respect to the optimal decisions of experts, bankers, and households. Next, I solve for the law of motion for endogenous state variables, wealth shares of experts and bankers based on market clearing conditions and first-order conditions. Lastly, I use first-order conditions and state variables' law of motion to establish a partial differential equation (PDE) that the price of physical capital satisfies. The solution of the PDE offers the full characterization of the equilibrium dynamics. At the end of this section, I will calibrate the model and highlight that economic dynamics would be significantly different if the bond market is shut down.

2.1 Households' Optimal Choices

Households have logarithmic preferences. In the following discussion, I will take advantage of two well-known properties of the logarithmic preference in the continuous-time setting: (i) a household's consumption c_t is ρ proportion of her wealth w_t^h, i.e.,

$$c_t = \rho w_t^h; \tag{7}$$

(ii) the Sharpe ratio of the risky investment equals the percentage volatility of the household's net worth implied the optimal portfolio choice.

A household's investment rate ι_t always maximizes $\Phi(\iota_t) - \iota_t/q_t$. The first-order condition implies that

$$\Phi'(\iota_t) = \frac{1}{q_t}, \tag{8}$$

which defines the optimal investment as a function of the price of physical capital $\iota(q_t)$.

Given the second property, it is straightforward to derive a household's optimal portfolio weight on physical capital x_t^h, which satisfies[5]

$$x_t^h + m_t \geq \frac{R_t^h - r_t}{(\sigma + \sigma_t^q)^2} \text{ with equality if } x_t^h > 0. \tag{9}$$

2.2 Experts' Portfolio Choices

According to the second property highlighted above, it is straightforward to characterize a safe expert's optimal bond-to-equity ratio[6]

$$1 + b_t^0 + m_t \geq \frac{R_t - r_t}{(\sigma + \sigma_t^q)^2} \text{ with equality if } b_t^0 > 0. \tag{10}$$

For a risky expert, both bond-to-equity ratio b_t^λ and loan-to-equity ratio l_t affect the percentage volatility of his/her wealth $(1 + (1-\lambda)b_t^\lambda + (1-\lambda)l_t + m_t)(\sigma + \sigma_t^q)$. Hence, optimal b_t^λ and l_t must satisfy

$$1 + (1-\lambda)b_t^\lambda + (1-\lambda)l_t + m_t \geq \frac{R - \lambda\kappa^d - r_t}{(1-\lambda)(\sigma + \sigma_t^q)^2} \text{ with equality if } b_t^\lambda > 0; \tag{11}$$

$$1 + (1-\lambda)b_t^\lambda + (1-\lambda)l_t + m_t \geq \frac{R - \lambda\kappa - r_t^\lambda}{(1-\lambda)(\sigma + \sigma_t^q)^2} \text{ with equality if } l_t > 0. \tag{12}$$

When the cost of bond financing equals the cost of bank financing, i.e., $\lambda\kappa^d + r_t = \lambda\kappa + r_t^\lambda$, individual risky experts are indifferent between bond financing and bank financing, and their portfolio choices are indeterminate. Without loss of generality, I assume that portfolio weights of both bond-financing and bank-financing, b_t^λ and l_t, are the same across all risky experts.

If the financing cost is too high or the return from holding physical capital R_t is too low, self-financing may become the only optimal choice for risky firms, i.e., $l_t = b_t^\lambda = 0$. As an expert's type shifts between being safe and risky constantly, his/her leverage moves discontinuously over time. However, this does not create any technical challenge for solving the dynamic programming problem because firms resell physical capital to repay debt every period and the key state variable for the expert W_t still has a continuous path.

[5]Sharpe ratio is $(R_t^h - r_t)/(\sigma + \sigma_t^q)$. The percentage volatility of the household's wealth is $(x_t^h + m_t)(\sigma + \sigma_t^q)$.

[6]In this case, the Sharpe ratio is $(R_t - r_t)/(\sigma + \sigma_t^q)$. The percentage volatility of the safe expert's wealth is $(1 + b_t^0)(\sigma + \sigma_t^q)$.

2.3 Banker's Optimal Choices

A banker's optimal portfolio weights on holdings of physical capital and loans satisfy

$$x_t^j + \lambda x_t + m_t \geq \frac{R_t^b - r_t}{(\sigma + \sigma_t^q)^2}, \text{with equality if } x_t^j > 0, \text{ and;} \tag{13}$$

$$x_t^j + \lambda x_t + m_t \leq (>) \frac{r_t^\lambda - r_t}{\lambda(\sigma + \sigma_t^q)^2}, \text{with equality if } 0 < x_t < \bar{x} \text{ (if } x_t = 0). \tag{14}$$

The loan rate r_t^λ depends on banks' exposure to aggregate risk $\lambda(\sigma + \sigma_t^q)$, banks' leverage x_t and x_t^j and also whether the capital requirement constraint is binding or not. If the constraint is binding, i.e., $x_t = \bar{x}$, then the positive Lagrange multiplier of the constraint implies that the loan r_t^λ is larger or equal to the level it would be if the constraint is not binding. The financing cost of bank loans for firms fluctuates endogenously for two reasons: the price volatility of physical capital changes over time, and banks' leverage varies across business cycles.

2.4 Market Clearing

Let W_t denote the total wealth that experts have in period t. Then, the total wealth of risky experts is $(1-\alpha)W_t$ given the reshuffling assumption regarding the risk-type of an expert made in Section 1.1. Hence, the total bank loans issued in equilibrium denoted by $x_t N_t$ satisfies

$$x_t N_t = (1-\alpha) W_t l_t, \tag{15}$$

where N_t denotes the total bank capital.

The demand for final goods comprises consumption and investment. The aggregate consumption of households is $\rho q_t K_t$. The market clearing condition with respect to final goods is

$$\rho q_t K_t = \frac{W_t}{q_t}(a - \iota_t)\big(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)\big) - \frac{N_t}{q_t}\iota_t x_t^j - \frac{q_t K_t - W_t - N_t}{q_t}\iota_t x_t^h \tag{16}$$

The market for physical capital clears if

$$\frac{W_t}{q_t}\big(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)\big) + \frac{N_t}{q_t}x_t^j + \frac{q_t K_t - W_t - N_t}{q_t}x_t^h = K_t. \tag{17}$$

Finally, the bond mutual fund's exposure to the aggregate risk must be shared by all agents $m_t q_t K_t = (1-\alpha)\lambda b_t^\lambda W_t$.

2.5 Dynamics of State Variables

Two endogenous state variables that characterize the dynamics of the economy are experts' wealth share $\omega_t = {}^{W_t}/_{(q_t K_t)}$ and bankers' wealth share $\eta_t = {}^{N_t}/_{(q_t K_t)}$. The decline of experts' wealth share naturally leads to a fall in average productivity since financial markets are incomplete and other agents are unproductive. If bankers' wealth share declines, then the supply of bank loans shrinks, and the interest rate on bank loans rises, which in turn lowers the aggregate productivity of the economy due to the increased financing cost for experts.

Given the reshuffling assumption, the total wealth of safe experts is αW_t and that of risky experts $(1-\alpha)W_t$. The dynamic budget constraints of individual experts (3) and (4) yield the law of motion for W_t

$$\frac{\mathrm{d}W_t}{W_t} = \left(R_t + \alpha b_t^0(R_t - r_t) + (1-\alpha)b_t^\lambda(R_t - \lambda\kappa^d - r_t) + (1-\alpha)l_t(R_t - \lambda\kappa - r_t^\lambda) - \frac{c_t}{W_t} - \chi\right)\mathrm{d}t + \left(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)(1-\lambda) + m_t\right)(\sigma + \sigma_t^q)\mathrm{d}Z_t. \quad (18)$$

A banker's dynamic budget constraint implies

$$\frac{\mathrm{d}N_t}{N_t} = \left(x_t^j R_t^b + x_t r_t^\lambda + (1 - x_t^j - x_t)r_t - \frac{c_t}{N_t} - \chi_\eta\right)\mathrm{d}t + (x_t^j + x_t\lambda + m_t)(\sigma + \sigma_t^q)\mathrm{d}Z_t. \quad (19)$$

Dynamics of η_t and ω_t also depend on the law of motion of the aggregate physical capital

$$\frac{\mathrm{d}K_t}{K_t} = \mu_t^K\mathrm{d}t + K_t\sigma\mathrm{d}Z_t, \text{where } \mu_t^K \equiv \Phi(\iota_t) - \delta - \eta_t x_t^j(\delta - \delta^b) - (1-\alpha)\omega_t\lambda(b_t^\lambda\kappa^d + l_t\kappa). \quad (20)$$

Given laws of motion of W_t, N_t, q_t, and K_t, I derive laws of motion for ω_t and η_t in equilibrium, which are summarized in the following lemma.[7]

Lemma 1 *In equilibrium, experts' wealth share ω_t evolves according to*

$$\frac{\mathrm{d}\omega_t}{\omega_t} = \mu_t^\omega\mathrm{d}t + \sigma_t^\omega\mathrm{d}Z_t, \text{ where} \quad (21)$$
$$\mu_t^\omega = R_t - \mu_t^q - \mu_t^K - \sigma\sigma_t^q + \alpha b_t^0(R_t - r_t) + (1-\alpha)b_t^\lambda(R_t - \lambda\kappa^d - r_t^\lambda) + (1-\alpha)l_t(R_t - \lambda\kappa - r_t^\lambda) - \left(\alpha b_t^0 + (1-\alpha)b_t^\lambda(1-\lambda) + (1-\alpha)l_t(1-\lambda) + m_t\right)(\sigma + \sigma_t^q)^2 - \rho - \chi$$
$$\sigma_t^\omega = \left(\alpha b_t^0 + (1-\alpha)b_t^\lambda(1-\lambda) + (1-\alpha)l_t(1-\lambda) + m_t\right)(\sigma + \sigma_t^q).$$

The state variable η_t evolves according to

$$\frac{\mathrm{d}\eta_t}{\eta_t} = \mu_t^\eta\mathrm{d}t + \sigma_t^\eta\mathrm{d}Z_t, \text{ where} \quad (22)$$
$$\mu_t^\eta = (x_t^j + \lambda x_t + m_t)(x_t^j - 1)(\sigma + \sigma_t^q)^2 + x_t(r_t^\lambda - r_t) + r_t - \mu_t^q - \mu_t^K - \sigma\sigma_t^q + (\sigma + \sigma^q)^2 - \rho - \chi_\eta$$
$$\sigma_t^\eta = (x_t^j + \lambda x_t + m_t - 1)(\sigma + \sigma_t^q)$$

The proof of Lemma 1 is in the appendix.

