Online Supplementary Materials for "Banking and Shadow Banking"

Ji Huang*
The Chinese University of Hong Kong

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This note collects supplementary materials for "Banking and Shadow Banking" (hereafter BSB). This note is not to be published. Section 1 contains the calibration of parameters and the source of aggregate securitization data that BSB uses. Section 2 develops a variant of the baseline model in which bankers can issue outside equity to some extent. In Section 3, we focus on an extension where households have Epstein-Zin preferences. Section 4 concentrates on a variant in which regular banks face the capital requirement constraint. In Section 5, we focus on the numerical challenge that we might have if we assume that the tax rate of regular bank debt is a constant. Proofs and figures are in the end of the note.

1 Calibration and Data

We set the time discount factor ρ to 3% to match the real interest rate estimated by Campbell and Cochrane (1999). Bankers' retirement rate χ is set at 15% to target the average Sharpe ratio. Without loss of generality, we set parameter σ as a rather small number 10^{-5} as it is the product $\chi(1-\sigma)$ that affects stochastic steady state of the economy (see equation 19 in BSB). We set bankers' productivity at 22.5% so that the average investment-to-capital ratio is close to 11% (He and Krishnamurthy, 2012). The productivity of less-productive households is chosen at 10% to match the fact that the Sharpe ratio during the 2007-09 financial crisis was approximately 15 times the average level (He and Krishnamurthy, 2012). Choices of the depreciation rate and the

*Contact details: 9/F, Esther Lee Building, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong, China. Email: huangjinku@gmail.com

Table 1: Moments[1]

Moment	Model	Target	Source
average Sharpe ratio	32.3%	40%	Wachter (2013)
$\frac{\text{highest Sharpe ratio}}{\text{average Sharpe ratio}}$	16.0	15	He and Krishnamurthy (2012)
average $\frac{\text{investment}}{\text{capital}}$ ratio	11.7%	11%	He and Krishnamurthy (2012)
average ratio of securitization	25.1%	25.1%	ratio of securitization in third quarter of 2006
bankers' overall leverage	2.8	3	He and Krishnamurthy (2012)
volatility of bankers' wealth growth rate			
in distress periods[2]	33.9%	31.5%	He and Krishnamurthy (2012)
in non-distress periods	18.2%	17.5%	He and Krishnamurthy (2012)

[1] We use the density of the stationary distribution to calculate all moments.
[2] The distress periods are those with highest 33% Sharpe ratio.

capital adjustment cost ϕ are standard in the macroeconomic literature (Christiano, Eichenbaum and Evans, 2005). We set the Poisson shock parameters to target the conditional volatility of the growth rate of bankers' wealth in distressed periods and in non-distressed periods. The distressed periods are defined as periods with the lowest 33% Sharpe ratios. The regulation parameter, tax rate τ, is set at 3% to target the average leverage of the entire banking sector. We set the intensity with which bankers can re-access shadow banking after default at 6% to target the ratio of securitization by non-agency issuers in the third quarter of 2006.

Table 2: Details of Securitization Data

	Outstanding	Securitized
Home Mortgages	FL383165105	FL673065105
Multifamily Residential Mortgages	FL143165405	FL673065405
Commercial Mortgages	FL383165505	FL673065505
Commercial and Industrial Loans[1]	FL253169255 FL263168005 FL263169255	FL673069505
Consumer Credit	FL153166000	FL673066000

[1] Becuase item FL253169255 is not available now, we use the ratio of securitization calculated by Loutskina (2011) to estimate the outstanding commercial and industrial loans.

Securitization. We follow Loutskina (2011) to compute the ratio of securitization. The difference is that we focus on securitization done by non-agency security issuers. All data are drawn from the "Flow of Funds Accounts of the United States". There are 5 loan categories. The details of items for each category are listed in Table 2.

2 Risky Shadow Bank Debt

In this section, we consider an extension of the baseline model in which bankers can issue outside equity up to ε proportion of their banks' outstanding equity. For shadow banks, this implies that investors of a shadow bank are willing to bear $1-\varepsilon$ proportion of asset losses that occur to the shadow bank. Let R_t^e denote the return of outside equity and r_t^s the return of shadow bank debt.

A banker's dynamic budget constraint is

$$dW_t = \left(W_t R_t + \frac{1-\varepsilon}{\varepsilon} W_t (R_t - R_t^e) + S_t (R_t - r_t - \tau_t) + S_t^* (R_t - r_t^s)\right) dt \\ + (W_t \pi_t - c_t) dt - \varepsilon \left(\frac{W_t}{\varepsilon} + S_t + (1 - \mathcal{D}_t) S_t^*\right) x_t^q dN_t.$$

The regular bank raise $(1-\varepsilon)W_t/\varepsilon$ outside equity. If a Poisson shock hits the economy, the banker only bears ε proportion of the loss to the regular bank. With respect to shadow banking, the regular bank extends implicit guarantees that cover ε proportion of the loss to the shadow bank.

Let E_t^h denote a household's holding of a regular bank's outside equity and O_t^h its holding of shadow bank debt. The dynamic budget constraint for a household is

$$dW_t^h = \left(W_t^h r_t + S_t^h \left(R_t^h - r_t\right) + E_t^h (R_t^e - r_t) + O_t^h (r_t^s - r_t) - c_t^h\right) dt \\ - \left(S_t^h + E_t^h (1 + \varepsilon s_t) + (1 - \varepsilon) O_t^h\right) x^q dN_t,$$

where s_t is the ratio of regular bank debt to inside bank equity. If a household contributes 1 dollar as outside equity, banker will match $\varepsilon/1-\varepsilon$ dollar as inside equity. Taking leverage into account, 1 dollar contribution of outside equity raises the size of a regular bank by $1+\varepsilon s_t/(1-\varepsilon)$. The risk exposure of 1 dollar outside equity is $(1+\varepsilon s_t)x_t^q$ if a Poisson shock hits the economy.

Households will take $\{r_t, R_t^h, R_t^e, r_t^s, t \geq 0\}$ as given and choose $\{S_t^h, E_t^h, O_t^h\}$ to maximize their expected life-time utility. First-order conditions with respect to E_t^h and O_t^h are

$$R_t^e - r_t = \lambda(1 + \varepsilon s_t)x_t^q,$$
$$r_t^s - r_t = \lambda(1 - \varepsilon)x_t^q.$$

If we plug in the expressions of R_t^e and r_t^s back to a banker's dynamic budget constraint, then we derive the dynamic budget constraint listed in BSB.

Next, we derive the optimality conditions of bankers. Given the dynamic budget constraint listed in BSB, first order conditions are

$$R_t - r_t - \tau_t - (1-\varepsilon)\lambda x_t^q \leq \frac{\varepsilon \lambda x_t^q}{1 - (1 + \varepsilon(s_t + s_t^*))x_t^q}, \text{ with equality if } s_t > 0$$
$$R_t - r_t - (1-\varepsilon)\lambda x_t^q \geq \frac{\varepsilon \lambda x_t^q}{1 - (1 + \varepsilon(s_t + s_t^*))x_t^q}, \text{ with equality if } s_t > 0$$

Since households are risk-neutral and bankers are risk-averse, bankers are willing to take higher leverage if they can share aggregate risks with households.

