Banking and Shadow Banking

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Abstract

This paper incorporates shadow banking modeled as off-balance-sheet financing in a continuous-time macro-finance framework. Regular banks pursue regulatory arbitrage via shadow banking, and they support their shadow banks with implicit guarantees. We show that an enforcement problem with implicit guarantees gives rise to an endogenous constraint on leverage for shadow banking. Our model captures that shadow banking is pro-cyclical and that shadow banking increases endogenous risk. Tightening bank regulation in our model increases the borrowing capacity of shadow banking and financial instability. Furthermore, we show that a limited degree of aggregate risk sharing does not improve financial stability in the presence of shadow banking.

Keywords: shadow banking; implicit guarantee; bank regulation; endogenous risk

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Introduction

The 2007-09 global financial crisis underlined the significance of shadow banking for both financial stability and bank regulation. Although shadow banking is not a well-defined concept, many experts agree that regulatory arbitrage is one of its main drivers (Acharya et al., 2012; Gorton and Metrick, 2010; Pozsar et al., 2010). Following this idea, we regard shadow banking in this paper as off-balance-sheet financing that banks use for economizing their regulatory capital.

Regulatory arbitrage offered by shadow banking is not free as it brings with it an enforcement problem. As credit enhancement, banks promise that they will protect their off-balance-sheet entities in trouble. These promises, typically referred to as implicit guarantees, have been widely used in various segments of the shadow banking sector such as asset-backed commercial paper (ABCP), money market mutual funds (MMF), and securitization.[1] The enforcement problem with implicit guarantees exists because these promises cannot be incorporated into any contract that a third party would enforce. If such promises were explicit, the arbitrage opportunity would disappear because regulatory authorities would consider off-balance-sheet debt in the same way as banks' on-balance-sheet obligations like deposits.

In this paper, we explore the implications of shadow banking for both financial stability and bank regulation within the continuous-time macro-finance framework proposed by Brunnermeier and Sannikov (2014). The framework offers an ideal setup for three reasons. First, it allows us to characterize the full dynamics of an economy with an enforcement problem. Second, the financial amplification effect, i.e., endogenous risk, emphasized in this framework is suitable for the analysis of financial instability caused by the amplification effect. As shadow banking activities affect financial amplification, the framework captures the link between shadow banking and financial stability. Third, the financial amplification effect leads to a market inefficiency that calls for bank regulation.

On the empirical side, our model captures two facts of shadow banking that we observed in the run-up to and during the 2007-09 financial crisis. First, shadow banking is pro-cyclical. Acharya et al. (2012) and McCabe (2010) document the boom and bust of ABCP and MMF markets respectively. According to the author's own calculations, the ratio of securitization undertaken by

[1] Gorton and Souleles (2007) have an extensive discussion on the institutional details of securitization and off-balance-sheet financing, and emphasize the enforceability problem of implicit guarantees for such financing. In practice, a large number of financial instruments or their investors have enjoyed implicit supports from sponsoring financial firms. For instance, HSBC spent $35 billion in order to bring assets of its off-balance-sheet structured investment vehicles (SIVs) onto its balance sheet in late 2007 (Goldstein, 2007); Citigroup moved $37 billion assets in SIVs back to its balance sheet (Moyer, 2007). In the money market, Securities and Exchange Commission reported that at least 44 MMFs received support from their sponsors to avoid breaking the buck during the 2007-09 financial crisis (McCabe, 2010).

private issuers in the U.S. rose from 13% in early 2003 to 22% in the middle of 2007.[2] However, this rising trend completely reversed in the 2007-09 financial crisis with the ratio dropping to 17% by the end of 2009. The second fact is the reintermediation process that shadow banks conducted fire sales of assets to traditional banks during the crisis. He et al. (2010) estimate that hedge funds and broker-dealers reduced their holdings of securitized assets by $800 billion, while the traditional banking sector raised its holding of securitized assets by $500 billion. Acharya et al. (2012) document that most losses on assets financed by ABCP remained with sponsoring banks because they absorbed the bulk of these assets when ABCP investors exited the market in 2007.

Our paper makes a number of theoretical contributions. We show that the enforcement problem with implicit guarantees gives rise to an endogenous leverage constraint on shadow banking. By exploring the dynamics of shadow banking, we highlight two key results: *i*) the pro-cyclicality of shadow banking increases endogenous risk, i.e., financial instability, and *ii*) strengthening regulation of regular banking raises the borrowing capacity of shadow banking, which in turn increases endogenous risk. In addition, we show that a certain degree of aggregate risk sharing does not necessarily improve financial stability in the presence of shadow banking. We next illustrate the main mechanisms driving these results.

In this paper, we model regular banking as a regular bank's *on-balance-sheet financing* and shadow banking as the regular bank's collateralized *off-balance-sheet financing*. Regular banking is subject to bank regulation that does not apply to shadow banking. In contrast with Brunnermeier and Sannikov (2014), aggregate risk in our model is driven by a jump process, which gives rise to credit risk as shadow banks may default. To enhance the safety of shadow bank debt, regular banks extend implicit guarantees, which in turn are subject to the enforcement problem.

Tightening regulation of regular banking raises the maximum leverage of shadow banking. As in the limited enforcement literature, we assume that a regular bank loses the opportunity of accessing shadow banking if it defaults on its shadow bank debt. Thus, the opportunity cost for the regular bank to default amounts to the present value of future benefits that shadow banking offers. Since more stringent regulation implies greater opportunities of regulatory arbitrage offered by shadow banking, the opportunity cost of default is larger in economies with tighter regulation, and thus the leverage of shadow banking is higher in such economies.

Since the leverage of shadow banking is endogenous, there exists an interesting feedback loop between the opportunity cost of default and the leverage of shadow banking. For instance, if the opportunity cost declines due to a loosening of bank regulation, the incentive to default rises and

[2] The data source is "Financial Accounts of the United States." See the online appendix for the details of how we construct the ratio of securitization one by private asset-backed security issuers.

the borrowing capacity of shadow banking declines. Thus, shadow banking offers fewer benefits to regular banks, which leads to an even lower opportunity cost of default. Because of this feedback loop, shadow banking could be unsustainable if the regulation of regular banking is sufficiently lenient. Furthermore, we show that if there is no such feedback loop in a model, tightening bank regulation does not necessarily lead to the expansion of the shadow banking system.

The dynamics of shadow banking are driven by the leverage constraint. In economic booms, high asset prices and the corresponding low return from holding assets lower the profitability of banking. Hence, regular banks are not inclined to leverage up via shadow banking and plan to default in the event of a negative shock. As a result, the leverage constraint is less tight in economic upturns, which permits the high borrowing capacity of shadow banking. In addition, the feedback loop emphasized above accelerates the expansion of the shadow banking sector in economic booms.

Shadow banking increases endogenous risk as a general equilibrium effect in our model. Since their borrowing capacity is pro-cyclical, shadow banks accumulate substantial amounts of assets in upturns. However, when the economy faces a recession, the borrowing capacity of shadow banks shrinks, which forces them to sell a large scale of assets to regular banks (i.e., reintermediation). Shadow banks have to sell these assets at fire-sale prices because bank regulation restrains regular banks from acquiring too many assets. Hence, asset prices have to drop a lot so that regular banks are willing to purchase the assets. Endogenous risk rises due to the asset fire-sale.

The relationship between bank regulation and financial instability is U-shaped. When regulation is loose enough, shadow banking is unsustainable. In such a situation, tightening regulation leads at first to lower instability. But when regulation becomes sufficiently tight, the borrowing capacity of shadow banking expands and a considerable number of banking activities shift to the shadow banking sector. Hence, more stringent regulation, in this circumstance, eventually gives rise to a larger shadow banking system and higher financial instability.

In addition, we find that in the presence of shadow banking financial stability does not improve substantially when a limited degree of aggregate risk sharing becomes possible in an economy. This is in contrast with the conventional wisdom that aggregate risk sharing yields better financial stability. The intuition for our result is that better risk sharing lowers a sponsor's incentive to default on its shadow bank debt. Thus, better risk sharing makes the leverage constraint less tight and the borrowing capacity of shadow banking higher. Since shadow banking increases endogenous risk, the expansion of the shadow banking sector dampens the positive effect of aggregate risk sharing.

Related Literature. The literature on shadow banking is swiftly growing and diverse. Different papers model shadow banking in drastically different ways, and Adrian and Ashcraft (2016) provide a thorough survey of this growing literature. In this paper, we attempt to categorize a few models of shadow banking along two dimensions: the motive for shadow banking and the type of negative externalities that shadow banking causes.

The existence of shadow banking can be demand/preference driven. For example, in Gennaioli et al. (2013), infinitely risk-averse households only value securities' worst scenario payoffs, and shadow banking can increase such payoffs by pooling different assets together. Meanwhile, in Moreira and Savov (2017), the preference specification of households leads directly to a demand for the liquid securities that shadow banking generates.

The second motive for shadow banking is regulatory arbitrage, as we discuss in this paper. Luck and Schempp (2014), Ordonez (2013), and Plantin (2014) are papers that fall into this category.

Models of shadow banking differ with respect to the type of the externalities that shadow banking causes. The first category includes non-pecuniary externalities. In Plantin (2014), shadow banking exposes the real sector to counter-productive uncertainty. In both Luck and Schempp (2014) and Gennaioli et al. (2013), creditors of shadow banks suffer from unexpected default that bank runs or crises cause. Generally, investments financed by shadow banking in these models have worse or more volatile fundamentals than investments financed by regular banking.

Unlike papers discussed in the previous paragraph, we do not assume that shadow banking involves any investments of inferior quality as in Moreira and Savov (2017). Instead, we focus on the pecuniary externality; that is, the leverage choices of individual shadow banks cause excessive endogenous risk because they do not internalize the price impact of their actions in the competitive equilibrium.

Although Plantin (2014) also touches upon the idea that tight regulation may have negative unintended consequences, our paper differs from his work in three critical aspects. First, we focus on the class of bank regulations that restricts the use of bank leverage; in contrast, Plantin (2014) examines regulation that prohibits banks from issuing outside equity. Second, we recognize financial instability as the endogenous risk that the financial market generates; for Plantin (2014), however, the riskiness of outside equity reflects the instability that is counterproductive for the real sector. Last, the dynamic general equilibrium setting of our framework allows us to characterize dynamic properties of shadow banking and to discuss the trade-off between economic growth and financial stability, for which the static setting in Plantin (2014) is not suitable.

This paper is also related to the literature on pecuniary externalities. One closely related paper is Bianchi (2011), whose quantitative examination of the pecuniary externality in a dynamic general equilibrium model highlights that raising borrowing costs can prevent excessive borrowing and improve welfare.

For our methodology in this paper, we follow the emerging literature (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2012b, 2013) that considers economies with financial frictions in a continuous-time setting. The methodology allows a full characterization of the entire dynamics of an economy. As a departure from this literature, we consider aggregate jump risks in our framework. The jump process makes it easy to model the insolvency risk of a shadow bank. Along with the insolvency risk, the tractability of the continuous-time method allows us to endogenize the leverage constraint on shadow banking and to explicitly solve for its debt capacity. Our contribution to this literature is to demonstrate how to solve a continuous-time macro-finance model with an enforcement problem.

The paper is structured as follows. We first establish our baseline model in Section 1. In Section 2, we then characterize the equilibrium of this baseline model and illustrate the main results with numerical examples. We explore the welfare implication of the baseline model in Section 3. Section 4 considers an extension of the model, in which debt issued by shadow banks is risky. Section 5 concludes.

1 The Baseline Model

In the baseline model, we introduce shadow banking into the macro-finance framework developed by Brunnermeier and Sannikov (2014), in which an economy is populated by productive bankers and less productive households. Unlike Brunnermeier and Sannikov (2014) that allows a single type of debt financing, bankers in our model can raise funds in two forms: regulated regular banking, modeled as on-balance-sheet financing, and unregulated shadow banking, modeled as off-balance-sheet financing.

1.1 Technology and Preferences

Time $t \in [0, \infty)$ is continuous. Let K_t be the aggregate "efficiency units" of physical capital in the economy and k_t the holding of a banker. A banker holding physical capital k_t produces consumption goods y_t at rate $y_t = ak_t$ over a short period of time dt, where a is a constant. We assume that bankers produce $\iota_t k_t$ units of new physical capital over dt with inputs $\iota_t k_t$ and capital adjustment costs $0.5\phi(\iota_t - \delta)^2 k_t$, both of which are paid in consumption goods. Parameter δ is the depreciation rate of physical capital and ϕ a positive constant.[3] Physical capital held by a banker evolves according to

$$dk_t = k_t \left(\iota_t - \delta\right) dt - k_t x dN_t,$$

where x is a positive constant and $\{N_t\}_{t=0}^{\infty}$ is a Poisson process with the arrival rate λ.[4] The Poisson shock is the only source of aggregate risk that the economy faces.

To have compact expressions, we let y_t denote the left limit of a stochastic process $\{y_s\}_{s=0}^{\infty}$ at time t and $\hat{y}_t$ denote the right limit. For instance, the amount of physical capital held by a banker will jump from k_t to $\hat{k}_t$ if the Poisson shock arrives, where $\hat{k}_t = k_t(1-x)$.