[7] I apply Ito's Lemma for this derivation in the continuous-time setting.

2.6 Markov Equilibrium

Like other continuous-time macro-finance models (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2012), my framework also has the property of scale-invariance with respect to total physical capital K_t. I focus on the equilibrium that is Markov in state variables ω_t and η_t. In the Markov equilibrium, dynamics of endogenous variables such as q_t can be characterized by laws of motion of ω_t and η_t and the mappings from the state (ω_t, η_t) to the values of endogenous variables, e.g., $q(\omega, \eta)$.

To solve for the full dynamics of the economy, I derive a partial differential equation with respect to $q(\omega, \eta)$. The PDE as well as its boundary conditions originate from equilibrium conditions and Ito's formula with $q(\omega, \eta)$. Ito's lemma with respect to the price volatility of physical capital implies that

$$q_t \sigma_t^q = q_\omega(\omega_t, \eta_t)\omega_t \sigma_t^\omega + q_\eta(\omega_t, \eta_t)\eta_t \sigma_t^\eta. \tag{23}$$

Given (q, ω, η), I can solve the equilibrium and derive all endogenous choice variables $(c, b^0, b^\lambda, l, x, x^h)$ and endogenous price variables $(r, r^\lambda, \mu^q, \sigma^q)$ as well as the lump-sum transfer related to the bond mutual fund m.[8] Then, equations (21) and (22) yield volatility terms of two state variables $(\sigma^\eta, \sigma^\omega)$. Hence, equation (23) is a well-defined PDE with respect to $q(\omega, \eta)$.

In addition to the differential equation, I need boundary conditions to solve for $q(\omega, \eta)$. The three boundaries are: $\{(\omega, \eta) : \omega = 0, 0 \leq \eta \leq 1\}$, $\{(\omega, \eta) : 0 \leq \omega \leq 1, \eta = 0\}$, and $\{(\omega, \eta) : 0 \leq \omega \leq 1, 0 \leq \eta \leq 1, \omega + \eta = 1\}$. On any of the three boundaries, one of the three agents has zero net worth and the differential equation (23) reduces to an ordinary differential equation, which I will solve first.

2.7 Calibration

Panel A: set parameters

ρ	time discount rate	2%
δ	capital depreciation	10%
ϕ	capital adjustment cost	3
$1/\bar{x}$	capital requirement	6%
θ	bank credit fraction of safe firms	0.28

Panel B: calibrated parameters

technology		
a	experts' productivity	0.15
δ_b	capital depreciation (bankers)	55%
σ	capital quality shock	3%
λ	idiosyncratic default likelihood	4.9%
α	the fraction of safe experts	15.2%
finance		
κ	bankruptcy cost (loan)	20%
κ_d	bankruptcy cost (bond)	64%
preference		
χ	experts' retirement rate	28%
χ_η	bankers' retirement rate	22%

Table 1: Parameters

[8] At this stage given (q, ω, η), I can only solve for $r - \mu^q$. However, it is straightforward to solve for r_t and μ^q after I derive the entire $q(\omega, \eta)$.

The calibration of the model mainly follows He and Krishnamurthy (2019) because the two models share the same continuous-time approach and both emphasize the role of the financial intermediary. I choose standard values from the real business cycle (RBC) literature for time discount rate ρ, the depreciation of physical capital held by experts and households δ, and capital adjustment cost parameter ϕ (Panel A of Table 6). I choose the minimum capital requirement $1/\bar{x}$ as 6% to be consistent with Elenev, Landvoigt and Van Nieuwerburgh (2018). The reason I use the capital adequancy ratio calculated by Elenev, Landvoigt and Van Nieuwerburgh (2018) is that both models are composed of a real sector and an explicit banking sector and thus the real world counterparts of bank assets in both models are corporate loans. Lastly, "safe" firms in reality also obtain bank credit. To take into account this fact, when calibrating the model, I assume that θ proportion of external credit raised by safe firms is in the form of bank loans and the risk-weight of safe loans is zero. Meanwhile, the risky loans have unit risk-weight and $1/\bar{x}$ is the risk-weighted capital ratio requirement. Thus, safe firms and banks still solve the same problems outlined in Section 1.[9] I set $\theta = 0.28$ according to Crouzet and Mehrotra (2020).[10]

For the nine calibrated parameters (Panel B of Table 6), I target nine key moments (see table 7), six of which are standard moments that He and Krishnamurthy (2019) and Elenev et al. (2018) consider and the other three of which are related to bond and bank financing: the ratio of bank credit to total credit, risk premiums on bank loans and corporate bonds. Their targets are from De Fiore and Uhlig (2011). The three variables' counterparts in my model are:

$$\text{the ratio of bank credit to total credit} : \frac{\alpha\theta b_t^0 + (1-\alpha)l}{\alpha b_t^0 + (1-\alpha)(l + b_t^\lambda)},$$
$$\text{risk premium on loans} : (r_t^\lambda - r_t + \lambda\kappa)\frac{(1-\alpha)l}{\alpha\theta b_t^0 + (1-\alpha)l},$$
$$\text{risk premium on bonds} : \lambda\kappa^d \frac{(1-\alpha)b_t^\lambda}{\alpha(1-\theta)b_t^0 + (1-\alpha)b_t^\lambda}.$$

Total bank loans while calculating moments include $\alpha\theta b_t^0$ loans obtained by safe firms and $(1-\alpha)l$ those obtained by risky firms. Note the risk premium on safe loans is zero. Hence, the risk premium on an average loan is risky loans' premium $r_t^\lambda - r_t + \lambda\kappa$ times the proportion of risky loans. Similarly, the risk premium on an average bond is risky bonds' premium $\lambda\kappa^d$ times the proportion of risky bonds. As the above expressions show, the three moments largely rely on the fraction of safe firms α, idiosyncratic default likelihood λ, bankruptcy costs of bond and bank finance κ and κ^d.

Table 7 reports the sample moments of model simulations. I simulate the economy for 4000 years and keep the results of the last 2000 years. The moments presented in Table 7 are based on the sample of 50,000 simulations. Like He and Krishnamurthy (2019), I solve for the global solution of the equilibrium and characterize the economy in both financially distressed states and non-distressed states. Since risky firms are the most important type of agents in the economy, I

[9]Note that if the risk-weight of safe loans is positive, safe experts' borrowing rate is higher than the risk-free rate when the capital requirement is binding.

[10]See Panel B of Table 1 in Crouzet and Mehrotra (2020).

use the Sharpe ratio of their investments

$$\text{Sharpe ratio}: \frac{R_t - \lambda\kappa^d - r_t}{(1-\lambda)(\sigma + \sigma_t^q)}$$

to measure to what extent the economy is financially constrained. As in He and Krishnamurthy (2019), I label states with the lowest 67% Sharpe ratio as the non-distressed states. The six standard moments involve Sharpe ratio, investment, bank equity, consumption, and output. Since Sharpe ratio is the key variable that captures the overall economic conditions. I calibrate parameters to match its first and second moments in the data as well as its value in crises (i.e. the highest Sharpe ratio). In addition, parameter values are calibrated so that the correlations of key economic variables generated by the model are consistent with the data: the covariance between Sharpe ratio and bank equity growth, and the coefficient of correlation between consumption growth and output growth.

Table 2: Targeted Moments[1]

This table presents the first and second moments of key endogenous variables (e.g., Sharpe ratio, investment-to-capital ratio, investment growth, consumption growth, bank credit to total credit ratio, and the risk premium of bank loans and corporate bonds) based on the sample of model simulations ("Model" column) and the real data ("Target" column). The last column "Source" lists the references of the target moments. The size of simulation sample is 50,000 and each simulation runs for 2,000 years.

Moment	Model	Target	Reference
Sharpe ratio (mean)	49%	48%	He and Krishnamurthy (2019)
investment-to-capital ratio (mean)[2]	9.7%	9%	He and Krishnamurthy (2019)
$\frac{\text{highest Sharpe ratio}}{\text{average Sharpe ratio}}$	15.6	15	He and Krishnamurthy (2019)
Sharpe ratio (volatility)[2]	29.9%	30%	He and Krishnamurthy (2019)
cov(Sharpe ratio, bank equity growth)[2]	0.6%	0.6%	He and Krishnamurthy (2019)
corr(consumption growth, output growth)[2]	0.89	0.88	Elenev et al. (2018)
bank credit to total credit ratio (mean)	0.394	0.401	De Fiore and Uhlig (2011)
risk premium on loans (mean)	1.77%	1.70%	De Fiore and Uhlig (2011)
risk premium on bonds (mean)	1.28%	1.43%	De Fiore and Uhlig (2011)

[1] I use the density of the stationary distribution to calculate all moments.
[2] These are moments conditional on non-distressed states, which are defined as states with lowest 67% Sharpe ratios.