The counterpart of the enforcement constraint (15) in the case is

$$\frac{1}{\rho}\Big(s_t\left(R_t - r_t - \tau_t\right) + s_t^*\left(R_t - r_t\right) - (1-\varepsilon)(s_t + s_t^*)\lambda x_t^q + \lambda \ln\left(1 - \left(1 + \varepsilon(s_t + s_t^*)\right) x^q\right)\Big) + \lambda \hat{h}_t$$
$$\geq \frac{1}{\rho}\Big(\tilde{s}_t\left(R_t - r_t - \tau_t\right) + \bar{s}_t^*\left(R_t - r_t\right) - (1-\varepsilon)(\tilde{s}_t + \bar{s}_t^*)\lambda x_t^q + \lambda \ln\left(1 - \left(1 + \varepsilon \tilde{s}_t\right) x^q\right)\Big) + \lambda \hat{h}_t^d.$$

We can simplify the enforcement constraint and derive the maximum leverage of shadow banking listed in BSB by following an argument that is similar to that in the Section 2.1.4 in BSB.

The procedure of solving for the equilibrium of this variant is the same as it is for the baseline model in BSB. Detailed discussion of key properties of this variant is contained in BSB as well.

3 Households with Epstein-Zin Preference

In this section, we consider a variant of the baseline model where households have Epstein-Zin preferences with the time discount rate ρ, the relative risk-aversion coefficient γ, and the elasticity of intertemporal substitution b^{-1}.

In the modified model, the household chooses $\{c_t^h, S_t^h, n_t, t \geq 0\}$ to maximize

$$U_0^h = E_0 \left[\int_0^\infty f\left(c_s^h, U_s^h\right) ds \right],$$

where

$$f\left(c^h, U^h\right) = \frac{1}{1-b} \left\{ \frac{\rho \left(c^h\right)^{1-b}}{\left((1-\gamma) U^h\right)^{\frac{\gamma-b}{1-\gamma}}} - \rho (1-\gamma) U^h \right\}$$

and

$$U_s^h = E_s \left[\int_s^\infty f\left(c_v^h, U_v^h\right) dv \right], \text{ for } s > 0,$$

subject to the dynamic budget constraint (5) in BSB.

In solving for the equilibrium of the modified model, we need to characterize the optimal consumption and portfolio choices of households. To do so, we conjecture that the continuation value of a household with net worth W_t^h has the following functional form

$$V_t^h = V\left(\zeta_t, W_t^h\right) \equiv \frac{\left(\zeta_t W_t^h\right)^{1-\gamma}}{1-\gamma},$$

where ζ_t follows

$$d\zeta_t = \zeta_t \mu_t^\zeta dt - (\zeta_t - \hat{\zeta}_t) dN_t.$$

ζ_t is interpreted as the continuation value multiplier of a household's net worth as the function $V\left(\zeta_t, W_t^h\right)$ is homogeneous of degree $1-\gamma$ with respect to W_t^h. Given our conjecture, the Hamilton-Jacobi-Bellman equation of the household's dynamic programming problem is

$$0 = \max_{c^h, S^h} \left\{ f\left(c_t^h, V_t^h\right) + \mathcal{D}^{c^h, S^h} V\left(\zeta_t, W_t^h\right) \right\}, \tag{1}$$

where

$$\begin{aligned} \mathcal{D}^{c^h, S^h} V\left(\zeta_t, W_t^h\right) &= \left(W_t^h r_t + S_t^h\left(R_t^h - r_t\right) - c_t^h\right) \zeta_t^{1-\gamma} \left(W_t^h\right)^{-\gamma} \\ &+ \mu_t^\zeta \left(\zeta_t W_t^h\right)^{1-\gamma} + \lambda \left(\frac{\left(\hat{\zeta}_t\left(W_t^h - S_t^h x_t^q\right)\right)^{1-\gamma}}{1-\gamma} - \frac{\left(\zeta_t W_t^h\right)^{1-\gamma}}{1-\gamma} \right). \end{aligned}$$

We summarize the key results of the problem in the following proposition.

Proposition 1 *Each household's optimal consumption weight* $\left\{\tilde{c}_t^h, t \geq 0\right\}$, *optimal portfolio weight*

$\{s_t^h, n_t, t \geq 0\}$, *and the process* $\{\zeta_t, t \geq 0\}$ *satisfy*

$$\left(\tilde{c}_t^h\right)^b = \frac{\rho}{\zeta_t^{1-b}}, \tag{2}$$

$$R_t^h - r_t \leq \frac{\lambda x_t^q}{\left(1 - s_t^h x_t^q\right)^\gamma} \left(\frac{\hat{\zeta}_t}{\zeta_t}\right)^{1-\gamma}, \text{with equality if } s_t^h > 0, \tag{3}$$

$$0 = \frac{\rho\left(\tilde{c}_t^h\right)^{1-b}}{(1-b)\zeta_t^{1-b}} - \frac{\rho}{1-b} + r_{t-} + s_t^h\left(R_t^h - r_t\right) - \tilde{c}_t^h + \mu_t^\zeta + \lambda\left(\frac{\left(\hat{\zeta}_t\left(1 - s_t^h x_t^q\right)\right)^{1-\gamma}}{1-\gamma} - \frac{1}{1-\gamma}\right) \tag{4}$$

where $\tilde{c}_t^h = c_t^h / W_t^h$ *and* $s_t^h = S_t^h / W_t^h$.

There are a few market clearing conditions in the modified model that are different from those in the baseline model. First, the risk-free rate is not constant, which instead is jointly determined by the optimal portfolio choices of both bankers and households (equation 3) and the dynamics of the continuation value multiplier $\{\zeta_t, t \geq 0\}$ (equation 4). Second, the consumption good market does not clear automatically in the modified model since households are risk-averse. Given households' optimal consumption choice condition (2), we have the market clearing condition for the consumption goods

$$a\psi_t + a^h(1-\psi_t) - g(\iota_t) = \left(\rho\omega_t + \rho^{\frac{1}{b}}\zeta_t^{\frac{b-1}{b}}(1-\omega_t)\right)q_t.$$

As in BSB, we focus on the Markov equilibrium of the modified model where shadow banking exists. The choice of parameter values is that $\rho = 4\%$, $\gamma = 2$, $b = 0.5$, $\chi = 0.1$, $a = 0.225$, $a^h = 0.1$, $\delta = 10\%$, $\phi = 3$, $\lambda = 1$, $x = 4\%$, $\tau = 1.2\%$, and $\xi = 5\%$. Figures 3 and 4 in Section 7 show that results found in the baseline model survive in the modified model.

In what follows, we study the welfare implication of bank regulation of the modified model. As in BSB, we suppose that the amount of physical capital in period 0 equals one and there are a representative banker and a representative household in the economy. The welfare pair of the representative household and banker is

$$\left(\frac{(\zeta_0(1-\omega_0)q_0)^{1-\gamma}}{1-\gamma}, \frac{\ln(\omega_0 q_0)}{\rho} + h_0\right),$$

where ζ_0 is the continuation multiplier of the representation household in period 0.

Different from the baseline model, the equilibrium risk-free rate in the modified model depends on the credit demand and supply. If the regulatory authority starts regulating regular banking and shadow banking is still unsustainable, the decrease in the banker's credit demand lowers the risk-free rate (Panel d in Figure 5), which hurts the household (Panel b in Figure 5). Naturally, the banker's welfare improves because of the low volatility of her wealth and the low borrowing cost. Overall, imposing tax on regular banking still improves the sum of two representative agents' welfare when shadow banking does not emerge (Panel c in Figure 5).