Households are less productive. Physical capital k_t^h held by a household generates consumption goods $y_t^h = a^h k_t^h$, where $a^h < a$. The capital adjustment costs $0.5\phi(\iota_t^h - \delta)^2 k_t^h$ apply to households' production of new physical capital. The law of motion for physical capital held by households is

$$dk_t^h = k_t^h \left(\iota_t^h - \delta\right) dt - k_t^h x dN_t.$$

Households are risk neutral with a time discount factor ρ. Although households may have negative consumption in the baseline model, main results that we will derive later still hold in the setup where households have Epstein-Zin preferences (see the online appendix).

Bankers have logarithmic preferences with the same time discount factor ρ. We assume that bankers retire and exit the economy with independent Poisson arrival rate χ and new bankers arrive at the same rate. When new bankers enter the economy, they inherit a positive σ proportion of retiring bankers' wealth. These assumptions limit the possibility that bankers take over all wealth in the economy and undo effects of financial frictions that Section 1.3 will cover. We assume that if a banker retires with wealth W_t, she will earn lifetime utility $J^r(W_t) = \frac{1}{\rho}\ln\big(\rho(1-\sigma)W_t\big)$. Hence, the expected lifetime utility of a banker is

$$E_0\left[\int_0^T e^{-\rho t} u\left(c_t\right) dt + e^{-\rho T} J^r\left(W_T\right)\right], \tag{1}$$

[3] We assume that capital adjustment cost to be quadratic in net investment, as consistent with Christiano et al. (2005), He and Krishnamurthy (2012a), and many other quantitative macroeconomic models.

[4] Poisson shocks adjust the "efficiency units" of physical capital held by a banker. In this setup, where capital quality shocks are proportional, the economy is scale-invariant with the aggregate "efficiency units" of physical capital. The scale-invariance property is useful for equilibrium characterization. See Gertler and Karadi (2011) and Gertler et al. (2012) for the same setting in discrete-time models.

where T denotes the stochastic retirement time that occurs at rate χ and

$$u(c) = \begin{cases} \ln(c), & \text{if } c > 0, \\ -\infty, & \text{otherwise.} \end{cases}$$

1.2 Return from Holding Physical Capital

The market for physical capital is frictionless. The price of physical capital is in units of consumption goods, denoted by q_t. The law of motion for q_t, which we will solve for in equilibrium, is

$$dq_t = q_t \mu_t^q dt - (q_t - \hat{q}_t) dN_t,$$

where μ_t^q is the growth rate of the price of physical capital. If a Poisson shock hits the economy at time t, the price of physical capital q_t jumps to $\hat{q}_t$.

At time t, in the absence of a negative shock, the return from holding a unit of physical capital includes the net output $a - \iota_t - 0.5\phi(\iota_t - \delta)^2$, the accumulation of physical capital $(\iota_t - \delta)q_t$, and the rise in the price of physical capital $\mu_t^q q_t$. In the presence of the Poisson shock, a unit of physical capital drops to $1 - x$ units and the price of physical capital jumps to $\hat{q}_t$. Hence, the total loss is $q_t - (1 - x)\hat{q}_t$. In summary, the rate of return for bankers from holding physical capital is

$$R_t dt - x_t^q dN_t, \text{ where} \tag{2}$$
$$R_t \equiv \frac{a - \iota_t - 0.5\phi(\iota_t - \delta)^2}{q_t} + \iota_t - \delta + \mu_t^q \text{ and } x_t^q \equiv \frac{q_t - (1 - x)\hat{q}_t}{q_t}.$$

We label x_t^q as endogenous risk. Similarly, the rate of return for households holding physical capital is

$$R_t^h dt - x_t^q dN_t, \text{ where } R_t^h \equiv \frac{a^h - \iota_t^h - 0.5\phi(\iota_t^h - \delta)^2}{q_t} + \iota_t^h - \delta + \mu_t^q$$

1.3 Equity Issuance Friction, Bank Regulation, and Shadow Banks

In this section, we introduce three sets of frictions with respect to bankers' external financing. Firstly, bankers face a constraint on equity issuance, which leads to a market inefficiency when they cannot trade contracts written on the aggregate state of the economy. Secondly, bank regulation is introduced to improve market efficiency. As a response to the regulation, bankers establish shadow banks to pursue regulatory arbitrage. The third friction is an enforcement problem that shadow banking is subject to.

The ***constraint on equity issuance*** is common in models with financial frictions. We could justify the constraint with agency problems between bankers and households as in He and Krishnamurthy (2012b) and Di Tella (2017). In the baseline model, we assume that bankers must retain 100 percent equity of their regular banks and the only channel for them to raise external funds is to issue short-term *risk-free* debt.[5] In Section 4, we relax the constraint and allow bankers to issue a limited amount of outside equity.

Given that financial markets are ***incomplete***, the constraint on equity financing leads to the lack of aggregate risk sharing and market inefficiency. Since bankers can only raise external funds by using leverage, their exposure to aggregate risks is disproportionately high when contingent securities are unavailable (Di Tella, 2017). Small shocks could have large effects on the economy due to the amplification through bankers' balance sheets. Moreover, individual bankers do not internalize these effects into their leverage decisions, which is a source of inefficiency (Lorenzoni, 2008; Stein, 2012). Thus, bank regulation is necessary for improving market efficiency.

Bank regulation in the model is a tax on regular banks' debt. We interpret the tax as the shadow cost of all bank regulations that banks face.[6] The rate is $\tau_t = \min\{\tau, \tau s_t\}$, where τ is a positive constant and s_t is the debt to equity ratio of the regular banking sector. We set the tax rate as $\min\{\tau, \tau s_t\}$ instead of a constant τ for two reasons. First, if $\tau_t = \tau$, the solution of the baseline model would have a kink at $s_t = 0$, and this kink would jeopardize our numerical algorithm.[7] Second, setting $\tau_t = \min\{\tau, \tau s_t\}$ simplifies the characterization of the leverage constraint on shadow banking. In Section 2.3, we argue that assuming $\tau_t = \tau$ would not change the qualitative predictions of the baseline model. To counterbalance the wealth effect of bank regulation on bankers, we assume that tax revenues are repaid back to regular banks instantly as lump-sum subsidies, and the ratio of subsidy to bank equity is π_t.

To circumvent the bank regulation, a banker sponsors a ***shadow bank*** and earns its residual value by charging a management fee in each period. In practice, this activity is referred to as off-balance-sheet financing. Similar to regular banks, shadow banks are subject to the constraint

[5] We can decompose any risky debt in our model into the risk-free component and the equity component. Since bankers cannot issue outside equity, households only hold risk-free debt in equilibrium.

[6] Kisin and Manela (2016) estimate the shadow cost of capital ratio requirement imposed on U.S. major banks. In Drechler et al. (2017), the opportunity cost of reserve requirement is equivalent to the tax on regular bank debt in our model.

[7] We define the solution of the baseline model in Section 2.2. The online appendix contains the detailed discussion of the numerical problem that arises if $\tau_t = \tau$.

on equity issuance. In the baseline model, households only accept risk-free debt issued by shadow banks. In Section 4, we allow shadow bank debt to be risky.

In contrast to regular banks, we assume that bankers cannot retain any equity of their shadow banks. The rationale of this assumption is that if they hold equity of their shadow banks, the regulatory authority will treat these shadow banks as regular ones. Given this assumption, shadow bank debt is backed by physical capital with no equity buffer. Investors of a shadow bank would bear any loss that occurs to the shadow bank unless its sponsor bails it out and absorbs the loss with her own wealth.

To enhance the safety of shadow bank debt, bankers extend implicit guarantees of bailing out their shadow banks in trouble. Offering explicit guarantees is not feasible as the regulatory authority will consider any debt with explicit guarantees as its sponsoring bank's on-balance-sheet debt. Since no third party would enforce implicit guarantees, shadow banking is subject to an *enforcement problem.* The enforcement problem does not apply to regular banking because the debt of a regular bank is senior to its equity.

To ensure that shadow bank debt is risk-free, households impose a leverage constraint on shadow banking: a shadow bank can borrow up to $\bar{s}_t^*$ times the wealth of its sponsor at time t. If a banker defaults on her shadow bank debt, we assume that she can re-enter the shadow bank debt market only if a random event occurs. This event arrives at a Poisson rate ξ. Based on this punishment scheme, households will fix $\bar{s}_t^*$ such that the continuation value of planning to default for a banker is the same as the continuation of not doing so. We will solve for $\bar{s}_t^*$ in Sections 2.1.3 and 2.1.4. To simplify the characterization of $\bar{s}_t^*$, we assume that when households lend to a shadow bank, they do not observe the leverage of its sponsor's regular bank.

1.4 Problems for Bankers and Households

Suppose a banker has wealth W_t. Denote the value of her regular bank debt by S_t. The excess return from holding physical capital funded by regular banking is $S_t \left(R_t - r_t - \tau_t\right) dt - S_t x_t^q dN_t$, where r_t is the risk-free rate. $R_t - r_t - \tau_t$ is the excess return earned by the bank in the absence of Poisson shocks. If a Poisson shock hits the economy, the bank loses $S_t x_t^q$. The banker also manages a shadow bank. We denote the value of the shadow bank debt by S_t^*. The leverage constraint on shadow banking implies

$$S_t^* \leq \bar{s}_t^* W_t. \tag{3}$$

The banker earns the difference between the asset return $R_t S_t^*$ and the interest expense $r_t S_t^*$. Let $\mathcal{D}_t$ denote the strategic decision. Like $\{S_t, S_t^*\}_0^\infty$, the process $\{\mathcal{D}_t\}_0^\infty$ is predictable with respect to

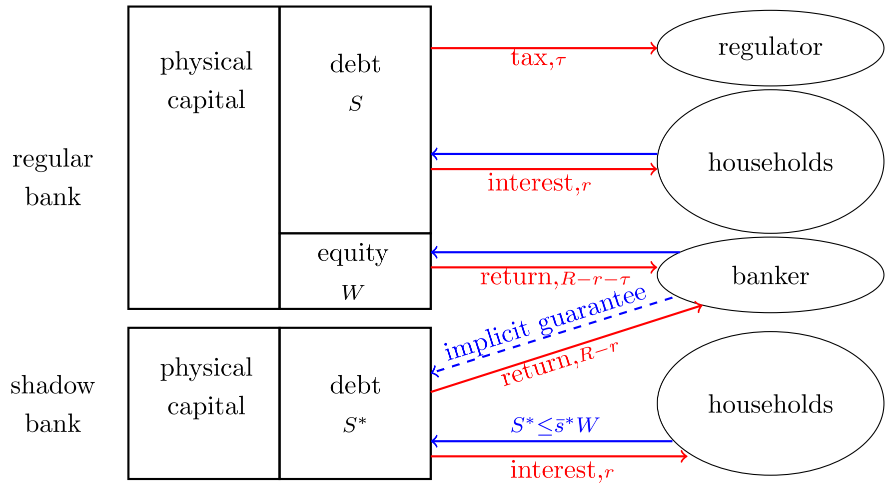

Figure 1: This figure shows the balance sheets of the regular and shadow banks managed by a banker.

the filtration generated by $\{N_t\}_0^\infty$. If $\mathcal{D}_t = 0$, the banker will bear the loss $S_t^* x_t^q$ for creditors of her shadow bank when the Poisson shock hits the economy; if $\mathcal{D}_t = 1$, the banker will not take the loss. Hence, the banker's dynamic budget constraint is

$$\begin{aligned} dW_t = & \left(W_t R_t + S_t \left(R_t - r_t - \tau_t\right) + S_t^* \left(R_t - r_t\right) + W_t \pi_t - c_t\right) dt \\ & - \left(W_t + S_t + \left(1 - \mathcal{D}_t\right) S_t^*\right) x_t^q dN_t, \end{aligned} \tag{4}$$

where c_t is the banker's consumption. In summary, the banker takes $\{q_t, r_t, \tau_t, \pi_t, \bar{s}_t^*\}_{t=0}^\infty$ as given and chooses $\{c_t, S_t, S_t^*, \iota_t, \mathcal{D}_t\}_{t=0}^\infty$ to maximize her expected lifetime utility (1) subject to the leverage constraint on shadow banking (3), the solvency constraint $W_t \geq 0$, and the dynamic budget constraint (4).

If a banker cannot issue shadow bank debt due to his previous default on shadow bank debt, his dynamic budget constraint becomes

$$dW_t = \left(W_t R_t + S_t \left(R_t - r_t - \tau_t\right) + W_t \pi_t - c_t\right) dt - \left(W_t + S_t\right) x_t^q dN_t.$$

The banker chooses $\{c_t, S_t, \iota_t\}_{t=0}^\infty$ to maximize his expected lifetime utility subject to solvency and dynamic budget constraints.

In addition to risk-free debt, households hold physical capital. Denote by S_t^h the value of physical capital that household h manages. The wealth W_t^h of the household evolves according to

$$dW_t^h = \left(W_t^h r_t + S_t^h \left(R_t^h - r_t\right) - c_t^h\right) dt - S_t^h x_t^q dN_t, \tag{5}$$

where c_t^h is the household's consumption. The household takes $\{q_t, r_t\}_{t=0}^{\infty}$ as given and chooses $\{c_t^h, S_t^h\}_{t=0}^{\infty}$ to maximize

$$U_0^h = E_0 \left[\int_0^\infty e^{-\rho t} c_t^h dt\right]$$

subject to the solvency constraint $W_t^h \geq 0$ and the dynamic budget constraint (5).