Table 3 presents the untargeted moments generated by the model and their counterparts in the data. The debt-to-asset ratio in the model is higher than that in the data. This is largely due to the setting that outside equity issuance is not allowed. The volatility of consumption growth generated by the model (5.7%) is too high compared to the data (1.5%). The reason for this outcome is that the economy is driven by a single exogenous aggregate shock, whose volatility σ influence several key moments such as Sharpe ratio and the volatility of Sharpe ratio. It is difficult to adjust one parameter to meet multiple objectives simultaneously. The volatility of bank equity growth turns out to be lower than its real-world counterpart. This problem also exists in He and Krishnamurthy (2019). I believe both the deregulation in 1980's of the banking sector and the concentration process afterwards contribute to the high volatility of bank equity growth. Our simple banking models, however, do not consider the deregulation and mergers and acquisitions in the banking industry. Lastly, the covariance of bank equity growth with Sharpe ratio, consumption growth, and investment growth in distressed states seem to be in line with their counterparts in the data, although the magnitude of the covariance with Sharpe ratio is too small.

Table 3: Untargeted Moments[1]

The size of simulation sample is 50,000 and each simulation runs for 2,000 years.

Moment	Model	Data	Reference
firms' debt-to-asset ratio (mean)	0.83	0.43 (U.S.) 0.64 (euro area)	De Fiore and Uhlig (2011)
vol of consumption growth	5.7%	1.5%	He and Krishnamurthy (2019)
vol of bank equity growth	5.5%	20.5%	He and Krishnamurthy (2019)
cov(bank equity growth, Sharpe ratio)[2]	-0.4	-8	He and Krishnamurthy (2019)
cov(− − −, consumption growth)[2]	0.5	0.2	He and Krishnamurthy (2019)
cov(− − −, investment growth)[2]	0.74	1.02	He and Krishnamurthy (2019)

[1] I use the density of the stationary distribution to calculate all moments.
[2] These are moments conditional on distressed states, which are defined as states with highest 33% Sharpe ratios.

2.8 Equilibrium Characterization

In this subsection, I highlight that the equilibrium of an economy highly depends on whether risky firms can directly issue bonds. To demonstrate the role of bond financing for the aggregate economy, I compare the benchmark economy with an otherwise identical economy with no bond market where I set the liquidation cost of bond financing sufficiently high. In the latter economy, safe firms obtain risk-free loans from banks. To ensure that the two economies are comparable, I assume that safe loans carry zero risk-weight, risky loans have unit risk-weight, and banks are subject to the risk-weighted capital ratio constraint in the economy without bond financing.[11] In Appendix F, I re-calibrate the model without bond financing and conduct the same comparison between the full economy and the bank-only economy, which yields similar conclusions.

First-Moment Comparison. The presence of bond financing makes the economy worse off according to several metrics (see Table 4). I run 50,000 simulations for both economies and each round of simulation continues for 4,000 years. Table 4 reports the average of nine key endogenous variables based on the sample of 50,000 draws. The table highlights one significant difference between the two economies, that is, bankers hold more wealth in the economy without a bond market. This is an intuitive result since bank financing is the only source of external credit for firms when they cannot tap the bond market, which yields more profits for the banking sector.

[11] Recall that safe firms' borrowing rate always equals the risk-free rate in the full model. If the risk-weight safe loans is positive, safe firms' borrowing rate could be higher than the risk-free rate when the capital requirement constraint becomes binding for banks. And, safe firms' borrowing rate would never be larger than the risk-free rate in the full model as they can always borrow in the bond market.

In particular, row 7 of Table 4 show that risky firms raise much more bank loans if the bond market shuts down, and row 8 indicates that the loan spread also increases by more than 40%. The improvement of the banking sector helps the real sector raise more external credit than it could in the economy with bond financing (see row 6 of table 4). On the real side, the economy without bond financing also outperforms: higher TFP, higher consumption to capital ratio, and higher investment to capital ratio (see rows 3-5 of Table 4), where the TFP is defined as experts' productivity times the fraction of physical capital they hold, i.e, $a \times \omega_t(\alpha(1 + b^0_t) + (1 - \alpha)(1 + b^\lambda_t + l_t))$. Notice that since the model is scale-invariant I focus on the consumption and investment to physical capital ratios.

Table 4: First-Moment Comparison[1]

This table reports the first moments of nine endogenous variables in the economies with and without bond financing: experts' wealth share, bankers' wealth share, TFP, consumption to capital ratio, investment to capital ratio, the real sector's liability, outstanding bonds and loans, and loan spreads. The size of simulation sample is 50,000 and each simulation runs for 4,000 years.

		with bond	without bond
1	experts' wealth share ω	0.1466	0.1571
2	bankers' wealth share η	0.0091	0.0302
3	TFP	0.1231	0.1277
4	consumption to capital ratio	0.0258	0.0261
5	investment to capital ratio	0.0973	0.1016
6	the real sector's liability	0.6652	0.6942
7	risky firms' loans	0.1511	0.4388
8	loan spreads	0.0177	0.0230

[1] I use the density of the stationary distribution to calculate all moments.

The above comparison indicates that the framework that omits bond financing *overstates* the beneficial role of the banking sector for the aggregate economy. If we take into account the effect of the bond market, banks enjoy less profits and the banking sector shrinks by 70%. The general equilibrium consequence is that the overall external credit that the real sector raise actually decreases in the long run if the alternative financing channel, bond issuance, becomes available (see row 6 of Table 4). Note that because of the outside equity financing constraint there exists pecuniary externality in my model where the financial market is incomplete. The leverage chosen by private agents tend to be higher than the socially optimal level (Lorenzoni, 2008; Bianchi, 2011). Shutting down the bond market essentially counteracts the over-borrowing of the real sector in the full model.

Dynamics. To illustrate the economic dynamics, I consider a hypothetical crisis scenario where the wealth shares of both experts and bankers suddenly decline by 20 percent. Figure 4 shows the transition paths of the sample median of key variables in the aftermath of the crisis. The sample includes 50,000 economies, and the simulation lasts for 2 years.

Before discussing economic dynamics in detail, let us review the *transmission mechanism* of the model. When a negative shock hits the economy, experts' dynamic budget constraints (3) and (4) imply that their net worth will decline disproportionately due to the leverage effect. On top of the exogenous shock, the decline in the price of physical capital causes additional losses to experts' net worth, as indicated by equations (3) and (4). The exogenous shock also affects bankers' net worth, which is the other state variable. Bankers' exposure to the aggregate risk comes from the collateral that backs their loans. When banks liquidate risky firms' physical capital, the exogenous shock affects the (efficient) units of physical capital seized by banks, and also the price at which they can sell the physical capital in the secondary market. Note that banks also take on high leverage and thus have high risk exposure to the exogenous shock as shown by equation (5). The decline in the net worth of both productive experts and financial intermediaries has persistent effects on the productivity, investment, asset prices, and external financing in the economy.