When shadow banking is sustainable, tightening bank regulation starts to improve the household's welfare. There are two underlying forces. First, the increased credit demand from shadow banks pushes up the risk-free rate, which benefits the household. Panel d in Figure 5 shows that this effect is so large that the average risk-free rate in an economy with a positive tax rate could be higher than it is in an economy without any tax. The underlying intuition is that if the banker has to pay a high tax rate for regular banking her willingness to pay a relatively high interest rate for shadow banking must be high as well.

The second force is that as the shadow banking sector expands the banker holds more fraction of capital goods in the economy and thus the household's exposure to the aggregate risk declines. Panel d in Figure 6 indicates that the volatility of the household's wealth declines as the tax rate increases until it hits 4%.

Overall, if we consider the sum of the two agent's welfare, we observe that strengthening bank regulation can raise the total welfare of the economy. Nevertheless, when regulation is too tight, tightening regulation makes social welfare worse off. This is primarily the consequence of the increased financial instability that the expansion of the shadow banking sector causes.

In summary, this section highlights novel channels, through which tightening regulation of regular banking improves the household's welfare. In particular, strengthening bank regulation helps the shadow banking sector expand, and the consequential increase in credit demand raises the return for the household to supply funds and lowers the household's exposure to the aggregate risk.

4 Quantity Control

In this section, we investigate a variant of the baseline model in BSB, in which the regular banking sector is subject to a quantity control. In particular, we consider the capital-requirement constraint, which imposes an upper bound $\bar{s}$ for a regular bank's debt-to-equity ratio.

With the price control replaced by the quantity control, a banker's dynamic budget constraint becomes

$$dW_t = \left(W_t R_t + \left(S_t + S_t^*\right)\left(R_t - r_t\right) - c_t\right) dt - \left(W_t + S_t + S_t^*\right) x^q dN_t.$$

In addition to the leverage constraint for shadow banking (4), the banker in the modified model faces the capital-requirement constraint $S_t \leq \bar{s} W_t$. Similar changes apply to bankers who cannot access shadow banking due to default.

We first focus on the dynamics of endogenous variables and then move to the regulatory smile result of the quantity-control model. A number of endogenous variables have dynamics similar to the baseline model (Panels a-e in Figure 7). However, the leverage dynamics of shadow banking change drastically. This is the consequence of the fact that when bankers' share of wealth is small the excess return is high and the incentives to build up leverage are strong. In these states, it is extremely costly to default on shadow bank debt because regular banking only allows for considerably low leverage. Therefore, when bankers' share of wealth is small, the enforcement problem is not severe and the leverage of shadow banking is high. This property is absent in the baseline model because bankers do not face a binding leverage constraint for regular banking.

The counterfactual result of shadow banking being counter-cyclical is primarily the consequence of the simplification that bankers also play the role of producers. In our model, when the capital misallocation is already severe in recessions, the time-invariant capital ratio requirements actually prevents more productive agents from raising external credit, which is apparently unwise and in the opposite of improving social welfare. In reality, what we often observe is that governments utilize all possible policy instruments in downturns to encourage banks' credit supply to the real sector.

Figure 8 shows that the regulatory smile result continues to hold in the modified model with quantity control. Very lenient bank regulation comes with the low leverage of shadow banking and high financial instability. As bank regulation tightens (i.e., the maximum leverage of regular banking $\bar{s}$ declines), financial instability initially diminishes. However, if the regulation is so tight that the shadow banking sector becomes sizeable, tighter regulation causes higher financial instability.

5 Numerical Challenge in the Case $\tau_t = \tau$

If $\tau_t = \tau$, the economy may evolve from a state where regular banks are marginal buyers of physical capital ($s_t > 0, s_t^* > 0$) to a state where shadow banks are marginal buyers ($s_t = 0, s_t^* > 0$). In other words, the first-order condition that determines μ_t^q changes from

$$\underbrace{\frac{a - g(\iota_t)}{q_t} + \iota_t - \delta + \mu_t^q}_{\equiv R_t} - r_t - \tau = \frac{\lambda x_t^q}{1 - (1 + s_t + s_t^*) x_t^q}$$

to

$$\underbrace{\frac{a - g(\iota_t)}{q_t} + \iota_t - \delta + \mu_t^q}_{\equiv R_t} - r_t \geq \frac{\lambda x_t^q}{1 - (1 + s_t + s_t^*) x_t^q}, \text{ with equality if } s_t^* < \bar{s}_t^*.$$

Recall that the derivative of $q(\omega)$ is given by the Ito's formula $q'(\omega) = \frac{q(\omega)\mu^q}{\omega\mu^\omega}$. Since shadow banks' marginal borrowing cost is lower than regular banks' borrowing cost by a constant τ, the change in marginal buyers leads to a discrete change in μ^q and thus a kink of the price of physical capital $q(\omega)$. The kink in the solution of a delay differential equation makes it very challenging to numerically solve for the solution, especially when we do not know the location of the kink. Chapters 1 and 2 of Bellen and Zennaro (2013) have a nice discussion of this challenge. The assumption $\tau_t = \min\{\tau, \tau s_t\}$ can smooth the path of μ_t^q as well as that of $q'(\omega)$. As the economy evolves from the state where $s_t > 0$ to the state $s_t = 0$, the difference between the marginal funding cost of a regular bank and a shadow bank τ_t gradually converges to zero. Therefore, we can avoid the discontinuity in μ_t^q and $q'(\omega_t)$.

6 Proofs

Proof of Proposition 1 in BSB. Without loss of generality, we focus on a banker with wealth W_t in period t and explicitly express her continuation value in different cases.

We start with the case that the banker is retired. Here, we explicitly derive the lifetime utility of a retired banker given that she still lives in the economy instead of taking the lifetime utility assumed in BSB as given. Since logarithmic agents only consumer ρ fraction of their wealth, the growth rate of her wealth is $r_{t+v} - \rho$ in period $t + v$. Hence, the banker's wealth in period $t + u$

will be $(1-\sigma)W_t \exp\left(\int_0^u r_{t+v} - \rho \, dv\right)$. The banker's continuation value at time t is

$$\begin{aligned}
& \int_0^\infty \exp(-\rho u)\left(\ln(\rho(1-\sigma)W_t) + \int_0^u r_{t+v} - \rho \, dv\right) du \\
= & \frac{\ln(W_t)}{\rho} + \frac{\ln(\rho(1-\sigma))}{\rho} + \int_0^\infty \exp(-\rho u) \int_0^u r_{t+v} - \rho \, dv \, du \\
= & \frac{\ln(W_t)}{\rho} + \frac{\ln(\rho(1-\sigma))}{\rho} + \frac{1}{\rho}\int_0^\infty \exp(-\rho v)\left(r_{t+v} - \rho\right) dv,
\end{aligned}$$

which is denoted by $\ln(W_t)/\rho + h_t^r$. Note that if the risk-free rate $r_t = \rho$, then the lifetime utility is exactly what we assume in BSB.