1.5 Equilibrium

Let $\boldsymbol{I} = [0,1]$ and $\boldsymbol{H} = (1,2]$ be sets of bankers and households, respectively. Individual bankers and households are indexed by $i \in \boldsymbol{I}$ and $h \in \boldsymbol{H}$.

Definition 1 *Given the initial endowments of physical capital* $\left\{k_0^i, k_0^h; i \in \boldsymbol{I}, h \in \boldsymbol{H}\right\}$ *to bankers and households such that*

$$\int_0^1 k_0^i di + \int_1^2 k_0^h dh = K_0,$$

an equilibrium is defined by a set of stochastic processes adapted to the filtration generated by $\{N_t, t \geq 0\}$*: the price of physical capital* $\{q_t\}$*, the risk-free rate* $\{r_t\}$*, the tax rate process* $\{\tau_t\}$*, the maximum leverage of shadow banking* $\{\bar{s}_t^*\}$*, the ratio of subsidy to bank equity* $\{\pi_t\}$*, wealth* $\left\{W_t^i, W_t^h\right\}$*, financing decisions* $\left\{S_t^i, S_t^{i,*}, S_t^h\right\}$*, investment decisions* $\left\{\iota_t^i, \iota_t^h\right\}$*, default decisions* $\left\{\mathcal{D}_t^i\right\}$*, and consumption* $\left\{c_t^i, c_t^h\right\}$ *of banker* $i \in \boldsymbol{I}$ *and household* $h \in \boldsymbol{H}$*; such that*

1. $\left\{W_0^i, W_0^h\right\}$ *satisfy* $W_0^i = q_0 k_0^i$ *and* $W_0^h = q_0 k_0^h$*, for* $i \in \boldsymbol{I}$ *and* $h \in \boldsymbol{H}$*;*

2. *households fix the maximum leverage of shadow banking* $\{\bar{s}_t^*\}$ *to ensure that shadow bank debt is risk-free, and they solve their problems given* $\{q_t, r_t\}$*;*

3. *bankers solve their problems given* $\{q_t, r_t, \tau_t, \pi_t, \bar{s}_t^*\}$*;*

4. *the budget of the regulatory authority is balanced*

$$\int_0^1 \tau_t S_t^i di = \int_0^1 \pi_t W_t^i di; \tag{6}$$

5. markets for both consumption goods and physical capital clear

$$\int_0^1 c_i^i di + \int_1^2 c_t^h dh = \int_0^1 \left(a - g\left(\iota_t^i\right)\right) k_t^i di + \int_1^2 \left(a^h - g\left(\iota_t^h\right)\right) k_t^h dh, \tag{7}$$

$$\int_0^1 k_t^i di + \int_1^2 k_t^h dh = K_t, \tag{8}$$

$$\text{where } dK_t = \left(\int_0^1 \left(\iota_t^i - \delta\right) k_t^i di\right) dt + \left(\int_1^2 \left(\iota_t^h - \delta\right) k_t^h dh\right) dt - xK_t dN_t,$$

$$q_t k_t^i = W_t^i + S_t^i + S_t^{i,*}, q_t k_t^h = S_t^h;$$

Given the definition, the market for risk-free debt clears by Walras' Law.

2 Bank Regulation and Financial Instability

In this section, we use numerical examples to characterize equilibria of the baseline model. With the model characterization, we will present our main result that the relationship between endogenous risk and bank regulation displays a U shape.

2.1 Equilibrium Characterization

Economic dynamics of the baseline model mainly depend on how the shadow banking sector evolves. In this section, we will characterize optimality conditions of households and bankers and then derive the maximum leverage of shadow banking.

2.1.1 Production of Physical Capital

Households choose the investment rate ι to maximize the rate of return from holding physical capital R_t^h; that is, the optimal ι solves

$$\max_{\iota} \frac{-\iota - 0.5\phi\left(\iota - \delta\right)^2}{q_t} + \iota.$$

The first-order condition implies that the optimal investment rate ι_t^h is a function of the price of capital q_t; that is, $\iota_t^h = \delta + \left(q_t - 1\right)/\phi$. Since bankers have the same investment technology as households do, in equilibrium

$$\iota_t = \iota_t^h = \delta + \frac{q_t - 1}{\phi}. \tag{9}$$

2.1.2 Households' Optimal Choices

Since households are risk-neutral and not financially constrained, the expected return they earn from holding any asset in equilibrium must equal their time discount factor ρ. Therefore, we have the following equilibrium conditions:

$$r_t = \rho, \tag{10}$$

$$R_t^h - \lambda x_t^q \leq r_t, \text{with equality if } S_t^h > 0. \tag{11}$$

Equation (11) indicates that if households hold physical capital in equilibrium, the expected rate of return equals the risk-free rate.

2.1.3 Bankers' Optimal Choices

Logarithmic bankers' consumption and portfolio decisions are "myopic," which simplifies the characterization of their strategic default decisions. In particular, we use two properties of the logarithmic preference: *i*) a banker's consumption c_t is ρ proportion of her wealth W_t; that is,

$$c_t = \rho W_t, \tag{12}$$

and *ii*) a banker's expected lifetime utility (i.e., continuation value) J_t satisfies

$$J_t \equiv E_t \left[\int_t^T e^{-\rho(u-t)} \ln(c_u) du + e^{-\rho(T-t)} J^r(W_T) \right] = \frac{\ln(W_t)}{\rho} + h_t,$$

where T denotes the random time that the banker retires, and h_t captures future investment opportunities and evolves endogenously according to $dh_t = h_t \mu_t^h dt - (h_t - \hat{h}_t) dN_t$. If a banker defaults, her expected investment opportunities will become worse and her continuation value becomes $J_t^d = \ln(W_t)/\rho + h_t^d$, where h_t^d follows $dh_t^d = h_t^d \mu_t^{h,d} dt - (h_t^d - \hat{h}_t^d) dN_t$. In the next section, we will characterize h_t and h_t^d.

We next illustrate the intuition of optimality conditions for bankers' portfolio and strategic default decisions. The formal derivation of these conditions can be found in Appendix A. Intuitively, a banker would like to maximize the expected growth rate of her continuation value $E[dJ_t]$. Given the law of motion for W_t (equation (4)), if we apply Ito's Lemma to $J_t = \ln(W_t)/\rho + h_t$, then

$$E_t[dJ_t] = \frac{1}{\rho}\Big(R_t + s_t(R_t - r_t - \tau_t) + s_t^*(R_t - r_t) + \lambda \ln\Big(1 - (1 + s_t + (1 - \mathcal{D}_t)\, s_t^*)x_t^q\Big)\Big)dt$$
$$+ \lambda\big((1 - \mathcal{D}_t)\hat{h}_t + \mathcal{D}_t\hat{h}_t^d\big)dt + O,$$

where $s_t = S_t/W_t$, $s_t^* = S_t^*/W_t$, and O denotes the sum of all other terms that are independent of $s_t, s_t^*, \hat{h}_t$, and $\hat{h}_t^d$. We label s_t and s_t^* as the leverage of her regular bank and shadow bank, respectively.

A banker's optimal portfolio choice depends on whether she plans to default on her shadow bank $\mathcal{D}_t$. Next, we will derive optimality conditions of (s_t, s_t^*) in both cases (i.e., $\mathcal{D}_t = 0$ and $\mathcal{D}_t = 1$). Given the optimal (s_t, s_t^*), we solve for the optimality condition of $\mathcal{D}_t = 0$; that is, the enforcement constraint for the banker.

No Intention of Default. Given that $\mathcal{D}_t = 0$, $E_t[dJ_t]$ reduces to

$$E_t[dJ_t] = \frac{1}{\rho}\Big(R_t + s_t(R_t - r_t - \tau_t) + s_t^*(R_t - r_t) + \lambda \ln\Big(1 - (1 + s_t + s_t^*)x_t^q\Big)\Big)dt + \lambda\hat{h}_t dt + O,$$

which shows that the banker is exposed to risks of her regular and shadow banks. First-order conditions with respect to (s_t, s_t^*) are

$$R_t - r_t - \tau_t \le \frac{\lambda x_t^q}{1 - (1 + s_t + s_t^*)\, x_t^q}, \text{ with equality if } s_t > 0, \tag{13}$$

$$R_t - r_t \ge \frac{\lambda x_t^q}{1 - (1 + s_t + s_t^*)\, x_t^q}, \text{ with equality if } s_t^* < \bar{s}_t^*. \tag{14}$$

Intuitively, the excess returns of regular banking $R_t - r_t - \tau_t$ and shadow banking $R_t - r_t$ must cover the risk premium of holding physical capital, which is on the right side of inequalities (13) and (14). The banker takes the tax rate τ_t as given because τ_t only depends on the debt to equity ratio of the regular banking sector rather the leverage of an individual bank. In addition, the portfolio choice (s_t, s_t^*) must be time consistent in the sense that if a Poisson shock indeed arrives, the banker still finds it optimal to honor her shadow bank debt. In Appendix A, we confirm the time-consistency of (s_t, s_t^*).

Intention of Default. If $\mathcal{D}_t = 1$, the banker does not bear any risk from shadow banking. Thus, she would borrow via shadow banking up to the limit (i.e., $s_t^* = \bar{s}_t^*$) if $R_t > r_t$. Recall that creditors of a shadow bank do not observe the leverage of its sponsor's regular bank. This assumption implies that investors of shadow bank debt cannot infer the banker's intention of default on shadow bank debt ex ante. Hence, the banker can freely choose the leverage of her regular bank $\tilde{s}_t$ to maximize

$$E_t\Big[dJ_t\Big] = \frac{1}{\rho}\Big(R_t + \tilde{s}_t(R_t - r_t - \tau_t) + \bar{s}^*_t(R_t - r_t)dt + \lambda \ln\Big(1 - (1+\tilde{s}_t)x^q_t\Big)\Big) + \lambda \hat{h}^d_t dt + O.$$

The first-order condition of $\tilde{s}_t$ is

$$R_t - r_t - \tau_t = \frac{\lambda x^q_t}{1 - (1+\tilde{s}_t)\, x^q_t}. \tag{15}$$

Since the banker assumes no risk from her shadow bank, the optimal leverage of her regular bank is relatively high ($\tilde{s}_t > s_t$).

Strategic Default. The enforcement constraint for the banker is to ensure that the expected growth rate of her continuation value $E_t[dJ_t]$ with no intention of default is not less than $E_t[dJ_t]$ with the intention of default; that is,

$$\begin{aligned} & \frac{1}{\rho}\Big(s_t\,(R_t - r_t - \tau_t) + s^*_t\,(R_t - r_t) + \lambda \ln\left(1 - (1 + s_t + s^*_t)\, x^q_t\right)\Big) + \lambda \hat{h}_t \\ \geq\;\; & \frac{1}{\rho}\Big(\tilde{s}_t\,(R_t - r_t - \tau_t) + \bar{s}^*_t\,(R_t - r_t) + \lambda \ln\left(1 - (1 + \tilde{s}_t)\, x^q_t\right)\Big) + \lambda \hat{h}^d_t. \end{aligned} \tag{16}$$

The opportunity cost of strategic default is that the banker cannot access shadow banking for a certain period and her future investment opportunities deteriorate ($\hat{h}^d_t < \hat{h}_t$). Hereafter, we let H_t denote $h_t - h^d_t$. The benefit of strategic default is that the banker can take higher leverage ($\tilde{s}_t + \bar{s}^*_t > s_t + s^*_t$).

2.1.4 The Maximum Leverage of Shadow Banking

In this section, we characterize the maximum leverage of shadow banking $\bar{s}^*_t$. Recall that households only accept debt that is risk-free in equilibrium. Thus, the maximum leverage of shadow banking $\bar{s}^*_t$ ought to be such that the enforcement constraint (16) holds as long as $s^*_t \leq \bar{s}^*_t$.

To find $\bar{s}^*_t$, we simplify the enforcement constraint (16) so that the leverage of shadow banking becomes a banker's only choice variable that enters the enforcement constraint. Since $s_t \geq 0$ and $s^*_t \leq \bar{s}^*_t$, four scenarios may occur in equilibrium: *i)* $s_t > 0$ and $s^*_t = \bar{s}^*_t$; *ii)* $s_t = 0$ and $s^*_t < \bar{s}^*_t$; *iii)* $s_t = 0$ and $s^*_t = \bar{s}^*_t$; and *iv)* $s_t > 0$ and $s^*_t < \bar{s}^*_t$. Scenario *iv* is inconsistent with a banker's optimality conditions (13) and (14). The assumption $\tau_t = \min\{\tau, \tau s_t\}$ excludes scenario *iii*.[8] Scenario *i* is close

[8] $s_t = 0$ implies that $\tau_t = \min\{\tau, 0\} = 0$. Thus, a banker is indifferent between shadow banking and

to what happens in reality; that is, regular banking is active ($s_t > 0$) and the leverage constraint on shadow banking is binding ($s_t^* = \bar{s}_t^*$). In scenario *i*, first-order conditions (13) and (15) imply that $s_t + s_t^* = \tilde{s}_t$, and thus the enforcement constraint reduces to

$$(\tilde{s}_t - s_t)\big(R_t - r_t - \tau_t\big) = s_t^*\big(R_t - r_t - \tau_t\big) \leq \rho\lambda\hat{H}_t.$$

If a banker plans to default on her shadow bank debt, she will take higher leverage and obtain an additional return $(\tilde{s}_t - s_t)\big(R_t - r_t - \tau_t\big)$. The simplified enforcement constraint shows that if the opportunity cost of default $\hat{H}_t$ is large enough, the banker would not plan to default.