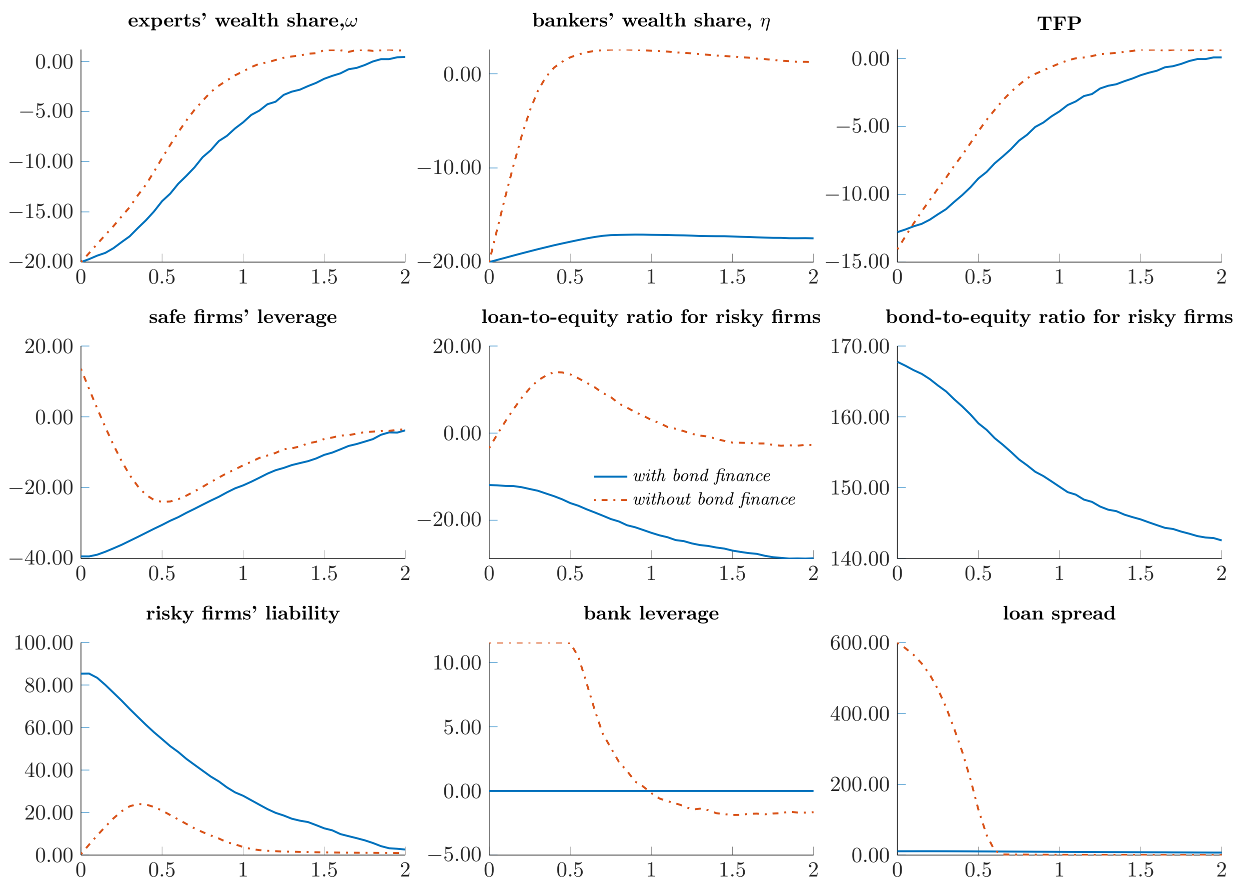

Figure 4: Dynamics
This figure shows the median sample path (deviation from the long-run level in percentage) of nine key aggregate variables in two economies over two years after a hypothetical crisis where state variables decline by 20%: experts' wealth share (upper-left), bankers' wealth share (upper-middle), TFP (upper-right), safe firms' leverage (middle-left), loan-to-equity ratio for risky firms (center), bond-to-equity ratio for risky firms (middle-right), risky-firm's liability (lower-left), bank leverage (lower-middle), loan spread (lower-right). The solid lines refer to an economy with bond financing, and the dashed lines refer to an economy without bond financing.

The key message of Figure 4 is that the economy recovers more quickly if firms cannot access the bond market (the upper-right and upper-middle panels). In addition, the negative impact of the crisis on the banking sector is more persistent in the presence of bond financing (the upper-middle panel). The intuition behind this result is straightforward. In the aftermath of the crisis, loan supply declines and loan spread soars (the lower-right panel). In the economy with bond financing, as risky firms can choose between bank credit and bond credit (the middle-right panel), the fall in loan demand is more significant (the solid line in the center panel) and the rise in loan spread is quite modest (solid line in the lower-right panel). By contrast, in the absence of bond financing firms have to borrow from banks (dashed line in the center panel) even although the loan spread are six times as high as its normal level. Therefore, banks enjoy much better profitability in the absence of bond financing, which is the key reason why the banking sector can recover so fast in the economy without a bond market.

As banks provide relatively cheap credit for firms, the firm sector recovers faster in the economy without bond financing (the upper-left panel of Figure 4). As a result, the average productivity also recovers faster in the economy without bond financing. Note that the initial decline in the productivity is deeper in the economy without bond financing. This is intuitive as firms in this economy has less financing options temporarily. The middle-left and lower-left panels of Figure 4 display the transition of safe and risky firms' liability in both economies.

3 Optimal Capital Requirement

In this section, I emphasize that the socially optimal level of capital ratio requirement highly depends on (i) the presence of bond financing *quantitatively*, (ii) the efficiency of an economy's bankruptcy procedure, and (iii) the distribution of borrowing firms' idiosyncratic default risk. The underlying mechanisms of the three results are related to the demand elasticity for bank loans, the key factor that determines the general equilibrium costs and benefits of bank capital requirements.

The welfare of an individual agent is the weighted sum of the agent's lifetime expected utility over all possible states of the economy. The weight of each state is its density of the long-run stationary distribution. The social welfare is the equal-weighted sum of the welfare of all agents.

3.1 The Consequences of Omitting Bond Financing

An economic model that omits bond financing overstates the benefit of capital ratio requirements, and thus prescribes an optimal requirement that is overly tight. I compare the social welfare of two economies — one with bond financing and the other without bond financing — under different degrees of capital requirement. Figure 5 clearly shows that the capital adequacy ratio $1/\bar{x}$ that maximizes social welfare is lower in the economy with a bond market than in the economy without a bond market. In other words, the socially optimal capital requirement should be more lenient in the presence of bond financing. This statement holds regardless of whether I focus on the welfare of experts, bankers, or households (see the upper-right, lower-left, and lower-right panels in Figure

5). Before expounding why this difference exists, I explain the channel through which capital requirement influences social welfare.

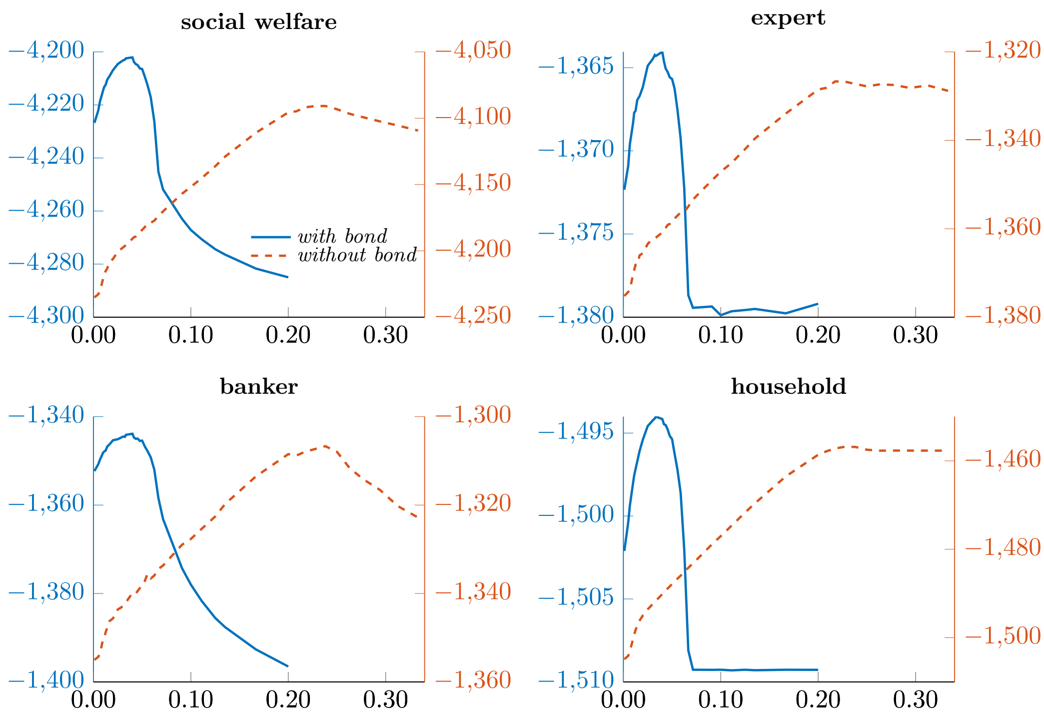

Figure 5: Welfare

This figure shows the relationship between the capital adequacy ratio $1/\bar{x}$ (horizontal axis) and the welfare of different types of agents in economies with and without bond financing. Solid lines refer to an economy with bond financing and dashed lines refer to an economy without bond financing. The aggregate welfare shown in the left panel is the sum of the welfare of the three types of agents. The welfare of an agent is the weighted sum of the agent's lifetime expected utility over all possible states of the economy. The weight of each state is its density of the simulated long-run stationary distribution. For the values of parameters other than $\bar{x}$, see Section 2.7.

Elenev, Landvoigt and Van Nieuwerburgh (2018) highlights that tightening the capital requirement shifts wealth from savers to borrowers. Here, I emphasize that part of the wealth is actually diverted to financial intermediaries. To illustrate this effect more clearly, first consider an economy without bond financing. The dashed lines in the two left panels in Figure 6 show that the wealth share of both experts and bankers rises as the capital ratio requirement tightens. Dashed lines in the lower-left and upper-right panels in Figure 6 clearly show why bankers' wealth share increases. Tightening the capital requirement lowers the supply of bank loans. Therefore, the loan spread that banks can charge increases accordingly. To some extent, the overall effect leads to the increase in bank profitability as shown by the dashed line in the lower-right panel in Figure 6. The cumulative effect of high bank profits naturally leads to an increasingly stronger banking sector, which translates to an improvement in bankers' welfare.

Tightening the capital requirement increases experts' wealth share as well as the welfare of both experts and households.[12] Lowering the maximum leverage of bankers limits the supply of bank credit. Given the excessive credit supply from unproductive households, the overall borrowing costs decrease, and thus experts' wealth share increases. In sum, the strengthening of both the firm sector and the banking sector increases the average productivity of the economy as highlighted in the lower-middle panel in Figure 6. The rise in the average TFP results in the improvement of households' welfare (the lower-right panel in figure 5). However, if the capital ratio requirement is too tight bank profitability ultimately declines due to the substantial decrease in loans that banks can originate. The aggregate productivity ultimately falls. So does the social welfare.