We use the same idea to express the continuation value of a banker who can access shadow banking. Given the banker's optimal portfolio choices $\left(s_{t+u}, s_{t+u}^*\right)$, if she does not retire in period $t+u$, her wealth in period $t+u$, W_{t+u} is

$$W_{t+u} \equiv W_t \exp\left(\begin{array}{c} \int_0^u \left(R_{t+v} + s_{t+v}\left(R_{t+v} - r_{t+v}\right) + s_{t+v}^*\left(R_{t+v} - r_{t+v}\right) - \rho\right) dv \\ + \int_0^u \ln\left(1 - \left(1 + s_{t+v} + s_{t+v}^*\right) x_{t+v}^q\right) dN_{t+v} \end{array}\right).$$

Since $E_t[N_{t+dt} - N_t] = \lambda dt$, we rewrite W_{t+u}

$$W_{t+u} = W_t \exp\left(\int_0^u G_{t+v} dv\right), \text{ where}$$

$$G_{t+v} \equiv R_{t+v} + s_{t+v}\left(R_{t+v} - r_{t+v}\right) + s_{t+v}^*\left(R_{t+v} - r_{t+v}\right) - \rho - \lambda \ln\left(1 - \left(1 + s_{t+v} + s_{t+v}^*\right) x_{t+v}^q\right).$$

Let $t+T$ denote the stopping time that the banker retires. Her continuation value at time t is

$$\begin{aligned}
& E_t\left[\int_0^T \exp(-\rho u)\left(\ln(\rho W_t) + \int_0^u G_{t+v} dv\right) du + \exp(-T\rho)\left(\frac{\ln(W_{t+T})}{\rho} + h_{t+T}^r\right)\right] \\
= & E_t\left[\int_0^T \exp(-\rho u)\left(\ln(W_t) + \ln(\rho) + \int_0^u G_{t+v} dv\right) du + \exp(-\rho T)\left(\frac{\ln(W_t)}{\rho} + \frac{1}{\rho}\int_0^T G_{t+v} dv + h_{t+T}^r\right)\right] \\
= & E_t\left[\int_0^T \exp(-\rho u)\left(\ln(W_t) + \ln(\rho) + \frac{1}{\rho} G_{t+u}\right) du + \exp(-\rho T)\left(\frac{\ln(W_t)}{\rho} + h_{t+T}^r\right)\right] \\
= & \frac{\ln(W_t)}{\rho} + E_t\left[\int_0^T \exp(-\rho u)\left(\ln(\rho) + \frac{1}{\rho} G_{t+u}\right) du + \exp(-\rho T) h_{t+T}^r\right] \\
= & \frac{\ln(W_t)}{\rho} + E_t\left[\int_0^\infty \chi \exp(-\chi T)\left(\int_0^T \exp(-\rho u)\left(\ln(\rho) + \frac{1}{\rho} G_{t+u}\right) du + \exp(-\rho T) h_{t+T}^r\right) dT\right] \\
= & \frac{\ln(W_t)}{\rho} + \frac{\ln(\rho)}{\rho + \chi} + E_t\left[\int_0^\infty \exp(-(\rho+\chi)u)\left(\frac{G_{t+u}}{\rho} + \chi h_{t+u}^r\right) du\right]
\end{aligned}$$

which we denote as $\ln(W_t)/\rho + h_t$. We have used integration by parts multiple times in the above derivation.

Finally, we consider the case that the banker who cannot use shadow banking but obtain such opportunity at intensity ξ. Let $\tilde{s}_t$ denote her optimal portfolio choice, which satisfies

$$R_t - r_t - \tau_t = \frac{\lambda x_t^q}{1 - (1 + \tilde{s}_t)\, x_t^q}, \tag{5}$$

and T_ξ the stopping when the banker obtain the access to shadow banking. Her continuation value at time t is

$$\begin{aligned}
&E_t \left[\begin{array}{c} \int_0^{\min\left(T,T_\xi\right)} \exp(-\rho u) \left(\ln(\rho W_t) + \int_0^u R_{t+v} + \tilde{s}_{t+v}\left(R_{t+v} - r_{t+v} - \tau_{t+v}\right) + s_{t+v}\tau_{t+v} - \rho\ dv\right) du \\ + \int_0^{\min\left(T,T_\xi\right)} \exp(-\rho u) \int_0^u \ln\left(1 - (1 + \tilde{s}_{t+v})\, x_{t+v}^q\right) dN_{t+v}\ du \\ + \exp(-T_\xi \rho) \left(\frac{\ln\left(W_{t+T_\xi}\right)}{\rho} + h_t \right) \mathbf{1}_{\left(\min\left(T_\xi,T\right)=T_\xi\right)} + \exp(-T\rho) \left(\frac{\ln(W_{t+T})}{\rho} + h_{t+T}^r \right) \mathbf{1}_{\left(\min\left(T_\xi,T\right)=T\right)} \end{array} \right] \\
=& E_t \left[\int_0^T \exp(-\rho u) \left(\ln(\rho W_t) + \int_0^u G_{t+v} \right) du + \exp(-T\rho) \left(\frac{\ln\left(\hat{W}_{t+T}\right)}{\rho} + h_{t+T}^r \right) \right] \\
&- E_t \left[\int_0^{\min\left(T,T_\xi\right)} \exp(-\rho u) \left(\int_0^u s_{t+v}^* \tau_{t+v} dv \right) du + \exp(-\rho \min(T,T_\xi)) \frac{1}{\rho} \int_0^{\min\left(T,T_\xi\right)} s_{t+v}^* \tau_{t+v} dv \right] \\
=& \ln(W_t)/\rho + h_t - H_t,
\end{aligned}$$

where $s_{t+v}\tau_{t+v}$ is the tax rebate to each banker in the economy and

$$\hat{W}_{t+T} \equiv W_t \exp \left(\begin{array}{c} \int_0^u \left(R_{t+v} + s_{t+v}\left(R_{t+v} - r_{t+v}\right) + s_{t+v}^*\left(R_{t+v} - r_{t+v}\right) - \rho\right) dv \\ + \int_0^u \ln\left(1 - \left(1 + s_{t+v} + s_{t+v}^*\right) x_{t+v}^q\right) dN_{t+v} \end{array} \right).$$

The first equation comes from the condition that $s_t + s_t^* = \tilde{s}_t$. Next, we verify that this condition always holds. If $s_t > 0$, the first-order equation (13) in BSB and equation (5) implies $s_t + s_t^* = \tilde{s}_t$. If $s_t = 0$, then $\tau_t = 0$ and the first-order equation (14) in BSB and equation (5) also give rise to $s_t + s_t^* = \tilde{s}_t$.

$$\begin{aligned}
H_t &= E_t \left[\int_0^{\min\left(T,T_\xi\right)} \exp(-\rho u) \left(\int_0^u s_{t+v}^* \tau_{t+v} dv \right) du + \exp(-\rho \min(T,T_\xi)) \frac{1}{\rho} \int_0^{\min\left(T,T_\xi\right)} s_{t+v}^* \tau_{t+v} dv \right] \\
&= E_t \left[\int_0^\infty \exp\left(-(\rho + \chi + \xi)\, u\right) \frac{\tau_{t+u}}{\rho} s_{t+u}^* du \right]
\end{aligned}$$

■

Proof of Theorem 1 in BSB. We will apply the contraction mapping theorem to show the uniqueness of the solution $H(\omega) = 0$. First, we define a complete metric space. Since the state variable ω is between 0 and $\bar{\omega}$, we focus on the space $B((0, \bar{\omega}])$ of bounded continuous functions $h : (0, \bar{\omega}] \to R$ under sup norm. Theorem 3.1 in Stokey et al. (1989) implies that $B((0, \bar{\omega}])$ is a complete metric space.