In scenario *ii*, the enforcement constraint (16) has the same simplified form.[9] Thus, to ensure that the enforcement constraint (16) holds, the maximum leverage of shadow banking $\bar{s}_t^*$ satisfies

$$\bar{s}_t^* = \frac{\rho\lambda\hat{H}_t}{R_t - r_t - \tau_t}. \tag{17}$$

The maximum leverage of shadow banking depends on the opportunity cost of default on shadow bank debt $\hat{H}_t$ and the profitability of banking $R_t - r_t - \tau_t$. Notice that bankers' portfolio choice determines the price of physical capital q_t, which in turn affects their return from holding physical capital R_t and the maximum leverage of shadow banking $\bar{s}_t^*$. However, bankers do not internalize this general equilibrium effect, which gives rise to a source of inefficiency.

Next, we characterize H_t to fully understand what affects the maximum leverage of shadow banking $\bar{s}_t^*$. The following proposition indicates that we can represent the opportunity cost of default H_t as the present value of future tax benefits $s_u^*\tau_u$, $u > t$. The discount factor of future tax benefits is the banker's time discount factor plus the "re-enter" rate ξ and the retirement rate χ. Once bankers re-enter the shadow bank debt market or retire, the opportunity cost of being prohibited from using shadow banking disappears.

Proposition 1 (Opportunity Cost of Default) *Probabilistic Representation of H_t:*

$$H_t \equiv h_t - h_t^d = E_t\left[\int_t^\infty \exp\left(-\left(\rho + \xi + \chi\right)(u - t)\right)\frac{s_u^*\tau_u}{\rho}\; du\right]. \tag{18}$$

Proof. See the online appendix. ■

regular banking. The binding leverage constraint on shadow banking $s_t^* = \bar{s}_t^*$ implies that a banker strictly prefers raising credit via regular banking, which contradicts with $s_t = 0$.

[9] The second scenario is not realistic and only exists for a small set of parameter values. If $s_t = 0$, then $\tau_t = 0$. First-order equations (14) and (15) imply that $s_t^* = \tilde{s}_t$. To plug $\tau_t = 0$ and $s_t^* = \tilde{s}_t$ into the enforcement constraint (16), we have the same simplified form.

There exists a crucial *feedback loop* between the maximum leverage of shadow banking $\{\bar{s}_t^*\}$ and the opportunity cost of default $\{H_t\}$. First, equation (17) implies that the maximum leverage of shadow banking increases with bankers' opportunity cost of default. Second, the probabilistic representation (18) indicates that the higher the maximum leverage of shadow banking is, the more costly it is for bankers to default on their shadow bank debt.

This feedback loop gives rise to an equilibrium where shadow banking does not exist. Let us conjecture that $\{\bar{s}_t^* = 0, t \geq 0\}$. The probabilistic representation (18) implies $\{H_t = 0, t \geq 0\}$, and equation (17) verifies the conjecture. Thus, we have the following proposition:

Proposition 2 (No Shadow Banking) *In the baseline model, there always exists an equilibrium where shadow banking does not exist; that is,* $\{\bar{s}_t^* = 0, H_t = 0, t \geq 0\}$.

In this degenerate equilibrium, productive bankers are unable to leverage up via shadow banking. By contrast, there may exist a non-degenerate equilibrium, where shadow banking exists. In this paper, we will focus on the non-degenerate equilibrium, given the importance of shadow banking.[10]

2.1.5 Market Clearing and Wealth Distribution

Since households are risk-neutral and they can have negative consumption, the market for consumption goods clears automatically as long as the risk-free rate equals households' time discount factor, $r_t = \rho$. The market for physical capital clears if the fractions of physical capital held by bankers and households sum to 1. Let ψ_t be the fraction of physical capital held by bankers. The budget of the regulatory authority is balanced if it transfers all tax revenues back to bankers; that is, $\pi_t = s_t \tau_t$.

Like other continuous-time macro-finance models, the wealth distribution matters for the dynamics of the economy. Later, we will capture the dynamics of an equilibrium with a state variable, the bankers' wealth share $\omega_t \equiv \int_0^1 W_t^i di \Big/ q_t K_t$. Lemma 1 characterizes how ω_t evolves.

Lemma 1 *The law of motion for* ω_t *is*

$$d\omega_t = \omega_t \mu_t^\omega dt - (\omega_t - \hat{\omega}_t)\, dN_t, \tag{19}$$

$$\text{where } \mu_t^\omega = R_t + s_t (R_t - r_t) + s_t^* (R_t - r_t) - \mu_t^q - \mu_t^K - \rho - \chi(1-\sigma), \tag{20}$$

$$\text{and } \hat{\omega}_t = \omega_t \frac{(1 + s_t + s_t^*)(1-x)\hat{q}_t - (s_t + s_t^*) q_t}{(1-x)\hat{q}_t}. \tag{21}$$

[10] There exist a continuum of sunspot equilibria, in which the economy may suddenly switch to the degenerate equilibrium where shadow banking disappears. Equilibrium selection is beyond the scope of this paper.

Proof. See Appendix A. ■

In the absence of Poisson shocks, the state variable ω_t grows at rate μ_t^ω. If a Poisson shock hits the economy, the state variable jumps from ω_t to $\hat{\omega}_t$.

2.2 Markov Equilibrium

Although we can characterize an equilibrium with equations (3) − (18), it is still challenging to solve for an equilibrium. Fortunately, our economy has the property of scale invariance. This means that the baseline model permits a Markov equilibrium with state variable ω_t, and the dynamics of all endogenous variables in the Markov equilibrium can be characterized by the law of motion for ω_t and functions $q(\cdot)$ and $H(\cdot)$. Hence, solving for the Markov equilibrium is equivalent to solving for $q(\cdot)$ and $H(\cdot)$. With Ito's Lemma, we can find a differential equation that defines $q(\cdot)$,

$$\mu_t^q = \frac{q'(\omega_t)}{q(\omega_t)}\omega_t\mu_t^\omega, \tag{22}$$

$$\hat{q}_t = q(\hat{\omega}_t). \tag{23}$$

Given the probabilistic representation of H_t, we know

$$\int_0^t \exp\left(-\left(\rho+\xi+\chi\right)u\right)\frac{s_u^*\tau_u}{\rho}\ du + \exp(-\left(\rho+\xi+\chi\right)t)H_t$$

is a martingale. To apply Ito's Lemma, we have

$$\left(\rho+\xi+\chi\right)H(\omega_t) = \frac{\min\{\tau,\tau s_t\}}{\rho}s_t^* + \omega_t\mu_t^\omega H'(\omega_t) + \lambda\left(H(\hat{\omega}_t) - H(\omega_t)\right). \tag{24}$$

Equations (22) and (24) indicate differential equations that $q(\cdot)$ and $H(\cdot)$ satisfy, respectively. It is easy to see that equation (24) is not an Ordinary Differential Equation as $H'(\omega)$ depends on the value of $H(\cdot)$ in state $\hat{\omega}_t$, where the economy will move to given a Poisson shock. Since $\hat{\omega}_t < \omega_t$ (i.e., Poisson shocks lower bankers' wealth share), equations (22) and (24) are Delay Differential Equations. In the following subsection, we will detail how to calculate $q'(\omega)$ and $H'(\omega)$ given the values of $q(\cdot)$ and $H(\cdot)$ over $(0,\omega)$ and also highlight boundary conditions for $q(\cdot)$ and $H(\cdot)$.

2.2.1 Numerical Calculation

Given $\{\omega, q(\tilde{\omega}), H(\tilde{\omega}), 0 < \tilde{\omega} \leq \omega\}$, we will compute $q'(\omega)$ and $H'(\omega)$ using the following procedure. First of all, we postulate that households also hold physical capital and find $s + s^*$ such that

$$\frac{a - a^h}{q} - \min\{\tau, \tau \max\{0, s + s^* - \bar{s}^*\}\} = \frac{\lambda x^q}{1 - (1 + s + s^*)x^q} - \lambda x^q, \tag{25}$$

equations (13), (14), (17), (21) and (23) hold. Equation (25) is the difference of the two Euler equations (11) and (13) with $\tau_t = \min\{\tau, \tau s_t\}$. While solving for $s + s^*$, we also derive $\hat{\omega}, \hat{q}, x^q, s, s^*$, and $\bar{s}^*$. Next, we compute $\psi = (1 + s + s^*)\omega$ and check if our conjecture is true, i.e., $\psi < 1$. If it is true, we calculate μ^q based on equation (13) and μ^ω according to equation (20). If $\psi < 1$ does not hold, then we set $\psi = 1$ and $s + s^* = 1/\omega - 1$. Given $s + s^*$, we derive $\hat{\omega}, \hat{q}, x^q, s, s^*, \bar{s}^*, \mu^q$, and μ^ω based on equations (13), (14), (17), (20), (21), and (23). Given μ^q and μ^ω, we compute $q'(\omega)$ according to equation (22). Finally, we derive $H'(\omega)$ according to equation (24).

Boundary conditions are *i)* $\mu^q(\bar{\omega}) = \mu^H(\bar{\omega}) = 0$ at $\bar{\omega}$, where $\mu^\omega(\bar{\omega}) = 0$; and *ii)* $q(0) = \underline{q}$ and $H(0) = 0$, where $\underline{q}$ satisfies $a^h - \delta + (\underline{q}-1)^2/2\phi - \rho\underline{q} = \lambda x\underline{q}$. The state $\bar{\omega}$ is a movable singular point such that $\mu^\omega(\bar{\omega}) = 0$. Ito's Lemma implies boundary conditions $\mu^q(\bar{\omega}) = 0$ and $\mu^H(\bar{\omega}) = 0$. The state $\omega = 0$ is a limit state where only households exist in the economy. We derive the asymptotic properties of $q(\omega)$ and $H(\omega)$ at $\omega = 0$ in Appendix B.

2.2.2 Equilibrium Uniqueness

Within the class of Markov equilibria, we can identify the condition under which the degenerate equilibrium is the unique equilibrium. To prove this result, we define a mapping Γ which takes the cost of default function $H(\cdot)$ as the input,

$$\Gamma H(\omega) = E_t\left[\int_t^\infty \exp(-(\rho + \xi + \chi)(u - t))\frac{\min\{\tau, \tau s_u\}}{\rho} s_u^* du \,\middle|\, \omega_t = \omega\right]$$

where

$$s_t^* \leq \frac{\rho\lambda H(\hat{\omega}_t)}{R(\omega_t) - r - \tau(\omega_t)},$$

and (s_u, s_u^*) are portfolio weights of a banker in the equilibrium of a hypothetical economy with exogenous $H(\cdot)$. To solve for $\Gamma H(\cdot)$, we follow the procedure illustrated in Section 2.2.1 and use the given $H(\cdot)$ to compute $\bar{s}^*$. The last step of the procedure yields $\Gamma H(\cdot)$.

The fixed point of the mapping Γ is $H(\cdot)$ that characterizes the Markov equilibrium. As we have noted in Section 2.1.4, the mapping Γ may have two fixed points: one corresponds to the non-degenerate equilibrium, and the other yields the degenerate equilibrium. The following theorem provides a sufficient condition that justifies the uniqueness of the degenerate equilibrium:

Theorem 1 (Uniqueness) *If $\tau < (\rho + \xi + \chi)\, x$, the mapping Γ is a contraction mapping with the fixed point $H(\omega) = 0$ for all $\omega \in (0, \bar{\omega}]$.*

Proof. See the online appendix. ■

To prove that Γ is a contraction mapping, we show that Γ satisfies Blackwell's sufficient conditions if $\tau < (\rho + \xi + \chi)x$. The feedback loop illustrated in Section 2.1.4 explains why Γ could be a contraction mapping. Suppose the tax on regular bank debt τ decreases. The probabilistic representation (18) implies that the opportunity cost of default drops. The enforcement constraint implies that the maximum leverage of shadow banking $\{\bar{s}_t^*\}$ declines accordingly (equation (17)). The decline in the leverage of shadow banking $\{\bar{s}_t^*\}$ reduces the opportunity cost of default $\{H_t\}$ further (the probabilistic representation (18)). This cycle makes shadow banking unsustainable in equilibrium if τ is small enough.

2.3 Economic Dynamics

In this section, we present main dynamic properties of the baseline model with a numerical example. Parameter values are $a = 22.5\%$, $a^h = 10\%$, $\delta = 10\%$, $\lambda = 1$, $x = 4\%$, $\phi = 3$, $\rho = 3\%$, $\chi = 15\%$, $\sigma = 10^{-5}$, $\tau = 3\%$, and $\xi = 6\%$. The calibration of parameter values is detailed in the online appendix.

With the continuous-time method, we can characterize full dynamics of the economy. First, we express all endogenous variables as functions of the state variable, bankers' wealth share ω_t. Second, we use the law of motion for ω_t (equation (19)) and Ito's formula to derive the law of motion for all endogenous variables. The stationary distribution of the state variable shown in the online appendix has a single peak where bankers hold around 38% of wealth in the economy.