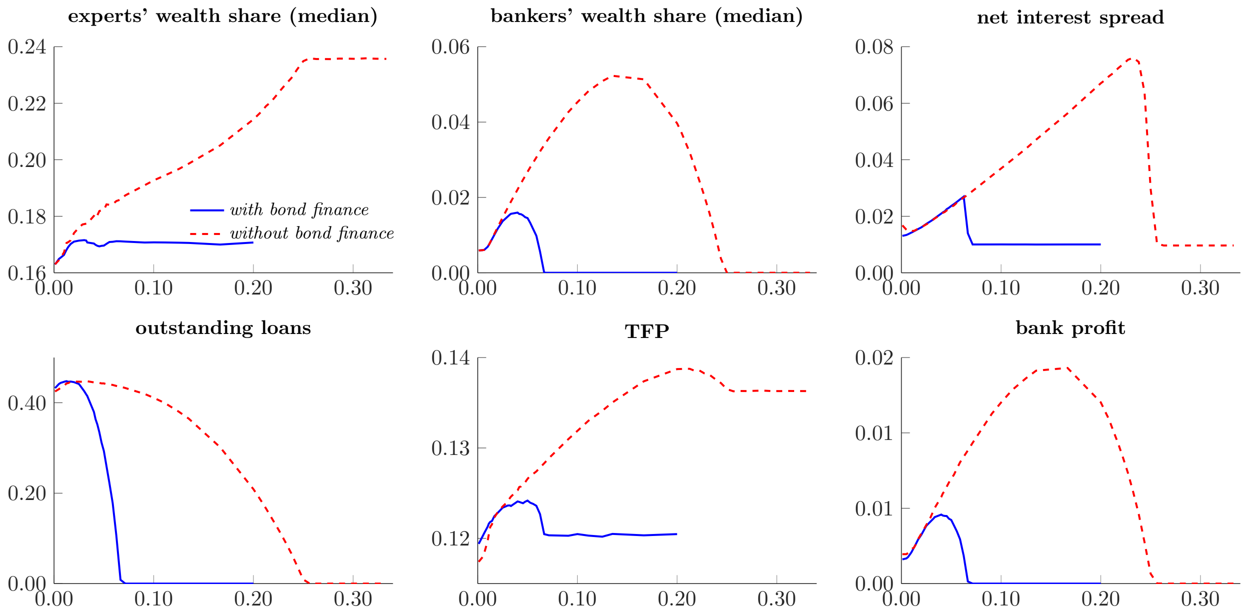

Figure 6: Wealth Distribution
This figure shows the relationship between the capital adequacy ratio $^1/_{\bar{x}}$ (horizontal axis) and the moments of six financial variables in the long-run stationary distribution: median experts' wealth share (upper-left), median bankers' wealth share (upper-middle), average net interest spread (upper-right), average outstanding loans (lower-left), average total factor productivity (lower-middle), and average bank profit (lower-right). For the values of parameters other than $\bar{x}$, see Section 2.7.

The presence of bond financing, however, can significantly dampen the wealth transfer effect of the capital ratio requirement. I now turn to an economy with a bond market. The solid line in the upper-middle panel in Figure 6 displays that bankers' wealth share increases as the capital ratio requirement increases up to around 4%. This increase is similar to their reactions to the regulatory change in the absence of bond financing. Nevertheless, if the capital adequacy ratio continues rising, the wealth share of the intermediary sector shrinks drastically until it completely vanishes. Due to the shrinkage of the loan supply, the real sector has to switch to more costly bond financing, which causes the decline in the average productivity as well as the fall in experts' welfare (see the lower-left and lower-middle panels of Figure 6).

[12] In my model, banking regulation mitigates pecuniary externalities and improves social welfare via the distributive effects emphasized by Dávila and Korinek (2017).

Why does the financial intermediary sector react so differently in the two economies? The key underlying reason is that firms have an alternative way of raising external credit in an economy with bond financing. Thanks to the alternative channel, firms can resort to bond financing when loan spreads rise. Hence, when bond financing is feasible, the decline in loan demand would be more substantial than in an economy where loan financing is the only option for the real sector (compare dashed lines and solid lines in the upper-right and lower-left panels of Figure 6). Therefore, bank profits are more likely to decline in an economy where firms have a second option for raising external credit (the lower-right panel of Figure 6). The decline in bank profitability, in turn, hurts bankers' wealth share and loan supply, which ultimately lowers the average productivity. In sum, tightening capital requirement in the presence of bond financing is more inclined to hurt the financial intermediary sector and the entire economy.

Quantitative Implications. The calibrated model indicates that the socially optimal capital adequacy ratio is 4%, which is much more lenient than the benchmark 6%. This is not a surprising result since, unlike Elenev, Landvoigt and Van Nieuwerburgh (2018) and others, my model takes into account an additional channel that dampens the positive effects of the capital requirement constraint. One caveat regarding the model's quantitative prediction is that the idiosyncratic default risk is exogenous in my model and thus, unlike papers such as Begenau (2019) and Dempsey (2018), the capital adequacy ratio in my model has no effects on banks' risk-taking from the micro-prudential perspective. My model only focuses on banks' excessive leverage-taking from the macro-prudential perspective.

The most crucial insight of the calibrated model is that the optimal level of capital ratio requirement is very sensitive to the presence of the bond market quantitatively. If I deprive the real sector of the bond financing option, the social welfare maximizing capital requirement would point to almost 24%, which is drastically different from 4% that the benchmark model with bond financing suggests. This result raises an important issue regarding the *robustness* of optimal bank regulation against alternative market forces.

3.2 Policy Experiments

The previous section shows that the discussion on the optimal capital requirement could be misleading if bond financing is omitted from the model. In this subsection, I conduct two policy experiments, and discuss whether and how the optimal capital requirement depends on the structure of the bond market. In the first experiment, I vary the liquidation cost of bondholders κ^d, and characterize the relationship between the optimal capital requirement and the development of the bond market (Djankov, Hart, McLiesh and Shleifer, 2008; Becker and Josephson, 2016). In the second experiment, I investigate the policy implication of the risk profile of borrowing companies in the bond market.

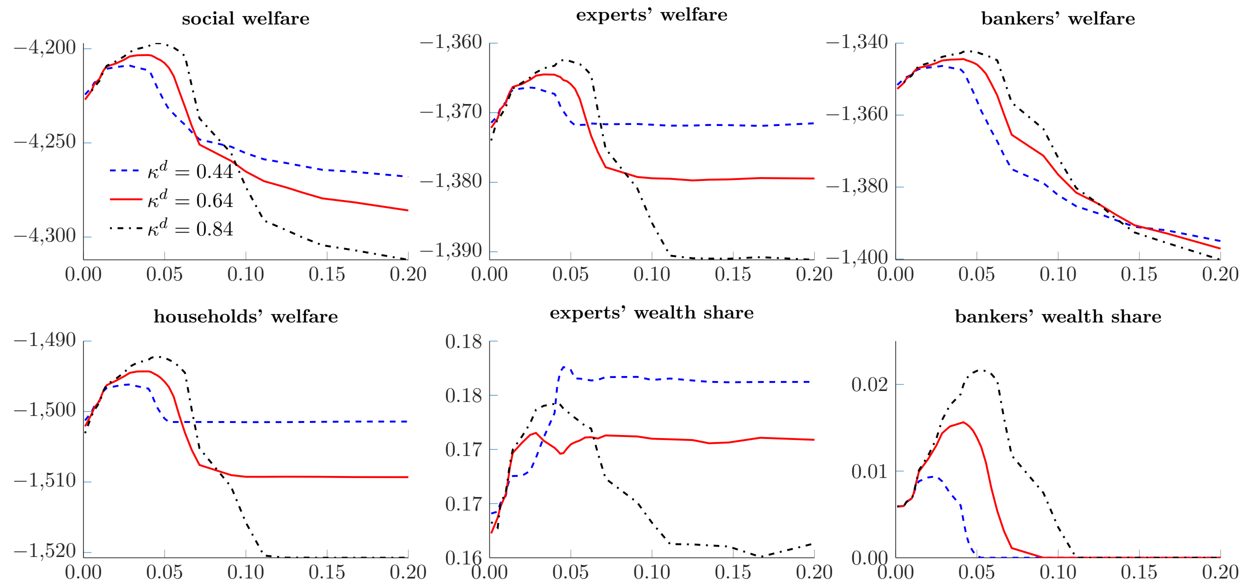

Figure 7: Development of bond market
This figure shows the welfare implications of a change in the capital adequacy ratio $1/\bar{x}$ (horizontal axis) for economies with different degrees of bond market development: more developed bond market ($\kappa^d = 0.44$), benchmark ($\kappa^d = 0.64$), less developed bond market ($\kappa^d = 0.84$). The bottom middle and right panels display effects of a change in $1/\bar{x}$ on the median wealth shares of experts and bankers. For the values of parameters other than $\bar{x}$ and κ^d, see Section 2.7.

3.2.1 Development of the Bond Market

Becker and Josephson (2016) emphasize that the efficiency differences in the processing of insolvency and bankruptcy cases (e.g., bankruptcy recoveries) can explain the cross-firm and also cross-country heterogeneity regarding the use of bond financing and bank financing. Their empirical evidence as well as theoretical results show that inefficient bankruptcy procedures in an economy is associated with less bond financing by risky firms. The efficiency of bankruptcy procedures, in turn, can be traced back to the legal origin and income per capita according to Djankov et al. (2008). Here, I treat bondholders' liquidation cost κ^d as an exogenous parameter that captures the efficiency of bankruptcy procedures in an economy. A lower liquidation cost κ^d signifies a more efficient bankruptcy system and a more developed bond market. Based on this assumption, I investigate how the optimal capital ratio requirement in a country depends on how developed its bond market is.

The upper-left panel in Figure 7 shows that the socially optimal capital requirement ought to be more stringent in an economy with a less developed bond market (i.e., higher κ^d). The intuition is the same as that in the previous analysis on the absence of bond financing. If the capital adequacy ratio rises, loan spreads increase; at the same time, the amount of loans originated by banks declines. In an economy with a less developed bond market, (i.e., higher liquidation cost κ^d), risky firms find it more costly to switch from bank financing to bond financing. Hence, the decrease in the amount of bank loans is not so significant; in fact, banks' overall profitability may actually increase when the loan spread increases. Given that the capital adequacy ratio is relatively low, raising this ratio actually increases bankers' wealth share in the economy. However, the lower-right panel in Figure 7 shows that if the capital requirement is too tight, the banking sector is more likely to vanish in an economy with a more developed bond market ($\kappa^d = 0.44$). The reason is that when risky firms switch to bond financing, there is a substantial decline in the quantity of bank loans and also a sizable drop in bank profitability. When the banking sector vanishes, the borrowing cost of the firm sector increases and its borrowing capacity declines substantially. Since the average productivity of the economy depends on to what extent firms can raise external funds, the capital ratio requirement affects social welfare through its impact on experts' borrowing.