We will use Blackwell's sufficient conditions to show Γ is a contraction mapping. Suppose both h, $\tilde{h} \in B((0, \bar{\omega}])$ and $h(\omega) \geq \tilde{h}(\omega)$, for all $\omega \in (0, \bar{\omega}]$, since

$$\bar{s}^*(\omega) = \frac{\rho\lambda H(\hat{\omega})}{R(\omega) - r - \tau(\omega)},$$

all portfolio choices permitted under $\tilde{h}(\cdot)$ are feasible under $h(\cdot)$. Hence, $\Gamma h \geq \Gamma\tilde{h}$, for all $\omega \in (0, \bar{\omega}]$. Next, we need to show that there exists a positive constant $\beta < 1$ such that $\Gamma(h + v) \leq \Gamma h + \beta v$, for all $h \in B((0, \bar{\omega}])$, $v \geq 0$, $\omega \in (0, \bar{\omega}]$. Consider

$$\Gamma(h + v)[\omega] = E_0 \left[\int_0^\infty \exp(-(\rho + \xi + \chi) u) \frac{\min\{\tau, \tau s_u\}}{\rho} s_u^* du \middle| \omega_0 = \omega \right],$$

where

$$s^*(\omega) \leq \frac{\rho\lambda h(\hat{\omega})}{R(\omega) - r - \tau(\omega)},$$

and (s, s^*) are portfolio weights of a banker in the equilibrium of a hypothetical economy with exogenous $h(\cdot)$. Since the lower bound of h is zero, then

$$\begin{aligned}
\bar{s}^*(\omega) &= \frac{\rho\lambda(h(\hat{\omega}) + v)}{R(\omega) - r - \tau(\omega)} = \frac{\rho\lambda h(\hat{\omega})}{R(\omega) - r - \tau(\omega)} + \frac{\rho\lambda v}{R(\omega) - r - \tau(\omega)} \\
&= \frac{\rho\lambda h(\hat{\omega})}{R(\omega) - r - \tau(\omega)} + \frac{\rho v(1 - (1 + s(\omega) + s^*(\omega))x^q(\omega))}{x^q(\omega)} \\
&= \frac{\rho\lambda h(\hat{\omega})}{R(\omega) - r - \tau(\omega)} + \rho v\left(\frac{1}{x^q(\omega)} - (1 + s(\omega) + s^*(\omega))\right) \\
&\leq \frac{\rho\lambda h(\hat{\omega})}{R(\omega) - r - \tau(\omega)} + \frac{\rho v}{x}
\end{aligned}$$

With the assistance of above inequality, we derive that

$$\begin{aligned}\Gamma\left(h+v\right)[\omega] &\le \Gamma h[\omega]+E_0\left[\int_0^\infty \exp\left(-\left(\rho+\xi+\chi\right)u\right)\frac{v}{x}\tau du\middle|\,\omega_0=\omega\right]\\ &\le \Gamma h[\omega]+\frac{\tau}{\left(\rho+\xi+\chi\right)x}v.\end{aligned}$$

If $\tau<\left(\rho+\xi+\chi\right)x$, Γ is a contraction mapping. ■

7 Figures

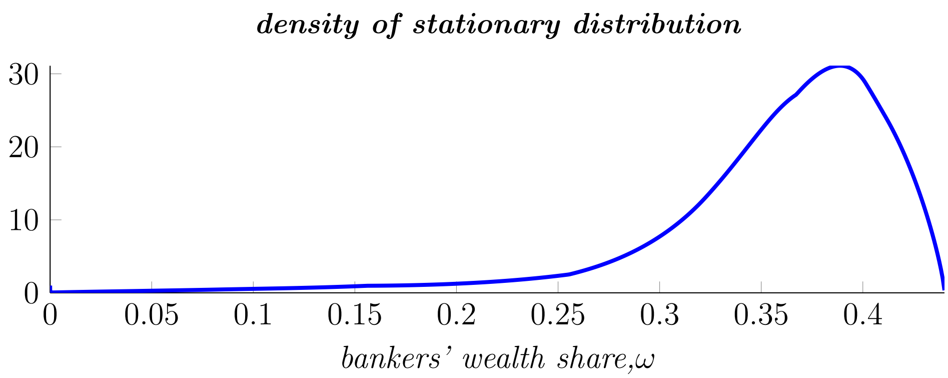

Figure 1: the density of stationary distribution. For parameter values, see Section 2.3 in BSB.