Our model inherits most dynamic features of Brunnermeier and Sannikov (2014). After a series of negative shocks, the economy enters downturns and bankers' wealth share diminishes due to their disproportionately high exposure to aggregate risks. As a result, bankers hold a declining fraction of physical capital and aggregate productivity deteriorates (Panel *a* of Figure 2). Hence, the price of physical capital declines (Panel *b* of Figure 2) and the endogenous risk x^q increases (Panel *d* of Figure 2). The endogenous risk reaches its highest level as bankers start asset fire sales and less productive households begin to hold physical capital. The profitability of banking $R_t - r - \tau_t$ rises

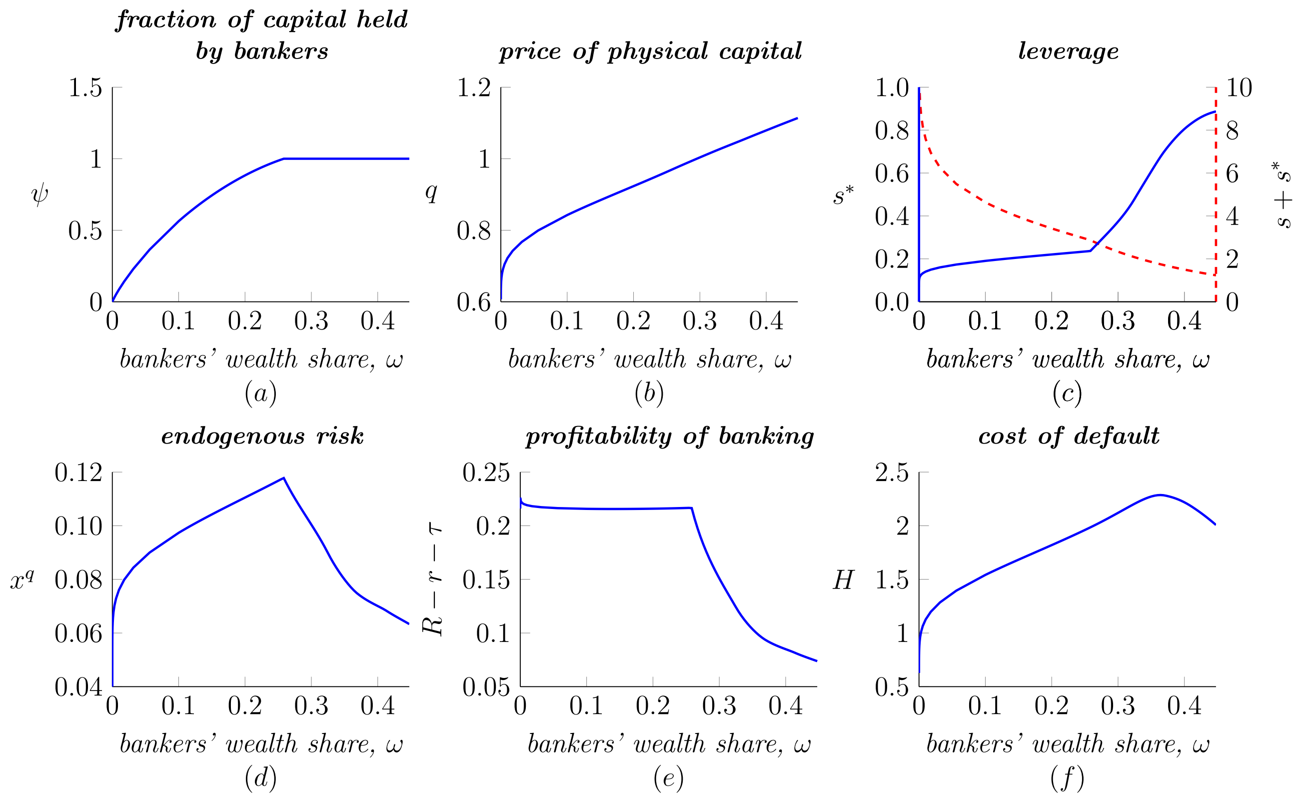

Figure 2: Economic Dynamics

This figure presents the fraction of physical capital held by bankers ψ, the price of physical capital q, the leverage of shadow banking s^* (solid line), a banker's overall leverage $s + s^*$ (dashed line), the endogenous risk x^q, the profitability of banking $R - r - \tau$, and the opportunity cost of default H as functions of the state variable ω (i.e., bankers' wealth share). For parameter values, see Section 2.3.

in downturns due to the low price of physical capital (Panel e of Figure 2), which explains why the overall leverage of a banker $s + s^*$ is counter-cyclical (the dashed line of Panel c of Figure 2).

Pro-cyclical Leverage of Shadow Banking. The leverage of shadow banking is pro-cyclical (Panel c of Figure 2). As the price of physical capital increases in economic upturns (Panel b of Figure 2), the profitability of regular banking $R_t - r_t - \tau_t$ declines (Panel e of Figure 2). Thus, the profitability of shadow banking and the opportunity cost of default are relatively high in upturns, which makes the enforcement constraint less tight (equation (17)). Since the enforcement constraint is more lax in upturns, bankers can take on higher leverage in the shadow banking sector.

The *feedback loop* that we highlight in Section 2.1.4 also contributes to the pro-cyclicality of shadow banking. Since the leverage of shadow banking is relatively high in upturns, the access to shadow banking helps bankers save a large amount of tax. Therefore, the opportunity cost of default on shadow bank debt is high in economic upturns (equation (18)) as default would deprive bankers of the benefit of tax saving. The higher opportunity cost of default, in turn, makes the enforcement constraint more lax, and thus increases the leverage of shadow banking further (equation (17)).

The dynamic properties of shadow banking and regular banking indicate that we do not lose the *generality of our results* by assuming that the tax rate on regular bank debt τ_t equals $\min\{\tau, \tau s_t\}$ instead of the constant τ. If regular banks' leverage is sufficiently high (i.e., $s_t > 1$) in downturns, the two setups yield the same result since $\min\{\tau, \tau s_t\} = \tau$. In upturns, if the maximum leverage of shadow banking is so high that bankers only use shadow banking (i.e., $s_t = 0$), the two setups also give the same result as the τ_t does not enter the first-order condition associated with shadow banking. The only difference between the two setups is in the intermediate case where $0 < s_t < 1$.

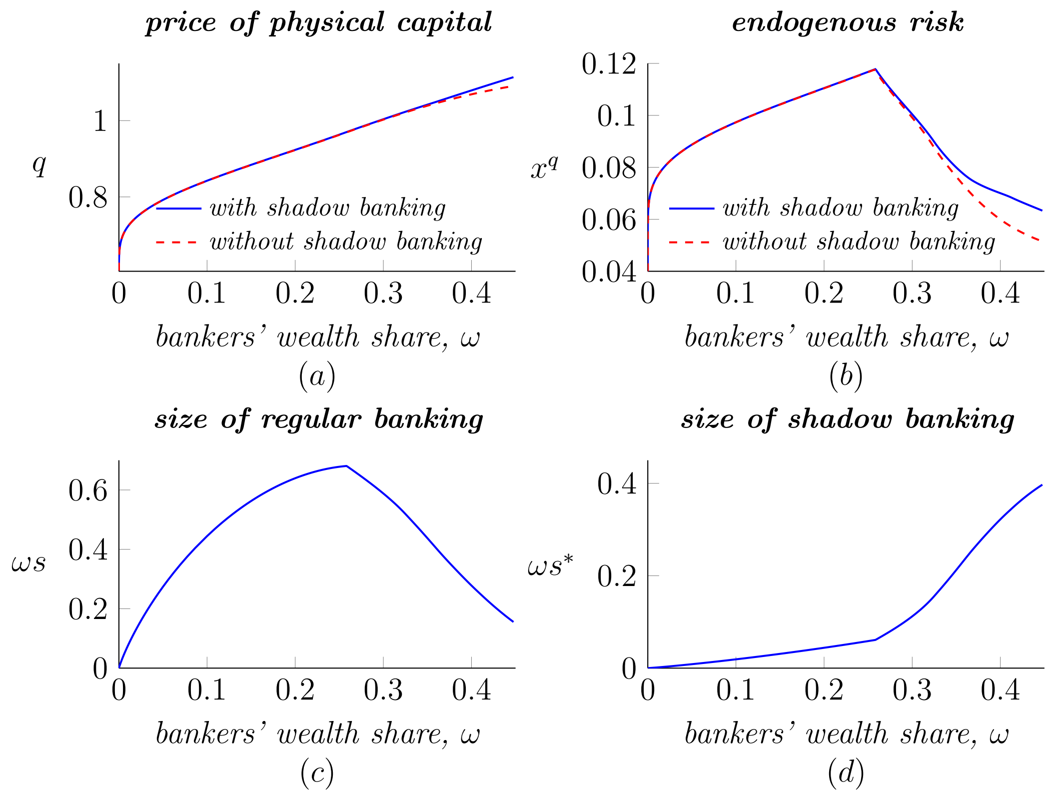

Figure 3: Financial Instability and Reintermediation
This figure presents the price of physical capital (solid line), the endogenous risk (solid line), the size of the regular banking sector, and the size of the shadow banking sector as functions of the state variable ω (i.e., bankers' wealth share) in an economy with shadow banking. For comparison, upper panels also show the price of physical capital (dashed line) and the endogenous risk (dashed line) for the same economy without shadow banking. For parameter values, see Section 2.3.

Financial Instability. Shadow banking increases financial instability. Upper panels of Figure 3 compare the baseline model (solid lines) with its simplified version without shadow banking. Shadow banking provides relatively cheap credit as it is not subject to regulation. Hence, bankers can hold more physical capital in an economy with shadow banking and the price of physical capital is also higher (Panel *a* of Figure 3). However, the benefit of shadow banking is not free. The endogenous risk is higher in the economy with shadow banking (Panel *b* of Figure 3).

Reintermediation. Shadow banking increases financial instability through the reintermediation process. The lower panels of Figure 3 illustrate the reintermediation process; that is, if the economy experiences a series of negative shocks, a large amount of assets migrate from the shadow banking sector to the regular banking sector. The reintermediation process raises endogenous risk for the following reason: shadow banks accumulate a large number of assets in economic upturns. The scale of this asset accumulation is larger than what the regular banking sector would pursue given its relatively high funding cost caused by bank regulation. If adverse shocks hit the economy, shadow banks have to divest large amounts of assets as their leverage constraints tighten, and regular banks are reluctant to acquire these assets because it is expensive to expand their balance sheets. As a result, the price of physical capital declines more than it would if there were no shadow banking.

Inefficiency. Shadow banking causes inefficiency through its *general equilibrium* effects. First of all, we notice that the operation of a single shadow bank neither lowers the quality of physical capital nor increases endogenous risk. In fact, shadow banking improves the welfare of a banker by providing cheap credit. Hence, it is the general equilibrium effects of shadow banking that cause economic inefficiency. The fundamental cause of the inefficiency is that in upturns when individual bankers accumulate large amounts of assets financed via shadow banking they do not internalize negative effects of reintermediation that occurs in recessions.

2.4 Regulatory Implications

In this section, we emphasize that the regulation of regular banking has nonlinear effects on financial instability in the presence of shadow banking. In addition, we show that the borrowing capacity of shadow banking being endogenous is crucial for our model to capture the nonlinear effects.

2.4.1 Regulatory Paradox

We present comparative-statics analyses to highlight that bank regulation may have unintended consequences when shadow banking plays a critical role. In particular, we vary the tax on regular bank debt τ, and stress the U-shaped relationship between financial instability and the regulation of the regular banking sector.

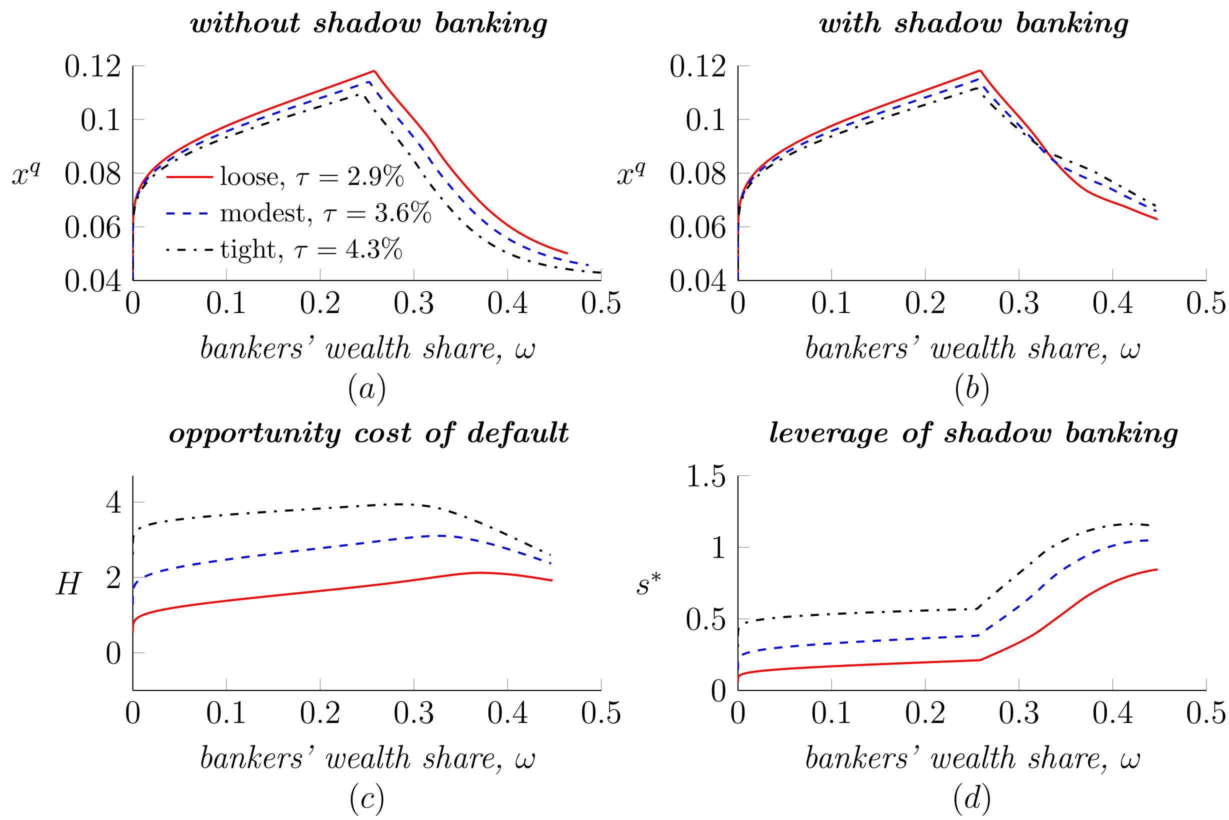

Figure 4: Regulatory Paradox

This figure shows endogenous risks in economies without shadow banking, endogenous risks in economies with shadow banking, the opportunity cost of default, and the leverage of shadow banking. Solid lines are for the economy with loose bank regulation ($\tau = 2.9\%$); dashed lines for the economy with modest regulation ($\tau = 3.6\%$); dash-dotted lines for the economy with tight regulation ($\tau = 4.3\%$). For other parameter values, see Section 2.3.