3.2.2 Average Firm Riskiness

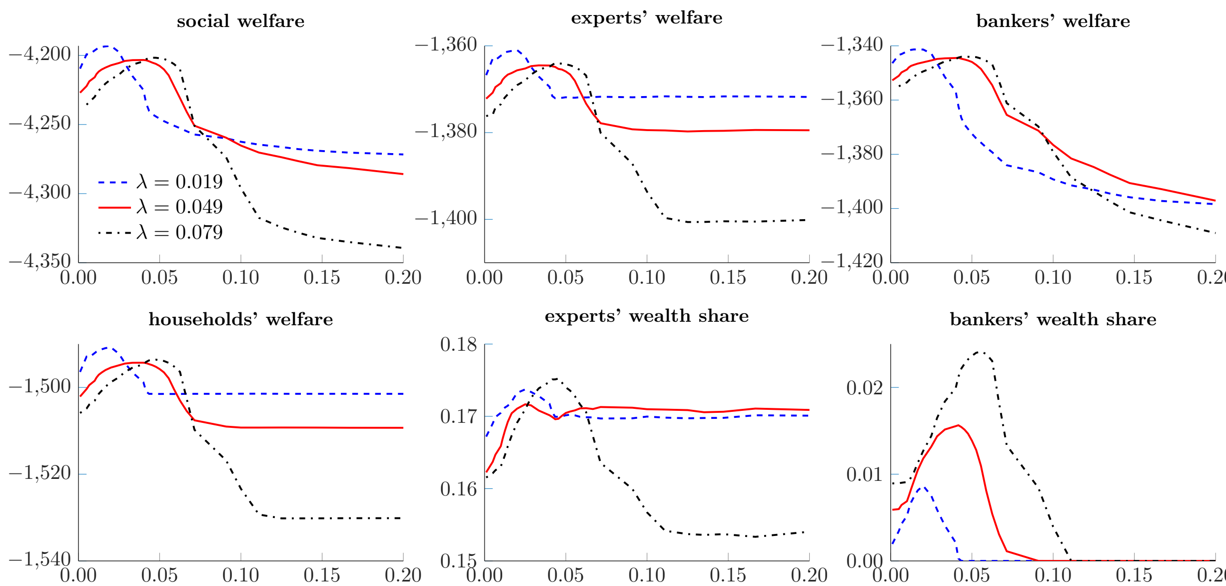

Figure 8: Riskier firms
This figure shows the welfare implications of a change in banks' capital adequacy ratio $^1/_{\bar{x}}$ (horizontal axis) for economies in which firms have different degrees of riskiness: less risky ($\lambda = 0.019$), benchmark ($\lambda = 0.049$), and more risky ($\lambda = 0.079$). The lower-middle and lower-right panels display effects of a change in $^1/_{\bar{x}}$ on the median wealth shares of experts and bankers. For the values of parameters other than $\bar{x}$ and λ, see Section 2.7.

Other than the efficiency of the bankruptcy process in an economy, the risk profile of its ultimate borrowers also affects the use of bond financing and bank financing. In particular, I investigate how the average riskiness of firms affects the optimal capital requirement. Keeping all parameters unchanged, I only vary the value of individual firms' bankruptcy probability λ.

The upper-right panel in Figure 8 shows that the socially optimal capital requirement is tighter in an economy where firms are riskier on average. The same conclusion holds regardless of whether I focus on the welfare of experts, bankers, or households (the upper-middle, upper-right, and lower-left panels in Figure 8). When a risky firm switches from bank financing to bond financing, equations

(11) and (12) imply that it has to pay an additional premium $\lambda(\kappa^d - \kappa)$ to compensate creditors for their losses in the event of firm liquidation. This switching cost is increasing in the likelihood of firm failure, i.e., λ. Hence, relative to safe firms, risky firms find it more costly to replace bank loans with bonds. When the capital requirement tightens, the decrease in the amount of bank loans is less significant in an economy with riskier firms. In such an economy, bank profitability is less likely to decline given the rise in the loan spread. Consequently, the banking sector is less likely to shrink (see the lower-right panels in Figure 8). Hence, the optimal capital requirement ought to be tighter in an economy with riskier firms.

4 Conclusion

In this paper, I point out that it is quantitatively important to incorporate bond financing into a dynamic general equilibrium framework that intends to assess the welfare implications of bank capital regulations. A model that omits the bond market overstates the benefit of capital requirements. In addition, I highlight two factors that affects the optimal level of bank capital requirements via their influences on the demand elasticity of bank loans: the efficiency of the bankruptcy system in an economy and the idiosyncratic default risks of firms in the real sector. In this paper, I have only considered the time-invariant capital requirement. A straightforward extension of my model can be used to assess the macro-prudential role of counter-cyclical capital buffer in the presence of bond financing.

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Appendix

A Proofs

Proof of Lemma 1.

The laws of motion for the price of physical capital (2) and the efficiency units of physical capital (20) imply

$$\frac{\mathrm{d}(q_t K_t)}{q_t K_t} = (\mu_t^q + \mu_t^K + \sigma\sigma_t^q)\mathrm{d}t + q_t K_t(\sigma + \sigma_t^q)\mathrm{d}Z_t$$

The equation above, together with equation (18), lead to

$$\begin{aligned}
\mathrm{d}\Big(\frac{W_{t+1}}{q_{t+1}K_{t+1}}\Big) = {} & \frac{W_t}{q_t K_t}\Big(R_t + \alpha b_t^0(R_t - r_t) + (1-\alpha)b_t^\lambda(R_t - \lambda\kappa^d - r_t) + (1-\alpha)l_t(R_t - \lambda\kappa - r_t^\lambda) - \frac{c_t}{W_t}\Big)\mathrm{d}t \\
& - \frac{W_t}{q_t K_t}(\mu_t^q + \mu_t^K + \sigma\sigma_t^q)\mathrm{d}t - \frac{W_t}{q_t K_t}\Big(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)(1-\lambda) + m_t\Big)(\sigma + \sigma_t^q)^2\mathrm{d}t \\
& + \frac{W_t}{q_t K_t}(\sigma + \sigma_t^q)^2\mathrm{d}t + \frac{W_t}{q_t K_t}\Big(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)(1-\lambda) + m_t\Big)(\sigma + \sigma_t^q)\mathrm{d}Z_t \\
& - \frac{W_t}{q_t K_t}(\sigma + \sigma_t^q)\mathrm{d}Z_t \\
\frac{\mathrm{d}\omega_t}{\omega_t} = {} & \mu_t^\omega \mathrm{d}t + \sigma_t^\omega \mathrm{d}Z_t,
\end{aligned}$$

Given one of the bankers' Euler equation (13), the law of motion for N_t can be rewritten as

$$dN_t = N_t\Big(x_t^j(x_t^j + \lambda x_t + m_t)(\sigma + \sigma_t^q)^2 + x_t(r_t^\lambda - r_t) + r_t - \frac{c_t}{N_t}\Big)dt + N_t(x_t^j + \lambda x_t + m_t)(\sigma + \sigma_t^q)dZ_t.$$

Hence,

$$\begin{aligned}
d\Big(\frac{N_{t+1}}{q_{t+1}K_{t+1}}\Big) &= \frac{N_t}{q_tK_t}\Big(x_t^j(x_t^j + \lambda x_t + m_t)(\sigma + \sigma_t^q)^2 + x_t(r_t^\lambda - r_t) + r_t - \frac{c_t}{N_t}\Big)dt - \frac{N_t}{q_tK_t}(\mu_t^q + \mu_t^K + \sigma\sigma_t^q)dt \\
&\quad - \frac{N_t}{q_tK_t}(x_t^j + \lambda x_t + m_t)(\sigma + \sigma_t^q)^2 dt + \frac{N_t}{q_tK_t}(\sigma + \sigma_t^q)^2 dt \\
&\quad + \frac{N_t}{q_tK_t}(x_t^j + \lambda x_t + m_t)(\sigma + \sigma_t^q)dZ_t - \frac{N_t}{q_tK_t}(\sigma + \sigma_t^q)dZ_t \\
\frac{d\eta_t}{\eta_t} &= \mu_t^\eta dt + \sigma_t^\eta dZ_t.
\end{aligned}$$

■

B Derivation of Optimality Conditions

I derive the optimal conditions of bankers in the section. The same derivation for the optimal choices of experts and households is almost the same. Let $J^b(n_t, \omega_t, \eta_t)$ denote the continuation value of a banker with net worth n_t at state (ω_t, η_t)

$$J^b(n_t, \omega_t, \eta_t) = E_0\left[\int_0^T e^{-\rho t}\ln(c_t)\,dt + e^{-T\rho}J^h(W_T, \omega_T, \eta_T)\right].$$