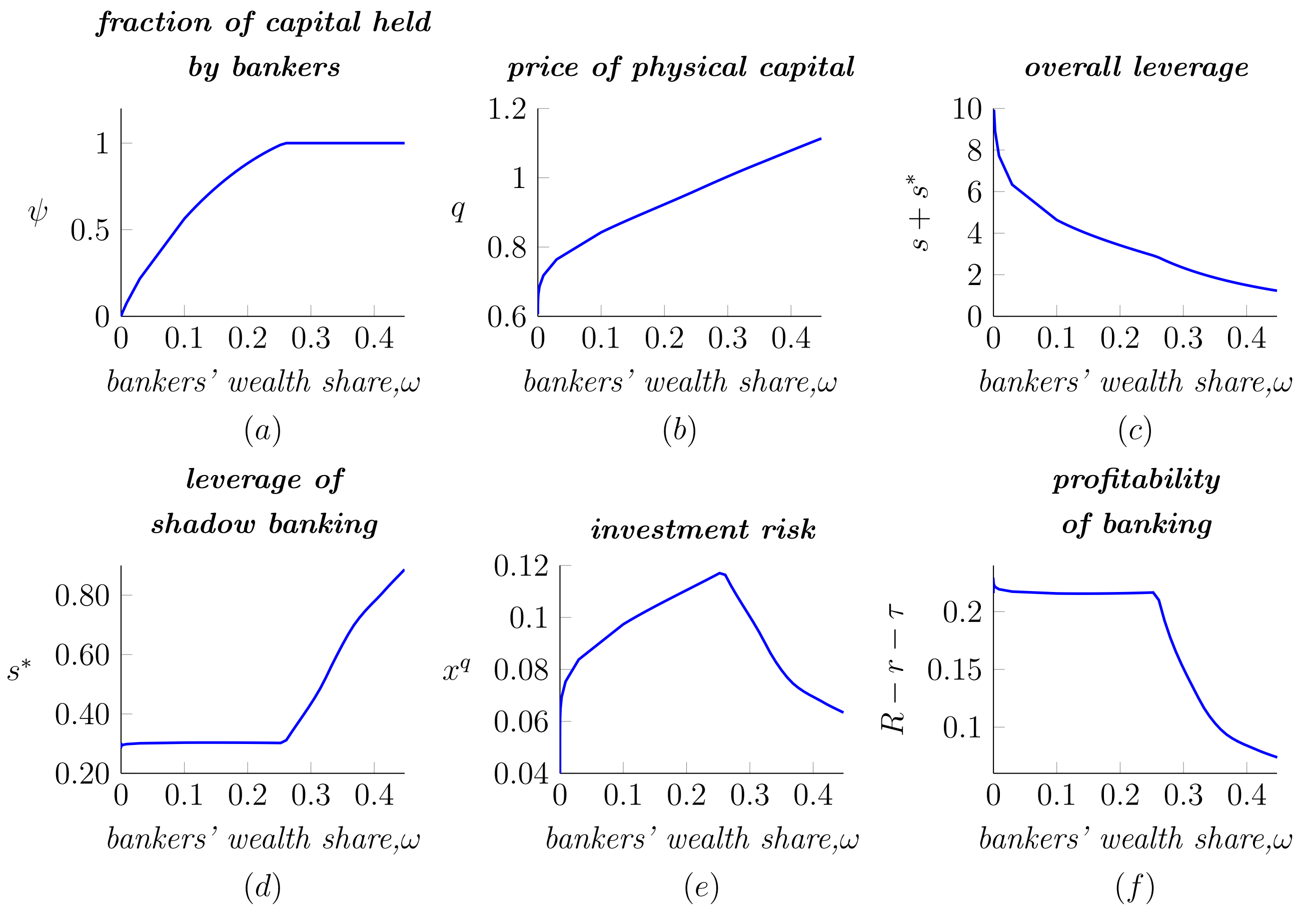

Figure 2: This figure presents the fraction of physical capital held by bankers ψ, the price of physical capital q, banker's overall leverage $s + s^*$, the leverage of shadow banking s^*, investment risk x^q, and profitability of banking $R - r - \tau$ as functions of the state variable ω (i.e., bankers' wealth share) in the modified model with a constant opportunity cost of default $\bar{H} = 2.1818$, which equals the average cost of default in the calibrated model in Section 2.3 in BSB. For other parameter values see the same section.

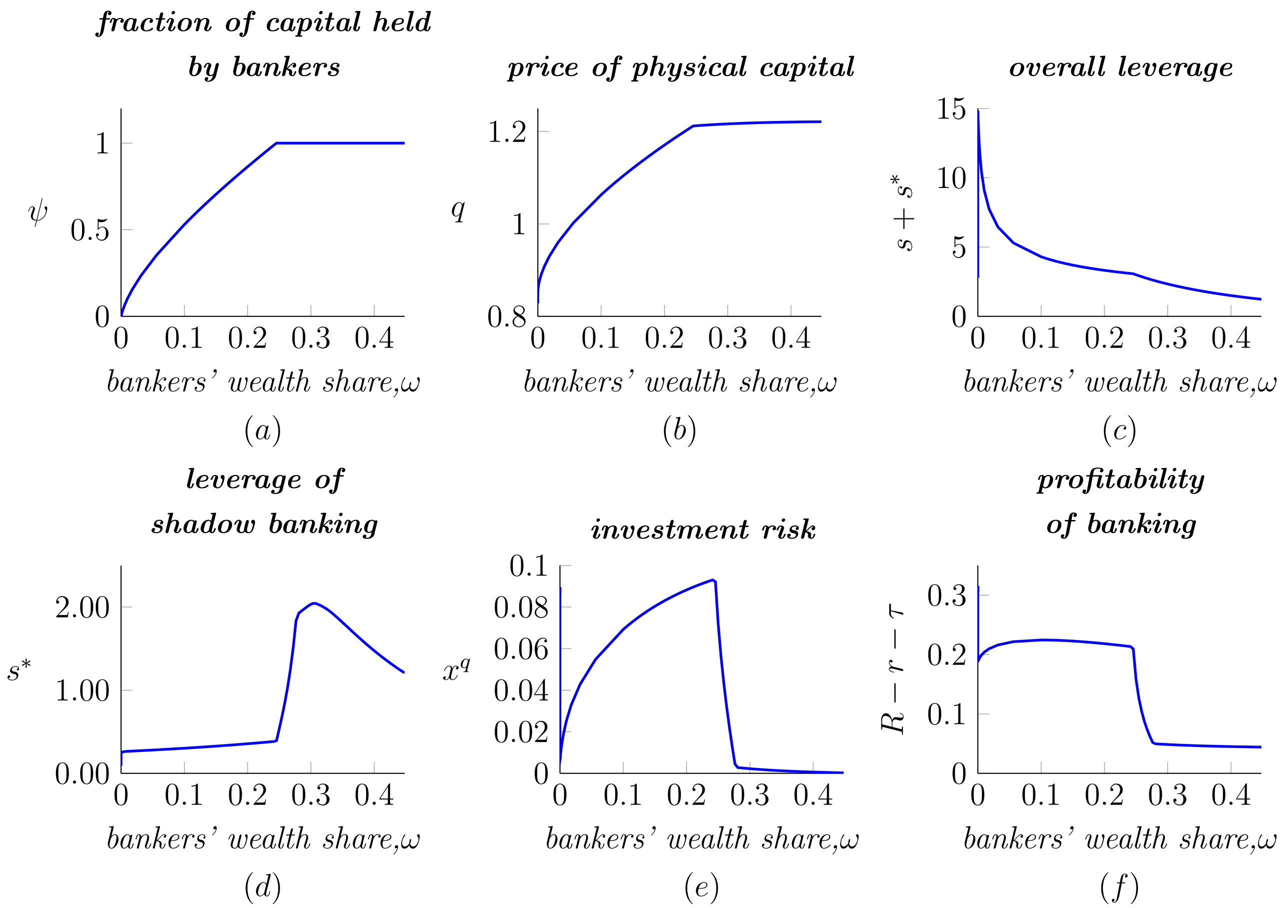

Figure 3: This figure presents the fraction of physical capital held by bankers ψ, the price of physical capital q, banker's overall leverage $s + s^*$, the leverage of shadow banking s^*, investment risk x^q, and profitability of banking $R - r - \tau$ as functions of the state variable ω (i.e., bankers' wealth share) in the modified model in which households have Epstein-Zin preferences. For the choice of parameter values, see Section 3.

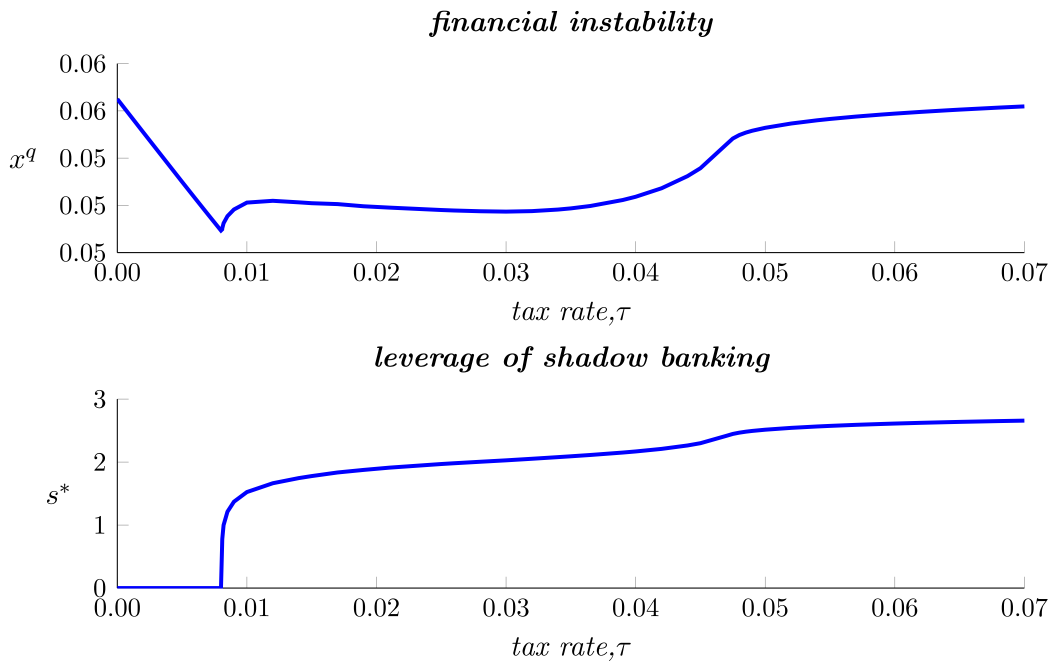

Figure 4: This figure shows how the change in the tax rate influences the investment risk x^q (the upper panel) and the leverage of shadow banking (the lower panel) at the stochastic steady states of the modified model with households of Epstein-Zin preference. We use the stationary distribution to calculate moments. For parameter values other than τ, see Section 3.