The conventional wisdom that tough regulation secures financial stability may not hold when shadow banking plays a role. In economies without shadow banking, if the regulatory authority tightens regulation by raising τ, banks' leverage and endogenous risk will decline (Panel *a* of Figure 4). However, in economies with shadow banking, it is those with tighter regulation that experience higher endogenous risk (Panel *b* of Figure 4). The intuition follows. Regular banks will face higher tax burdens if regulation becomes tighter. Since if a banker defaults on her shadow bank debt, she can only use regular banking, tighter regulation gives rise to a larger opportunity cost of default (Panel c of Figure 4). Furthermore, the enforcement constraint implies that the larger opportunity cost of default leads to the higher leverage of shadow banking (equation (17)). Hence, the shadow banking sector is larger in economies with more stringent regulation (Panel d of Figure 4). Since shadow banking adds to financial instability, tough regulation imposed on regular banks can deteriorate financial stability, as Panel b of Figure 4 presents.

Regulatory Smile. The regulatory paradox result holds only when the regulatory restriction is so tight that the shadow banking sector becomes sizable. Recall the feedback loop discussed in Section 2.1.4. If the regulatory authority lowers the tax rate τ, the opportunity cost of default on shadow bank debt declines. This, in turn, lowers the maximum leverage of shadow banking and further reduces the opportunity cost of default. The feedback loop is very effective. The solid line in the lower panel of Figure 5 shows that as τ declines from 2.5% to 2.3%, the shadow banking system disappears. In the regime where the shadow banking system is absent, the conventional wisdom is still true; that is, tightening regulation secures financial stability (the solid line in the upper panel of Figure 5). In summary, our model stresses that regulatory restrictions on regular banks have nonlinear effects on financial instability in an economy where the scale of shadow banking varies endogenously.

2.4.2 Exogenous Leverage Constraint on Shadow Banking

The endogenous leverage constraint on shadow banking is essential for the "regulatory paradox" result. To illustrate this point, we modify the baseline model by replacing the endogenous cost of default $\{H_t, t \geq 0\}$ with a constant $\bar{H}$.

The "regulatory paradox" result does not hold in the modified model (the dashed line of the upper panel of Figure 5), although it preserves many other dynamic features of the baseline model.[11] The intuition follows. If the opportunity cost of default on shadow bank debt is an exogenous constant, the borrowing capacity of shadow banking will be unrelated to the degree of bank regulation. Hence, the size of the shadow banking sector will not change much as the degree of regulatory restriction varies (the dashed line in the lower panel of Figure 5). If regulatory authorities raise the tax on regular banking, not many banking activities will migrate to the shadow banking sector. As a result, the scale of the reintermediation does not change as significantly as it does in the baseline model. Hence, financial instability does not rise as the tax rate increases.

The comparison between the baseline model and its variant underlines two points. First, if the leverage constraint on shadow banking is exogenously given in a model, the model may not

[11] We characterize the dynamic properties of the modified model in the online appendix.

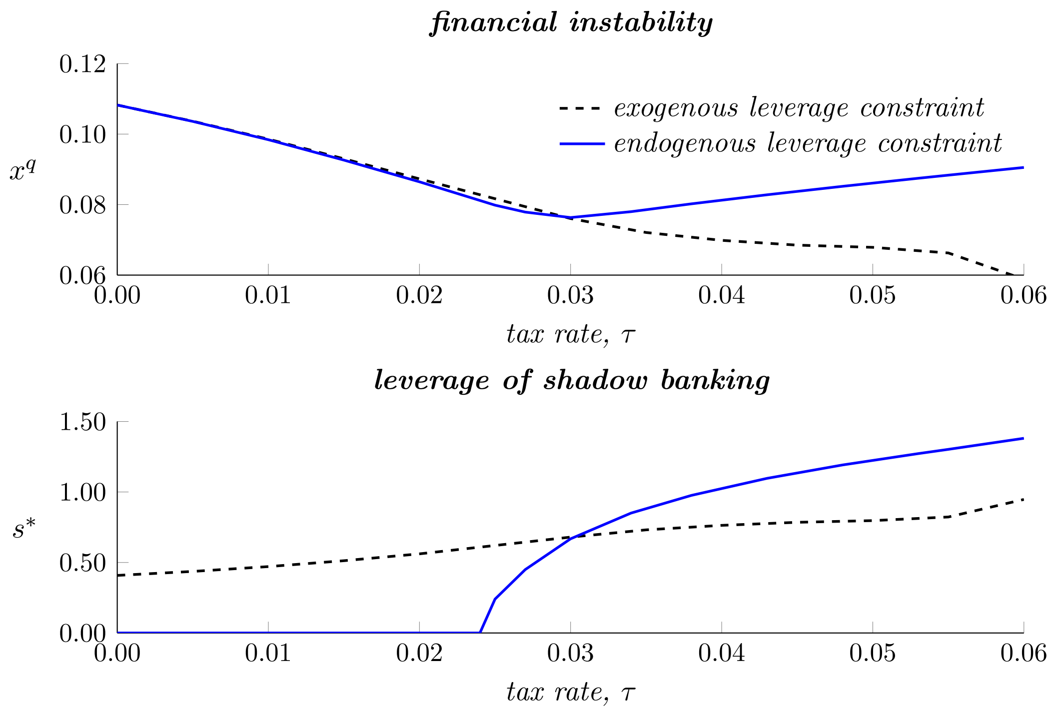

Figure 5: Financial Instability and Bank Regulation.
This figure shows how the change in the tax rate influences the endogenous risk x^q and the leverage of shadow banking at the stochastic steady states in the baseline model (solid lines) and in the modified model with an exogenous leverage constraint for shadow banking (dashed lines). The stochastic steady state is the state where $\omega\mu^{\omega} - \lambda(\omega - \hat{\omega}) = 0$. The exogenous cost of default H equals 2.1818, which is the average cost of default in the calibrated model in Section 2.3. For other parameter values, see Section 2.3.

capture the U-shaped relationship between financial instability and bank regulation. Second, it is not true that tightening regulation always squeezes a large amount of banking activities into the shadow banking sector. This is because shadow banking faces more frictions than regular banking does. The enforcement problem is one example. Due to these frictions, creditors must impose some leverage constraints on shadow banking. These constraints would restrain shadow banks from acquiring too many assets that regular banks have to unload due to tight regulation.

3 Welfare Analysis

In this section, we highlight that tightening bank regulation can decrease bankers' welfare because it leads to the expansion of the shadow banking sector and the deterioration of financial stability. This channel is in contrast to the traditional view that stringent bank regulation depresses economic growth, which in turn may lower social welfare.

To simplify welfare aggregation, we assume that all bankers have the same level of initial wealth. Without loss of generality, we normalize the aggregate amount of physical capital at time 0 to be 1. Note that bankers' wealth share ω_0 is exactly the fraction of physical capital that they own. Thus, a banker's initial wealth is $\omega_0 q_0$, and the initial wealth of the household sector is $\big(1-\omega_0\big)q_0$. The welfare pair of a representative household and a banker in period 0 is $\big((1-\omega_0)q_0, \ln(\omega_0 q_0)/\rho + h_0\big)$. It is straightforward to see that a household's welfare only depends on the price of physical capital since it is risk-neutral. Upper panels of Figure 6 display the welfare of a banker and a household in the state $\omega_0 = 0.38$ across economies with different degrees of bank regulation.[12] Solid lines correspond to economies with shadow banking and dashed lines to those without shadow banking.

When there is *no shadow banking*, implementing bank regulation improves bankers' welfare (the dashed line in Panel a of Figure 6) but lowers households' welfare (the dashed line in Panel b of Figure 6). As the tax rate τ increases from zero, both the volatility and the growth rate of bankers' wealth decline (dashed lines in Panel c and d of Figure 6). The benefit of the low wealth volatility dominates if the tax rate is not too high. From a banker's perspective, the unregulated competitive equilibrium ($\tau = 0$) is suboptimal because she does not internalize the negative impact of her leverage choice on endogenous risk x_t^q. If the tax rate is too high, tightening regulation lowers the welfare of the banker because the negative effect of the low wealth growth dominates (the dashed line in Panel a of Figure 6). Households' welfare always worsens as bank regulation tightens. This is because tight regulatory restriction prevents productive bankers from raising credit for financing their holdings of physical capital, which lowers the aggregate productivity and the price of physical capital.

Lower panels of Figure 6 show that as *shadow banking emerges*, tightening bank regulation increases both the growth rate and the volatility of bankers' wealth. This is in contrast with how the change in regulatory restriction affects the two terms when shadow banking is absent. Strengthening bank regulation increases the size of the shadow banking sector. On the one hand, bankers can access more cheap credit as there is no tax on shadow bank debt. This benefits the growth of their wealth. On the other hand, a large shadow banking sector leads to high endogenous risk, which makes bankers' wealth more volatile.

When shadow banking exists, bank regulation has nonlinear effects on bankers' welfare. If bank regulation is not too stringent, tightening regulation leads to the growth of shadow banking, which benefits bankers' welfare (the solid line in Panel a of Figure 6). However, if regulation is

[12]The economy mostly stays around the state $\omega_0 = 0.38$.

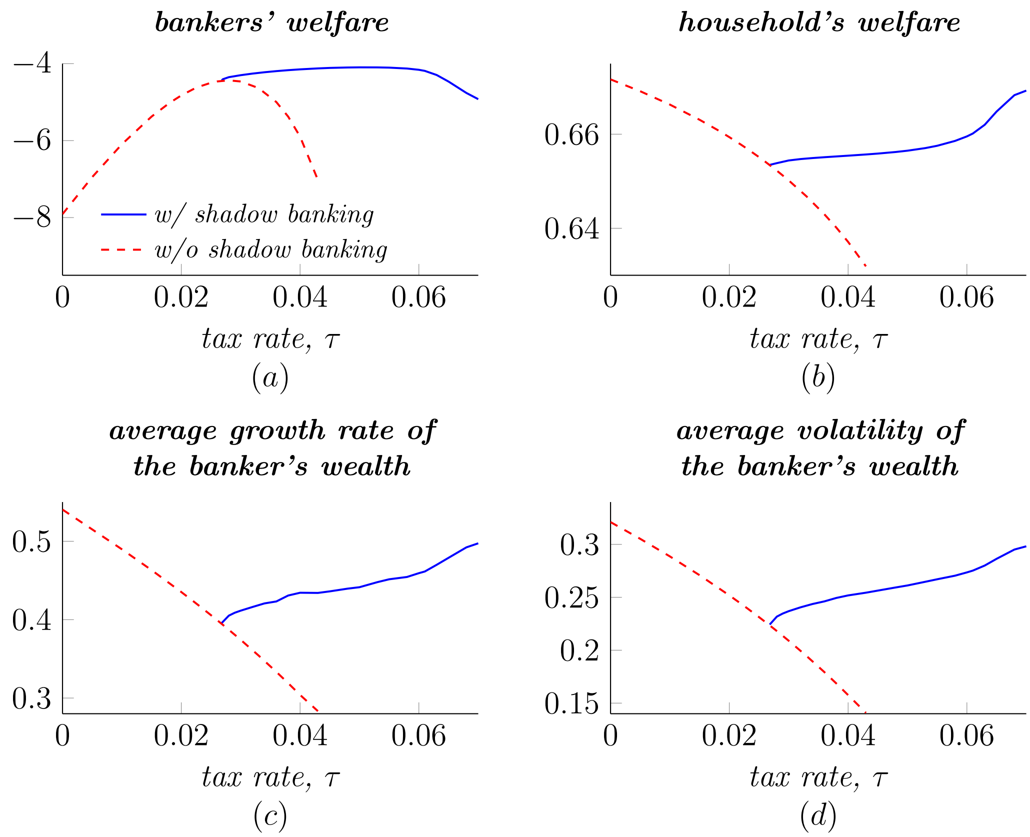

Figure 6: Welfare and Dynamics of Banker's Wealth
This figure shows how the change in tax rate affects the welfare of a banker and a household in period 0, the average growth rate of bankers' wealth, and the average volatility of bankers' wealth in an economy without shadow banking (dashed lines) and in an economy with shadow banking (solid lines). For agents' welfare, we focus on state $\omega_0 = 0.38$. We use the stationary distribution to calculate moments. For parameter values other than τ, see Section 2.3.

too strict, the negative effect of high volatility dominates, and tightening regulation hurts bankers' welfare. Even though tightening bank regulation eventually lowers bankers' welfare in both types of economies, the underlying mechanisms are different: in the economy without shadow banking, it is the low wealth growth that causes the low welfare; in the economy with shadow banking, it is the high volatility of bankers' wealth that drives the result.