As in the literature, $J^b(n_t, \omega_t, \eta_t)$ is specified as

$$J^b(n_t, \omega_t, \eta_t) = \frac{1}{\rho}\ln(n_t) + j^b(\omega_t, \eta_t).$$

The Hamilton-Jacobi-Bellman equation is

$$\begin{aligned}
\rho\left(\frac{1}{\rho}\ln(n_t) + j^b(\omega_t, \eta_t)\right) = \max_{c_t, x_t^j, x_t \le \bar{x}_t} \Bigg\{ &\ln(c_t) + \frac{1}{\rho}\left(r_t + x_t^j(R_t^b - r_t) + x_t(r_t^\lambda - r_t) - \frac{c_t}{n_t}\right) \\
&- 0.5\frac{1}{\rho}\left(x_t^j + \lambda x_t + m_t\right)^2(\sigma + \sigma_t^q)^2 + j_\omega^b\omega_t\mu_t^\omega + j_\eta^b\eta_t\mu_t^\eta \\
&+ 0.5\left(j_{\omega\omega}^b(\omega\sigma_t^\omega)^2 + 2j_\omega^b j_\eta^b\omega_t\eta_t\sigma_t^\omega\sigma_t^\eta + j_{\eta\eta}^b(\eta_t\sigma_t^\eta)^2\right)\Bigg\}
\end{aligned}$$

The first-order conditions with respect to c_t, x_t^j and x_t yield the corresponding optimal conditions. Note that optimal choices of a banker are independent of the function $j^b(\omega, \eta)$, which, in turn, is a solution of the following PDE

$$\rho j^b(\omega_t,\eta_t) = \ln(\rho) + \ln(c_t) + \frac{1}{\rho}\left(r_t + x_t^j(R_t^b - r_t) + x_t(r_t^\lambda - r_t) - \rho\right) - 0.5\frac{1}{\rho}\left(x_t^j + \lambda x_t + m_t\right)^2 (\sigma + \sigma_t^q)^2$$
$$+ j_\omega^b \omega_t \mu_t^\omega + j_\eta^b \eta_t \mu_t^\eta + 0.5\left(j_{\omega\omega}^b(\omega\sigma_t^\omega)^2 + 2 j_\omega^b j_\eta^b \omega_t \eta_t \sigma_t^\omega \sigma_t^\eta + j_{\eta\eta}^b(\eta_t\sigma_t^\eta)^2\right).$$

The existence and uniqueness conditions of $j^b(\omega_t,\eta_t)$ are quite technical. See Crandall, Ishii and Lions (1992) for detailed discussion.

C Numerical Procedure

I first simplify the final goods' market clearing condition so as to illustrate the numerical procedure more clearly. Let ψ_t denote the fraction of physical capital held by experts, the market clearing condition reduces to

$$a\psi_t = \rho q_t + \iota_t = \rho q_t + \frac{q_t - 1}{\phi}, \tag{24}$$

where the second equality uses the result of equation (8). The connections between ψ_t and the leverage of households and experts follow

$$\psi_t = \omega_t(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)), \text{ and} \tag{25}$$
$$x_t^h = \frac{\omega_t + \eta_t - \psi_t}{1 - \omega_t - \eta_t}.$$

The numerical procedure of solving for the equilibrium consists of three parts: *i*) three boundary solutions of $q(\omega,\eta)$, where $\omega = 0$, $\eta = 0$, or $\omega + \eta = 1$, *ii*) the interior solutions of $q(\omega,\eta)$ where $\psi(\omega,\eta) = 1$, *iii*) the interior solutions of $q(\omega,\eta)$ where $\psi(\omega,\eta) < 1$. While solving for $q(\omega,\eta)$, the algorithm will also yield solutions of all other endogenous variables such as $b^0, b^\lambda, l, x^j, x$, and extra. I discretize the space as $\{(\omega_{i,j},\eta_{i,j})$, where $i = 1,...,N, j = 1,...,I(j), \omega_{i,j} = \omega_{i,h}$ for any j and $h, \eta_{i,j} = \eta_{h,j}$ for any i and $h,$ and $\omega_{i,J(i)} + \eta_{i,J(i)} = 1\}$.

(*i*) **boundary solutions**. I first solve for $q(\omega,\eta)$ along the three boundaries, that is, $q(\omega_{1,j} = 0, \eta_{1,j})$, $q(\omega_{i,1}, \eta_{i,1} = 0)$, and $q(\omega_{i,J(i)}, \eta_{i,J(i)})$. In the boundary cases, equation (23) yields ordinary differential equations (ODEs). The numerical scheme of solving the three ODEs is a simplified version of the one used for the interior solutions of $q(\omega,\eta)$ where $\psi(\omega,\eta) < 1$, i.e., part (*iii*). Hence, I do not repeat the description of the scheme for the simpler case.

(*ii*) $\psi(\omega,\eta) = 1$, **experts hold all physical capital**. This part starts from $i = N$ towards $i = 1$. Given that i and $q(\omega_{i,J(i)}, \eta_{i,J(i)})$ solved in part (*i*), I start solving for $q(\omega_{i,j}, \eta_{i,j})$ from $j = J(i) - 1$ towards $j = 1$. The numerical scheme begins with the conjecture $\psi(\omega,\eta) = 1$ that I will verify later. The conjecture and the final goods' market clearing condition (24) imply that $q(\omega,\eta) = \frac{a\phi+1}{\rho\phi+1}$. Since $q(\omega,\eta)$ is a constant in the neighborhood of $(\omega_{i,j},\eta_{i,j})$, the volatility of q is zero, i.e., $\sigma_q = 0$. Then, I can solve for b^0, b^λ, l, and x given the optimality conditions of experts and bankers (10), (11), (12), and (14) as well as equation (25). In the end, I check the optimality conditions of households and bankers regarding their holdings of physical capital (9) and (13) to verify if neither of them prefer holding physical capital. If the conjecture is verified, then I move to $j-1$; otherwise, the algorithm proceeds to $i-1$ and let $\Psi(i)$ denote $\min\{j+1, J(i)\}$, the minimum $\tilde{j}$ such that $\psi(\omega_{i,\tilde{j}}, \eta_{i,\tilde{j}}) = 1$.

(iii) $\psi(\omega, \eta) < 1$, **experts hold a fraction of physical capital**. I start from $i = 2$ to $i = N$, and for each i, I will solve for $q(\omega = \omega_{i,j}, \eta)$ as the solution of an ODE. The ODE is based on Equation (23). To ensure the stability of the numerical procedure, I use the implicit method that involves the root-finding of a sixth order polynomial with respect to the unknown $q(\omega_{i,j}, \eta_{i,j})$. I rearrange equation (23) to illustrate how to formalize the polynomial

$$\begin{aligned}
q\sigma^q &= q_\omega(\omega,\eta)\omega\sigma^\omega + q_\eta(\omega,\eta)\eta\sigma^\eta \\
\frac{q\sigma^q}{\sigma+\sigma_t^q} &= q_\omega\omega\big(\alpha b_t^0 + (1-\alpha)b_t^\lambda(1-\lambda) + (1-\alpha)l_t(1-\lambda) + m_t\big) + q_\eta\eta(x_t^j + \lambda x_t + m_t - 1) \\
q^2\sigma^2 &= (q - q_\omega\omega\big(\alpha b_t^0 + (1-\alpha)b_t^\lambda(1-\lambda) + (1-\alpha)l_t(1-\lambda) + m_t\big) - q_\eta\eta(x_t^j + \lambda x_t + m_t - 1))^2(\sigma+\sigma^q)^2.
\end{aligned}$$

Notice that I can express b^0, b^λ, l, x^j, x, and $(\sigma + \sigma^q)^2$ as polynomial functions of the unknown $q(\omega_{i,j}, \eta_{i,j})$ by rearranging and combining the market clearing condition and optimality conditions of different agents. I calculate $q_\omega(\omega, \eta)$ according

$$\frac{q_{i,j} - q_{i-1,j}}{\omega_{i,j} - \omega_{i-1,j}}$$

where $q_{i,j}$ denotes $q(\omega_{i,j}, \eta_{i,j})$. Note that I have already derived $q(\omega_{i-1,j}, \eta_{i-1,j})$ for $j = 1, 2, ..., J(i-1)$ from the $i-1$'th round. Hence, if I fix $\omega = \omega_{i,j}$, the above equation is an ODE of $q(\omega = \omega_{i,j}, \eta)$.

I use finite difference method to calculate $q_\eta(\omega_{i,j}, \eta_{i,j})$. The "upwind scheme" will dictate whether I use forward or backward difference. If $x_t^j + \lambda x_t + m_t \geq 1$, then I use backward difference

$$q_\eta(\omega_{i,j}, \eta_{i,j}) = \frac{q_{i,j} - q_{i,j-1}}{\eta_{i,j} - \eta_{i,j-1}}$$

and start the update of $q_{i,j}$ from $j = 2$ towards $\Psi(i)$. As the updating of $q_{i,j}$ proceeds, $x_t^j + \lambda x_t + m_t$ will be less than one if $\eta_{i,j}$ is large enough. Then, I will update $q_{i,j}$ starting from $\Psi(i) - 1$ towards $j = 2$ and use forward difference to calculate

$$q_\eta(\omega_i, \eta_j) = \frac{q_{i,j+1} - q_{i,j}}{\eta_{i,j+1} - \eta_{i,j}}.$$

D Robustness

To show the robustness of the main result of my paper, I vary each of the calibrated parameters by 10% except κ^d. Table 5 shows that the main result still holds in the neighborhood of the calibrated parameters.