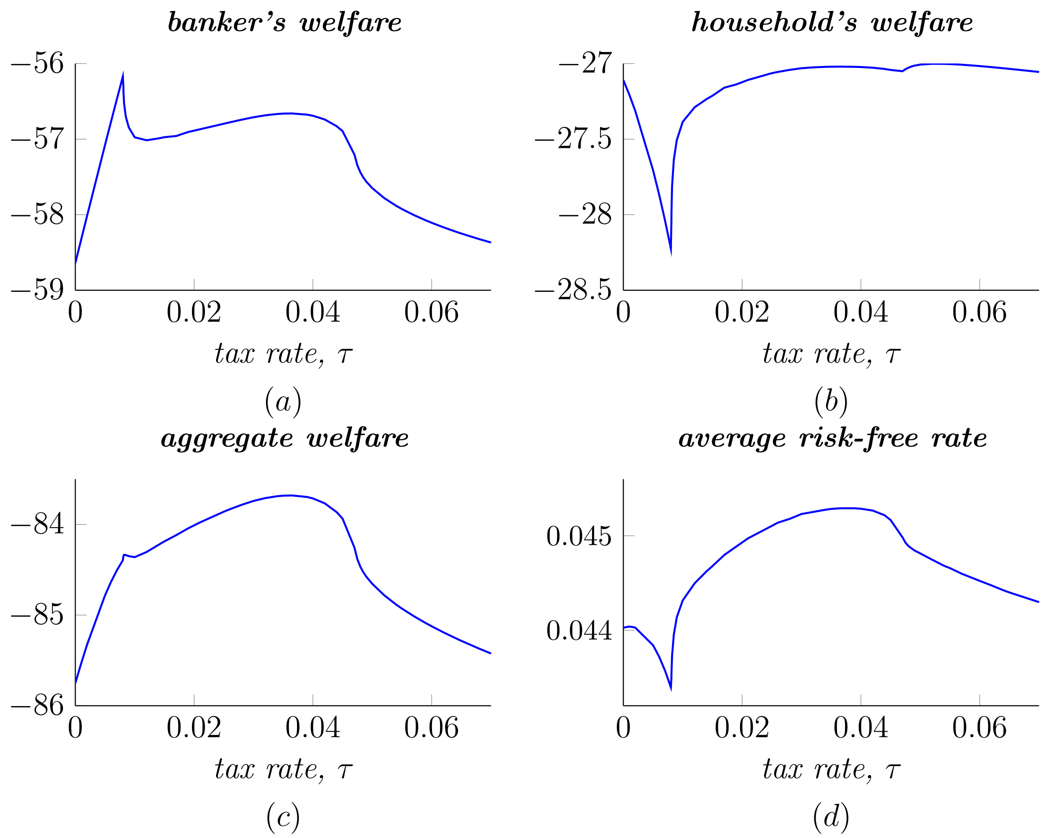

Figure 5: Welfare
This figure shows how the representative banker's welfare (panel a), the representative household's welfare (panel b), the sum of the two agents' welfare (panel c), and the average risk-free rate (panel d) change with the tax rate. For agents' welfare, we focus on state $\omega_0 = 0.38$. For parameter values other than τ, see Section 3.

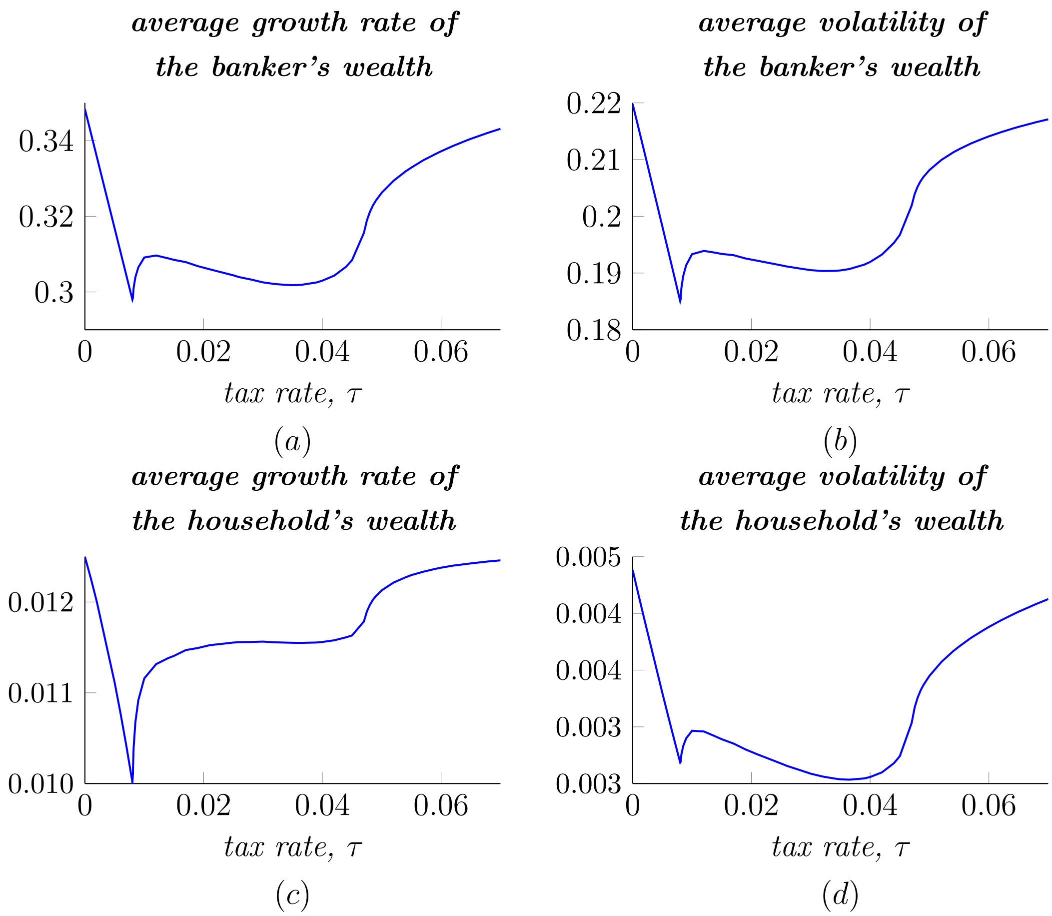

Figure 6: Welfare

This figure shows how tax rate influences the average growth rate (left panels) and the average volatility (right panels) of both the representative banker's wealth (upper panels) and the representative household's wealth (lower panels). We use the stationary distribution to calculate moments. For parameter values other than τ, see Section 3.