As the shadow banking sector expands, productive bankers can raise more cheap credit to fund their holdings of physical capital. Hence, the aggregate productivity increases and the price of physical capital appreciates. Therefore, tightening bank regulation benefits households in the presence of shadow banking (the solid line in Panel b of Figure 6).

4 Robustness

We have done three types of robustness checks for the "regulatory paradox" result. In this section, we only concentrate on the first one, in which we allow both regular and shadow banks to issue outside equity up to a certain proportion. The first robustness check also shows that better aggregate risk sharing does not necessarily improve financial stability in the presence of shadow banking.

The detailed discussion of the other two robustness exercises is in the online appendix. In the second one, we assume that households have Epstein-Zin preferences instead of risk-neutral ones. In the third robustness check, we model bank regulation as a capital adequacy constraint. In the third exercise, the dynamic features of both shadow banking and regular banking are inconsistent with facts observed during the 2007-09 financial crisis.

4.1 Risky Shadow Bank Debt

We relax the restriction on equity issuance such that bankers must retain ε proportion of a bank's outstanding equity. For shadow banking, the equity issuance means that investors of a shadow bank expect its sponsor to bear only ε fraction of the total loss to the shadow bank.

Given the relaxed restriction on equity issuance, a *banker's dynamic budget constraint* becomes

$$dW_t = \left(W_t R_t + \frac{1-\varepsilon}{\varepsilon} W_t (R_t - r_t) + S_t (R_t - r_t - \tau_t) + S_t^* (R_t - r_t) - (1-\varepsilon)\left(\frac{W_t}{\varepsilon} + S_t + S_t^*\right) \lambda x_t^q \right) dt \\ + (W_t \pi_t - c_t) dt - \varepsilon \left(\frac{W_t}{\varepsilon} + S_t + (1-\mathcal{D}_t) S_t^*\right) x_t^q dN_t.$$

We only explain two terms related to outside equity since other terms have the same interpretation as they do in the economy model (see equation (4)). A banker with wealth W_t can obtain external equity financing ${}^{(1-\varepsilon)W_t}/_{\varepsilon}$. In addition to paying the opportunity cost r_t, the banker must pay external shareholders a premium for their exposure to the potential loss $(1-\varepsilon)\left({}^{W_t}/_{\varepsilon} + S_t + S_t^*\right) x_t^q$, of which investors of the shadow bank bear $(1-\varepsilon) S_t^* x_t^q$. Since households are risk-neutral, the premium is the expected loss $(1-\varepsilon)\left({}^{W_t}/_{\varepsilon} + S_t + S_t^*\right) \lambda x_t^q$.

Relaxing the equity issuance restriction also makes the enforcement constraint less binding. This is because investors of shadow banks share risks with their sponsors, which lowers sponsors' incentives to default. In particular, the maximum leverage of shadow banking (17) becomes

$$\bar{s}_t^* = \frac{\rho \lambda \hat{H}_t}{R_t - r - \tau_t - (1-\varepsilon)\lambda x^q}.$$

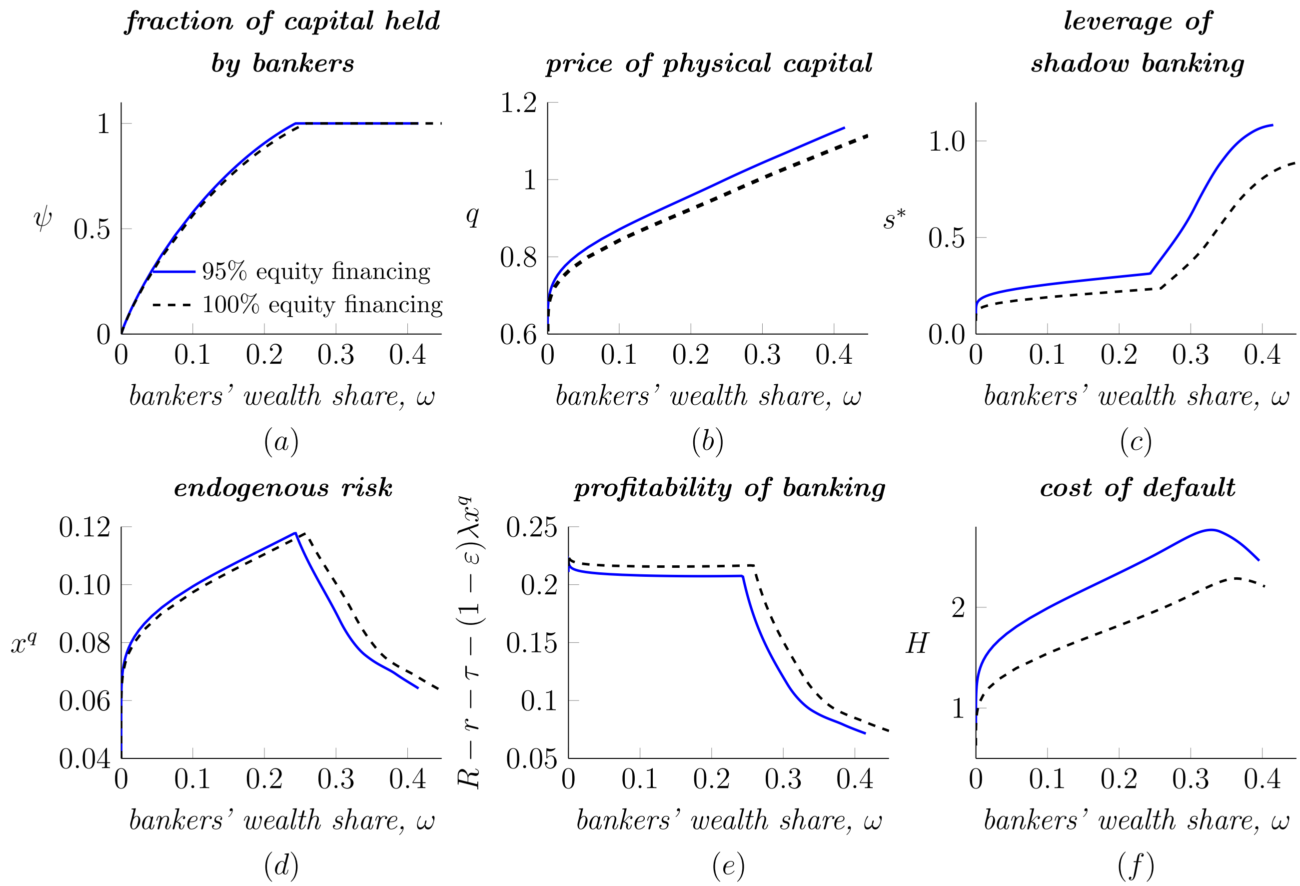

Figure 7: Economic Dynamics

This figure presents the fraction of physical capital held by bankers ψ, the price of physical capital q, the leverage of shadow banking s^*, the endogenous risk x^q, the profitability of banking $R - r - \tau - (1 - \varepsilon)\lambda x^q$, and the opportunity cost of default H as functions of the state variable ω (i.e., bankers' wealth share) in two different models. Solid lines correspond to the extended model where bankers only need to retain 95% of their banks' equity shares. For a shadow bank, retaining 95% of their equity shares means investors only expect sponsoring banks to bear 95% of the loss that occurs to the shadow bank. Dashed lines correspond to the baseline model. For parameter values, see Section 2.3.

We next focus on key properties of the model (readers can find other equilibrium conditions in the online appendix). First of all, outside equity financing improves the risk sharing between bankers and households as well as the efficiency of capital allocation (Panel a and b of Figure 8). However, due to the expansion of shadow banking (Panel c of Figure 8), endogenous risk does not change much compared to the baseline model (Panel d of Figure 8). Since bankers share profits with external shareholders, the profitability of banking declines (Panel e of Figure 8), which explains the decline in bankers' incentives to default on their shadow bank debt.

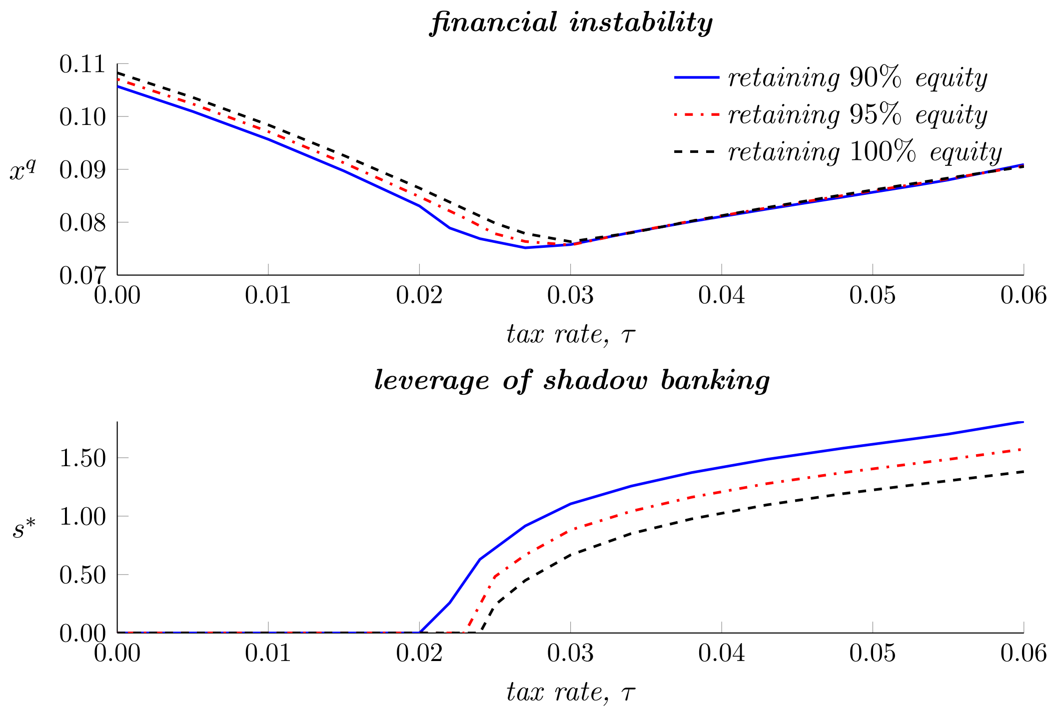

Figure 8: Financial Instability, Bank Regulation, and Outside Equity.
This figure shows how the change in the tax rate influences the endogenous risk x^q and the leverage of shadow banking at the stochastic steady states in the baseline model (dashed lines) and in two extended models where bankers must retain 95% (dash-dotted lines) or 90% (solid lines) of their banks' equity shares. For a shadow bank, retaining ε proportion of their equity shares means investors only expect its sponsor to bear ε proportion of the loss that occurs to the shadow bank. The stochastic steady state is the state where $\omega\mu^{\omega} - \lambda(\omega - \hat{\omega}) = 0$. For other parameter values, see Section 2.3.

The "regulatory paradox" result still holds. If the tax on regular banks' liability is sufficiently low, shadow banking is unsustainable. Since the enforcement problem is less severe in economies with better risk sharing, shadow banking is more likely to emerge in such economies. The lower panel of Figure 8 shows that shadow banking emerges if the tax rate τ is larger than 0.2 in the economy where bankers must retain 90% of equity. However, this threshold is around 0.25 in the baseline model. If shadow banking is relatively small, financial instability is significantly lower in economies where bankers can share more aggregate risks with households (the upper panel of Figure 8). If the regulation of regular banks is so stringent that the shadow banking sector is relatively large, tightening bank regulation raises financial instability as in the baseline model.

The striking result presented by Figure 8 is that the expansion of shadow banking can neutralize positive effects of aggregate risk sharing. The lower panel of Figure 8 shows that the size of the shadow banking sector is larger in an economy where bankers can share a larger proportion of aggregate risk with households. Since shadow banking increases endogenous risk, economies with different degrees of aggregate risk sharing have almost the same level of financial instability as the economy without aggregate risk sharing has (the upper panel of Figure 8).

5 Conclusions

In this paper, we emphasize the enforcement problem with shadow banking and endogenize the borrowing capacity of shadow banking based on this friction. By modeling shadow banking in a continuous-time macro-finance framework, our paper captures dynamics of shadow banking and uncovers the general equilibrium mechanism through which shadow banking adds to financial instability. Since the borrowing capacity of shadow banking is endogenous in our framework, our paper highlights that tightening regulation of regular banking actually helps shadow banks increase their debt capacity. The general equilibrium framework that we use is suitable for welfare and policy discussions in the modern economy where the unregulated shadow banking sector plays a critical role.