This table shows the robustness of the main result the presence of bond financing is crucial for the optimal level of capital requirement. The "+" superscript refers to the case where the relevant parameter value raised by 10% from its calibrated level shown in Table 1 in the updated version. The "−" superscript refers to the case the relevant parameter value lowered by 10%.

case	with bond	without bond	case	with bond	without bond
"a^+"	3.3%	25%	"a^-"	4.6%	22.7%
"α^+"	3.3%	22.7%	"α^-"	4.2%	22.7%
"χ^+"	3.3%	23.8%	"χ^-"	3.3%	21.7%
"χ_η^+"	3.3%	22.7%	"χ_η^-	4%	25%
"δ_b^+"	2.9%	23.8%	"δ_b^-"	4%	23.8%
"κ^+	3.3%	22.7%	"κ^-	2.9%	22.7%
"λ^+"	4.2%	23.8%	"λ^-"	3.3%	22.7%
"σ^+"	4.6%	22.7%	"σ^-"	2.9%	23.8%

Table 5: Robustness

E Economy with Bond Financing

Panel A: set parameters

ρ	time discount rate	2%
δ	capital depreciation	10%
ϕ	capital adjustment cost	3
$1/\bar{x}$	capital requirement	6%
θ	bank credit fraction of safe firms	0.28
α	the fraction of safe experts	15.2%

Panel B: calibrated parameters

technology		
a	experts' productivity	0.145
δ_b	capital depreciation (bankers)	625%
σ	capital quality shock	2%
λ	idiosyncratic default likelihood	7.1%
finance		
κ	bankruptcy cost (loan)	15.5%
preference		
χ	experts' retirement rate	37.5%
χ_η	bankers' retirement rate	17%

Table 6: Parameters

Table 7: Targeted Moments[1]

This table presents the first and second moments of key endogenous variables (e.g., Sharpe ratio, investment-to-capital ratio, investment growth, consumption growth, bank credit to total credit ratio, and the risk premium of bank loans and corporate bonds) based on the sample of model simulations ("Model" column) and the real data ("Target" column). The last column "Source" lists the references of the target moments. The size of simulation sample is 50,000 and each simulation runs for 2,000 years.

Moment	Model	Target	Reference
Sharpe ratio (mean)	58%	48%	He and Krishnamurthy (2019)
investment-to-capital ratio (mean)[2]	9%	9%	He and Krishnamurthy (2019)
$\frac{\text{highest Sharpe ratio}}{\text{average Sharpe ratio}}$	17	15	He and Krishnamurthy (2019)
Sharpe ratio (volatility)[2]	30.6%	30%	He and Krishnamurthy (2019)
cov(Sharpe ratio, bank equity growth)[2]	0.5%	0.6%	He and Krishnamurthy (2019)
corr(consumption growth, output growth)[2]	0.83	0.88	Elenev et al. (2018)
risk premium on loans (mean)	1.64%	1.70%	De Fiore and Uhlig (2011)

[1] I use the density of the stationary distribution to calculate all moments.

[2] These are moments conditional on non-distressed states, which are defined as states with lowest 67% Sharpe ratios.

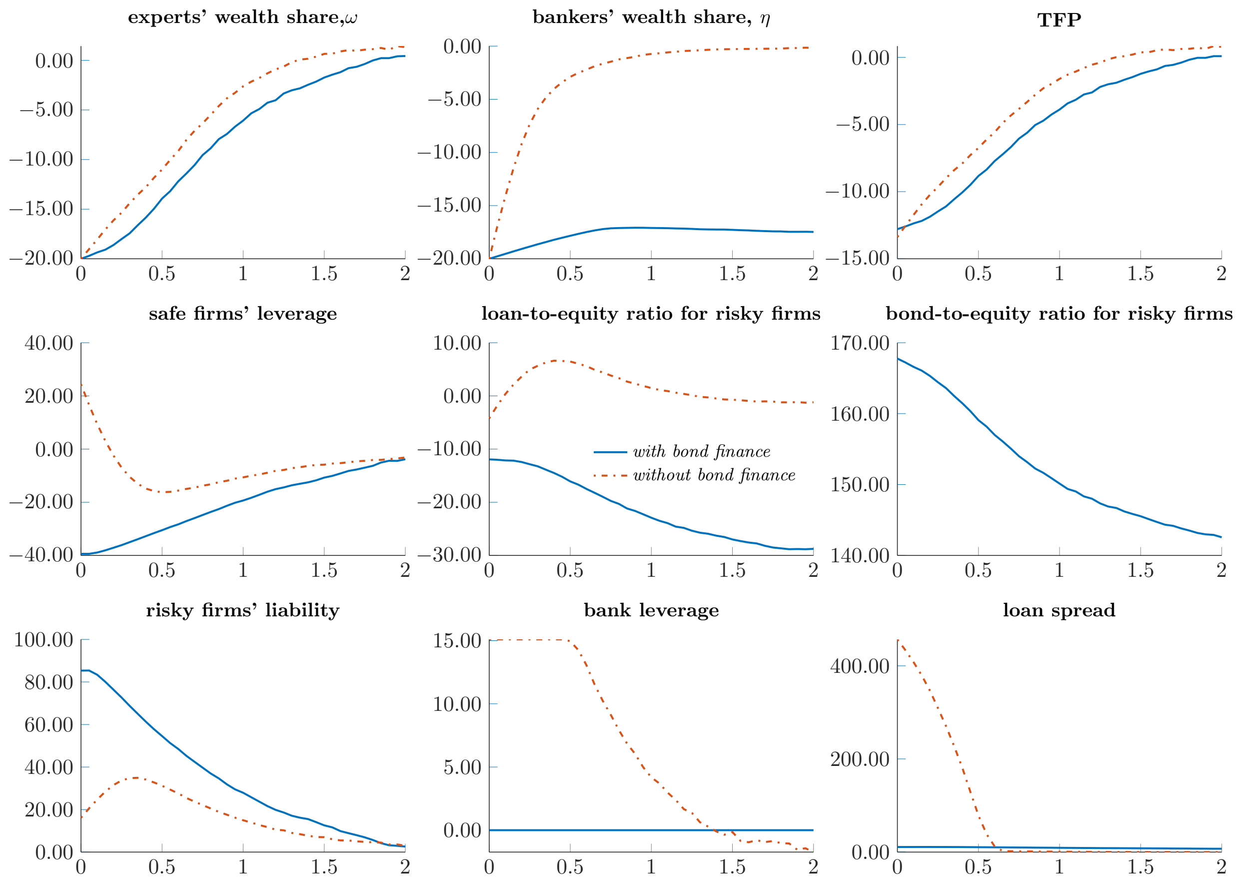

Figure 9: Dynamics

This figure shows the median sample path of nine key aggregate variables in two economies over two years after a hypothetical crisis where state variables decline by 20%: experts' wealth share (top left), bankers' wealth share (top middle), TFP (top right), consumption-to-physical capital ratio (middle left), investment-to-capital ratio (center), risky firms' liability (middle right), outstanding bonds (bottom left), outstanding loans (bottom middle), loan spread (bottom right). The solid lines refer to the economy with bond financing, and the dashed lines refer to the calibrated economy without bond financing.

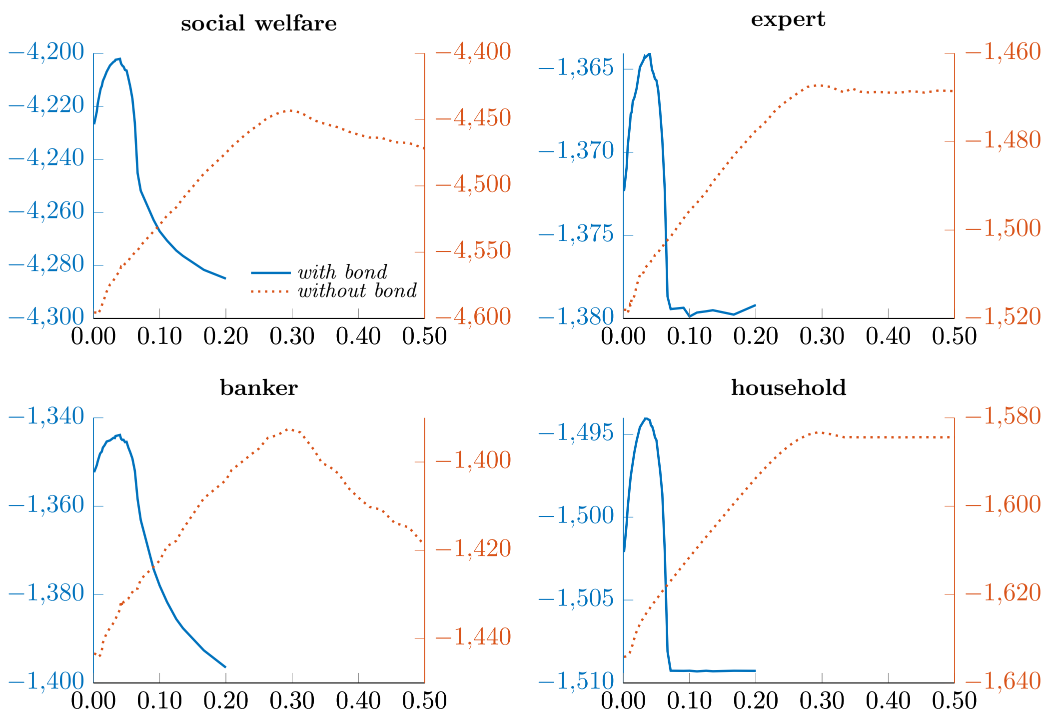

Figure 10: Welfare

This figure shows the relationship between the capital adequacy ratio $1/\bar{x}$ (horizontal axis) and the welfare of different types of agents in economies with and without bond financing. Solid lines refer to the calibrated economy with bond financing and dashed lines refer to the calibrated economy without bond financing. The aggregate welfare shown in the left panel is the sum of the welfare of the three types of agents. The welfare of an agent is the weighted sum of the agent's lifetime expected utility over all possible states of the economy. The weight of each state is its density of the simulated long-run stationary distribution. For the values of parameters other than $\bar{x}$, see Section 2.7.