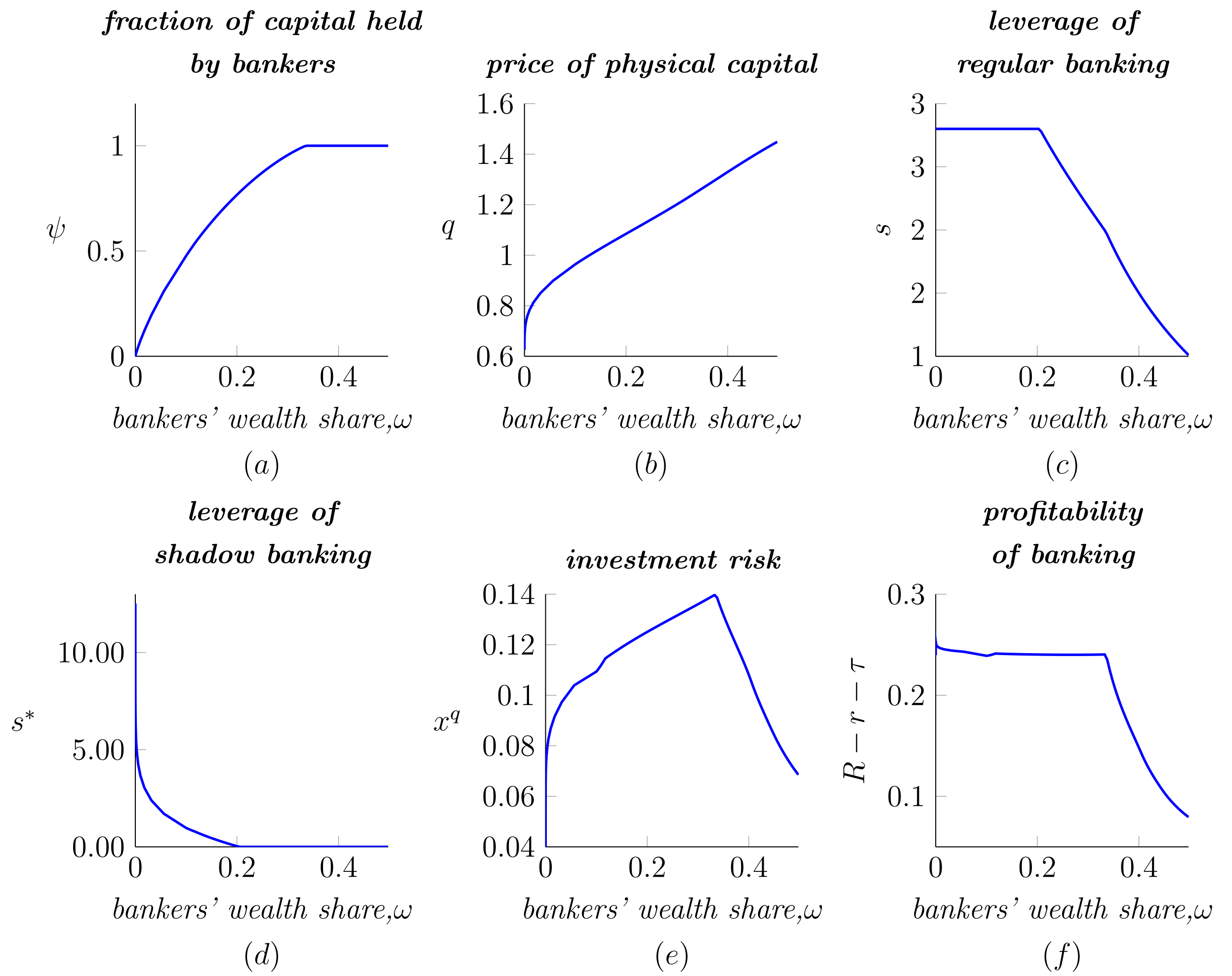

Figure 7: This figure presents the fraction of physical capital held by bankers ψ, the price of physical capital q, the leverage of regular banking $s + s^*$, the leverage of shadow banking s^*, investment risk x^q, and profitability of banking $R - r - \tau$ as functions of the state variable ω (i.e., bankers' wealth share) in the modified model with the capital requirement constraint. The choice of parameter values follows: $\rho = 3\%$, $\chi = 1\%$, $a = 0.225$, $a^h = 0.1$, $\delta = 10\%$, $\phi = 3$, $\lambda = 1$, $x = 4\%$, and $\bar{s} = 2.8$.

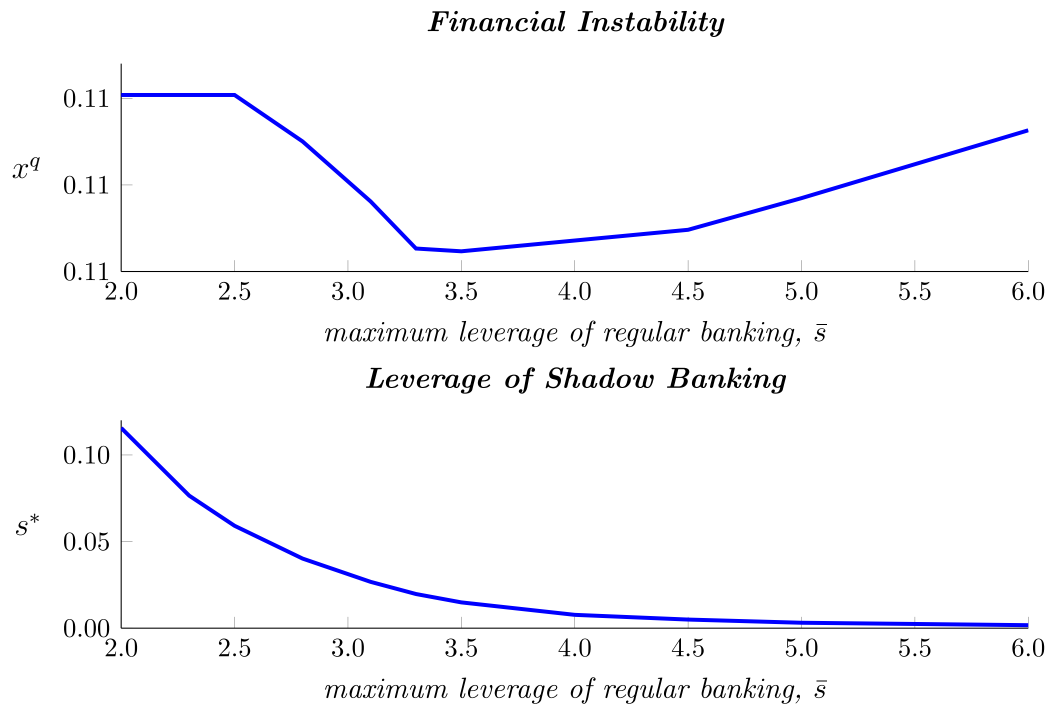

Figure 8: This figure shows how the change in capital-requirement constraints influences the investment risk x^q (the upper panel) and the leverage of shadow banking (the lower panel) at the stochastic steady states of the modified model. The stochastic steady state is the state where $\omega\mu^\omega - \lambda(\omega - \hat{\omega}) = 0$. The maximum leverage of regular banking $\bar{s}$ is 2.8. For the choice of other parameter values, see Figure 7.

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