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Appendix

A Proofs

Proof of Lemma 1. Let W_t^a denote $\int_0^1 W_t^i di$. In a Markov equilibrium, bankers' dynamic budget constraint (4), the optimal portfolio choice of bankers, and the balanced budget of the regulatory authority imply that

$$\begin{aligned} dW_t^a &= W_t^a \left(\left(R_t + s_t \left(R_t - r_t \right) + s_t^* \left(R_t - r_t \right) - \rho - \chi(1-\sigma) \right) dt \right) \\ &\quad - \left(W_t^a - \left((1 + s_t + s_t^*)(1-x) \frac{\hat{q}_t}{q_t} W_t^a - (s_t + s_t^*) W_t^a \right) \right) dN_t, \end{aligned}$$

where $(1+s_t+s_t^*)(1-x)\frac{\hat{q}_t}{q_t}W_t^a$ is the asset values of all physical capital that bankers hold and $(s_t+s_t^*)W_t^a$ is the value of liabilities that bankers have. Note bankers retire at the intensity χ. Given

$$d\left(q_tK_t\right)=q_tK_t\left(\mu_t^q+\mu_t^K\right)dt-\left(q_tK_t-(1-x)K_t\hat{q}_t\right)dN_t,$$

the scaling factor $1/\left(q_tK_t\right)$ evolves according to

$$d\left(\frac{1}{q_tK_t}\right)=-\frac{1}{q_tK_t}\left(\mu_t^q+\mu_t^K\right)dt-\left(\frac{1}{q_tK_t}-\frac{1}{(1-x)K_t\hat{q}_t}\right)dN_t.$$

Then, Ito's lemma implies that

$$\begin{aligned} d\omega_t &= \omega_t\mu_t^\omega dt-(\omega_t-\hat{\omega}_t)dN_t \\ \text{where } \mu_t^\omega &= R_t+s_t\left(R_t-r_t\right)+s_t^*\left(R_t-r_t\right)-\mu_t^q-\mu_t^K-\rho-\chi(1-\sigma) \\ \text{and } \hat{\omega}_t &= \omega_t\frac{(1+s_t+s_t^*)(1-x)\hat{q}_t-(s_t+s_t^*)q_t}{(1-x)\hat{q}_t}. \end{aligned}$$

■

Lemma 2 *The optimal choice of a banker $\{s_t,s_t^*,\mathcal{D}_t\}$ characterized in Section 2 is time consistent.*

Proof. The Hamilton-Jacobi-Bellman (HJB) equation for the banker's optimal control problem

$$0=\max_{c_t,S_t,S_t^*\le\bar{s}_t^*W_t,\mathcal{D}_t}\left\{(1-\mathcal{D}_t)\mathbf{HJB}_\mathcal{N}+\mathcal{D}_t\mathbf{HJB}_\mathcal{D}\right\},\text{ where} \tag{26}$$

$$\begin{aligned} \mathbf{HJB}_\mathcal{N} &\equiv \max_{c_t,S_t,S_t^*\le s_t^*W_t}\left\{\begin{array}{c}\ln(c_t)-\rho J_t+\frac{\mu_W}{\rho}+h_t\mu_t^h+\chi\left(J_t^r(W_t)-J_t\right)\\ +\lambda\left(\frac{1}{\rho}\ln\left(W_t-\left(W_t+S_t+S_t^*\right)x_t^q\right)+\hat{h}_t-J_t\right)\end{array}\right\}, \\ \mathbf{HJB}_\mathcal{D} &\equiv \max_{c_t,S_t,S_t^*\le s_t^*W_t}\left\{\begin{array}{c}\ln(c_t)-\rho J_t+\frac{\mu_W}{\rho}+h_t\mu_t^h+\chi\left(J_t^r(W_t)-J_t\right)\\ +\lambda\left(\frac{1}{\rho}\ln\left(W_t-\left(W_t+S_t\right)x_t^q\right)+\hat{h}_t^d-J_t\right)\end{array}\right\}, \\ \mu_W &\equiv \frac{1}{W_t}\left(W_t(R_t+\pi_t)+S_t\left(R_t-r_t-\tau_t\right)+S_t^*\left(R_t-r_t\right)-c_t\right). \end{aligned}$$

While choosing her portfolio and consumption at time t, the banker also decides whether she would default on her shadow bank obligations ($\mathbf{HJB}_\mathcal{D}$) in the event of an adverse shock or not ($\mathbf{HJB}_\mathcal{N}$). Because of the time-consistency problem, a banker's portfolio choice (S_t,S_t^*) with respect to both $\mathbf{HJB}_\mathcal{N}$ and $\mathbf{HJB}_\mathcal{D}$ must satisfy their time-consistency constraints:

$$\frac{\ln\left(W_t-\left(W_t+S_t+S_t^*\right)x_t^q\right)}{\rho}+\hat{h}_t\ge\frac{\ln\left(W_t-\left(W_t+S_t\right)x_t^q\right)}{\rho}+\hat{h}_t^d \tag{27}$$

for $\mathbf{HJB}_\mathcal{N}$ and

$$\frac{\ln\left(W_t-\left(W_t+S_t+S_t^*\right)x_t^q\right)}{\rho}+\hat{h}_t\le\frac{\ln\left(W_t-\left(W_t+S_t\right)x_t^q\right)}{\rho}+\hat{h}_t^d \tag{28}$$

for $\mathbf{HJB}_{\mathcal{D}}$. First-order conditions with respect to portfolio choices are given by (13) and (14) for $\mathbf{HJB}_{\mathcal{N}}$, and (15) for $\mathbf{HJB}_{\mathcal{D}}$. The banker finds it optimal to honor her shadow bank debt if $\mathbf{HJB}_{\mathcal{N}} \geq \mathbf{HJB}_{\mathcal{D}}$.

Next, we will show that if the leverage constraint on shadow banking is satisfied bankers' HJB equation can reduce to $0 = \mathbf{HJB}_{\mathcal{N}}$ with the time-consistency constraint (27) being satisfied. First, we know that the optimal choice of $\mathbf{HJB}_{\mathcal{D}}$ is dominated by that of $\mathbf{HJB}_{\mathcal{N}}$ by how we define the maximum leverage of shadow banking in equation (17). Thus, what we need to verify is that if the portfolio choice (s_t, s_t^*) satisfies the leverage constraint for shadow banking, it also satisfies the time-consistency constraint (27). Given the leverage constraint $s_t^* \leq \bar{s}_t^*$ and the first-order condition with respect to (s_t, s_t^*), we have

$$s_t^* \leq \rho\big(\hat{h}_t - \hat{h}_t^d)\big)\frac{1-(1+s_t+s_t^*)x_t^q}{x_t^q}.$$

Since $x > \ln(1+x)$ for $x > 0$ and $(1+s_t+s_t^*)x_t^q < 1$ by the solvency constraint,

$$\ln\left(1+\frac{s_t^* x_t^q}{1-(1+s_t+s_t^*)x_t^q}\right) < \frac{s_t^* x_t^q}{1-(1+s_t+s_t^*)x_t^q} \leq \rho\big(\hat{h}_t - \hat{h}_t^d\big).$$

Hence, we show that the time-consistency constraint (27) is satisfied. ■

B Algorithm

The formal delay differential equations that characterize the Markov equilibrium are

$$q'(\omega) = \frac{\mu^q(\omega, q_0^\omega, H_0^\omega)}{\omega\mu^\omega(\omega, q_0^\omega, H_0^\omega)}q(\omega), \text{and } H'(\omega) = \frac{\mu^H(\omega, q_0^\omega, H_0^\omega)}{\omega\mu^\omega(\omega, q_0^\omega, H_0^\omega)}H(\omega),$$

where $q_0^\omega \equiv \{q(v), 0 \leq v \leq \omega\}$, $H_0^\omega \equiv \{H(v), 0 \leq v \leq \omega\}$ and μ^q, μ^H, and μ^ω are functional operators that we illustrate in Section 2.2.1.

As ω approaches zero, $q(\hat{\omega})$ is close to $q(\omega)$ and the expression of the endogenous risk (2) implies that x^q converges to x. As the marginal buyer of physical capital, households' first-order condition (11) fixes $q(0) = \underline{q}$, which satisfies

$$\frac{a^h - \delta}{\underline{q}} + \frac{(\underline{q}-1)^2}{2\phi\underline{q}} - \rho = \lambda x.$$

As ω converges to zero, $H(\hat{\omega})$ becomes close to $H(\omega)$. The differential equation (24) for $H(\omega)$ implies that $H(\omega)$ converges to ${\tau s^*}/{\rho(\rho+\xi+\chi)}$. Given the expression for the maximum leverage of shadow banking (17), we can see that $H(\omega)$ converges to zero as ω becomes arbitrarily close to zero.

The critical difference between delay differential equations and ordinary differential equations is that $q'(\omega)$ and $H'(\omega)$ not only depend on $q(\omega)$ and $H(\omega)$ but also rely on values of the two functions on $[0, \omega]$. Therefore, to numerically solve for $q(\omega)$ and $H(\omega)$ initial conditions that we need are $q(\omega) = \underline{q} + q_\epsilon$ and

$H(\omega) = H_\epsilon \frac{\omega}{\omega^0}$ on an interval $[0, \omega^0]$, where q_ϵ, H_ϵ, and ω^0 are small constants.

Numerical Procedure

We set positive constants $q_\epsilon^l = 0$, $q_\epsilon^h = 0.4$, $H_\epsilon^l = 0$, $H_\epsilon^h = 0.4$, $\omega^0 = 2 \times 10^{-5}$, and define $q_\epsilon = 0.5(q_\epsilon^l + q_\epsilon^h)$ and $H_\epsilon = 0.5(H_\epsilon^l + H_\epsilon^h)$. We discretize the state space $[0, 1]$ such that $\omega(1) = 0, \omega(2), ..., \omega(m) = \omega^0, ..., \omega(N) = 1$. We follow the following steps to solve the differential equations for $\omega(n)$, $n = m, m+1, ...$

(i) For each $n \geq m$, we need to identify $\hat{\omega}$, where ω moves to if a Poisson shock arrives. For any candidate $\hat{\omega}$ within the interval $(0, \omega(n))$, we find $\hat{q} = q(\hat{\omega})$ and $\hat{H} = H(\hat{\omega})$. Next, we solve for $s + s^*$ based on equation (21) and $\bar{s}^*$ based on equations (13) and (17). Hence, we also fix s. To test if the candidate $\hat{\omega}$ is correct, we check if equation (25) holds. If we find such $\hat{\omega}$, then proceed to step (ii)

(ii) If $s + s^*$ satisfies $\omega(1 + s + s^*) \leq 1$, then proceed to step (iii); otherwise, we search for smaller $\hat{\omega}$ such that $\omega(1 + s + s^*) = 1$ holds and proceed to step (iii)

(iii) Given $\hat{\omega}$ as well as x^q, s, and s^*, we use equation (20) to compute μ^ω and use equation (13) to compute μ^q. Given μ^ω and μ^q, equation (22) yields $dq/d\omega$. Hence, we compute $q(n+1)$

$$q(n+1) = q(n) + \frac{dq}{d\omega}(\omega(n+1) - \omega(n)),$$

and proceed to step (iv)

(iv) To update $H(n+1)$, we first compute $\hat{H} = H(\hat{\omega})$ and use equation (24) to compute $dH/d\omega$. Then, we compute $H(n+1)$ according to

$$H(n+1) = H(n) + \frac{dH}{d\omega}(\omega(n+1) - \omega(n))$$

and proceed to step (v).

(v) Check five conditions: 1) both μ^q and μ^ω are negative; 2) μ^q is negative and μ^ω is positive; 3) μ^ω is negative and μ^q is positive; 4) $\psi(n) < \psi(n-1)$ while $\psi(n-1) = 1$; 5) $\mu^q(n) > \mu^q(n-1)$ while $\psi(n-1) = 1$.

If condition 1 holds, proceed to step (vi); if condition 2 holds, we lower q_ϵ by setting $q_\epsilon^h = q_\epsilon$ and restart step (i) from $n = m$ without changing q_ϵ^l, H_ϵ^h, or H_ϵ^l; if either of conditions 3 - 5 holds, then we raise q_ϵ by setting $q_\epsilon^l = q_\epsilon$ and restart step (i) from $n = m$ without changing q_ϵ^h, H_ϵ^h, or H_ϵ^l.

If none of the five conditions hold, we proceed to step (i) from $n + 1$.

(vi) Check 4 conditions: 1.1) $|\mu^H(n)| < 10^{-3}$; 1.2) $\mu^H > 10^{-3}$; 1.3) $\mu^H < -10^{-3}$; 1.4) there exists $k \leq n$ such that $\mu^H(k) < 0$ and the maximum absolute value of $\mu^H(k) - \mu^H(k-1)$ for all $k \leq n$ is higher than 1.5.

If condition 1.1 holds, we exit the whole procedure; if condition 1.2 holds, we lower H_ϵ by setting $H_\epsilon^h = H_\epsilon$ and restart step (i) from $n = m$ with $q_\epsilon^l = 0$, $q_\epsilon^h = 0.4$, and not changing H_ϵ^l; if either conditions 1.3 or 1.4 holds, we raise H_ϵ by setting $H_\epsilon^l = H_\epsilon$ and restart step (i) from $n = m$ with $q_\epsilon^l = 0$, $q_\epsilon^h = 0.4$, and not changing H_ϵ^h.

We identify conditions 4, 5, and 1.4 from experimenting a number of numerical procedures and initial conditions. Higher-order methods, such as Runge-Kutta, can replace the Euler method used steps (*iii*) and (*iv*).