

Systemic Run on Shadow Banks: The Minsky Moment

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Abstract

In this paper, we modernize Minsky's "Financial Instability Hypothesis" within a continuous-time macro-finance model in which the financial market consists of both regular banking and shadow banking. Specifically, we model shadow banking as off-balance-sheet financing. A friction exists with off-balance-sheet financing, inasmuch as banks can strategically default on securities generated via off-balance-sheet financing. This friction constrains the borrowing capacity of shadow banking, although shadow banking provides cheaper credit than regular banking does. We show that the decline of asset volatility in economic upturns speeds up the growth of shadow banking. The overheated expansion of shadow banking exposes the economy to the risk of a self-fulfilling systemic run; in turn, this risk then may lead to the sudden collapse of the shadow banking system (i.e., the Minsky moment in this model). We show that this systemic risk arises because individual banks fail to internalize the impact of their leverage choices and, in so doing, expose the entire economy to the systemic risk.

This model provides a framework that could quantify the likelihood of a systemic run in a dynamic setting. Our results show that the chance of a systemic run on the US shadow banking sector occurring in the subsequent year increased from 0 percent in 2003 to 25 percent in 2007.

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Introduction

“In many cases the expansion of credit often resulted from the development of substitutes for what previously had been the traditional monies”

—*Kindleberger’s Manias, Panics, Crashes*

After the 2007-2009 global financial crisis, the rise and fall of the US shadow banking system popularized the “Financial Instability Hypothesis” that Hyman Minsky developed in the 1980s (McCulley, 2009). However, some thirty years later, such an appealing theory for explaining financial crises cannot find its place within the dynamic general equilibrium framework that is widely used in macroeconomic research and policy discussions. Hence, in this paper, we intend to recast the Financial Instability Hypothesis by analyzing shadow banking within the continuous-time macro-finance framework recently developed by He and Krishnamurthy (2012a) and Brunnermeier and Sannikov (2014).

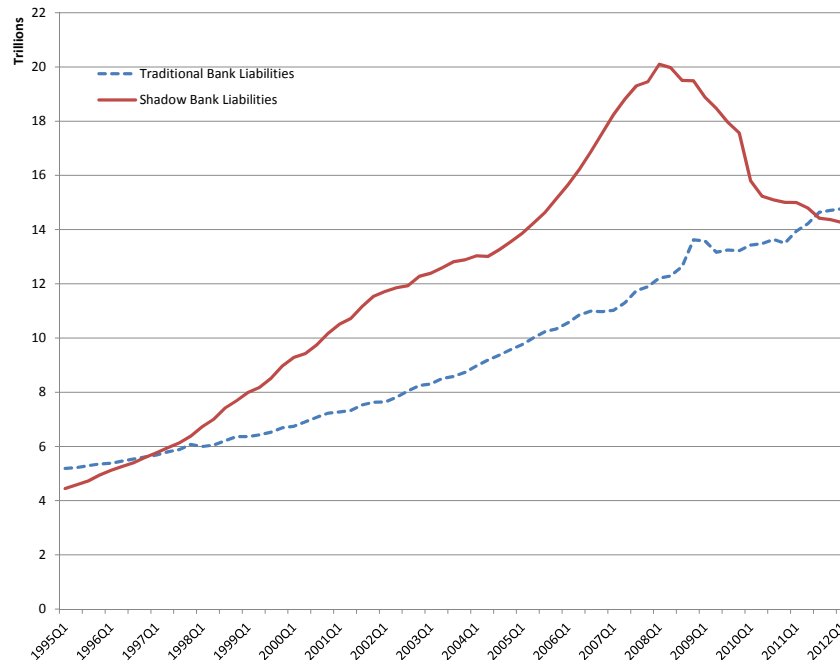


Figure 1: The Liability of Traditional Banking (blue dashed line) and the liability of Shadow Banking (red solid line) from the first quarter of 1995 to the first quarter of 2012.

Source: Board of Governors of the Federal Reserve System, “Flow of Funds Accounts of the United States”¹

¹To follow Pozsar et al. (2010), traditional bank liabilities refer to total liabilities of the commercial banking sector (line 19 of Table L.109). Shadow bank liabilities (netted from overlaps with Table L.109) refer to the sum of total outstanding open market paper (line 1 of Table L.208), total repo liabilities (line 1 of Table L.207), net securities loaned (line 20 of Table L.130), total GSE liabilities and pool securities (lines 21 and 6 of Tables L.124 and L.125, respectively), total liabilities of ABS issuers (line 11 of Table L.126), and total shares outstanding of money market mutual funds (line 14 of Table L.121).

In this paper, we model shadow banking as off-balance-sheet financing and regular banking as on-balance-sheet financing (Pozsar, Adrian, Ashcraft, and Boesky, 2010).

Although regular banks specialize in screening potential borrowers and originating loans, they bear a managerial cost for holding loans to maturity since their employees monitor debtors' financial conditions and collect proceeds, and also because different regulators require inspection.

By exploiting the specialization in the capital market, regular banks can save the managerial costs of holding loans and free up their balance sheets by moving loans that they originate to their off-balance-sheet vehicles, which are more efficient for warehousing loans and processing proceeds. Such off-balance-sheet vehicles are simply shadow banks in the model I use in this paper.

To enhance credit for their shadow banks, regular banks typically promise to protect creditors of shadow banks from potential losses due to any credit event. However, an enforceability problem exists, as regular banks may or may not fulfill their promises. Consequently, the size of the shadow bank that a regular bank can establish is constrained by its creditors so that the cost of keeping its promise is not too large for the regular bank.

Since the bailout cost is smaller when the asset price is less volatile, the size of shadow banking moves inversely with asset volatility. Thus, shadow banking expands as asset volatility declines during economic booms. However, the shadow banking sector's expansion of credit potentially destabilizes financial markets, as the risk of a systemic run on the shadow banking system increases.

A systemic run on the shadow banking sector could lead to the simultaneous default of all shadow banks. When the economy panics about shadow banking and a large number of shadow banks must liquidate assets and repay creditors, asset prices necessarily depreciate, as regular banks hold these assets reluctantly because of managerial costs (as discussed earlier). In the process, the market liquidity of assets deteriorates, thereby heightening shadow banks' credit risk. If assets become very illiquid and shadow banks must sell them at rather low prices to repay creditors, then shadow banks are more likely to fail, as sponsoring regular banks find it too expensive to bail out shadow banks. In turn, these decisions intensify widespread panic in the economy. This self-fulfilling banking panic eventually leads to the collapse of the shadow banking system, as experienced in 2007 and 2008.

The tractability of this model allows us to apply the global game device to uniquely select an equilibrium, when the self-fulfilling banking panic equilibrium is one of them. More importantly, the equilibrium selection mechanism endogenizes the link between the likelihood of a systemic run on shadow banks and their creditors' exposure to this systemic

risk. Intuitively, the banking panic is more likely to happen if creditors' exposure to shadow banking is larger.

Following the Financial Instability Hypothesis, declining asset volatility during prolonged economic upturns accelerates the growth of the shadow banking sector, which in turn increases creditors' exposure to the systemic risk that the shadow banking system may suddenly collapse. When the size of the shadow banking system crosses the endogenously-determined threshold in this model, the economic system becomes unstable, as the possibility that the shadow banking sector collapses increases.

Within a modern dynamic general equilibrium framework, this model adds values to the Financial Instability Hypothesis in the following aspects. First, this model formally characterizes the condition under which the economy or the financial system becomes unstable. As a result, we are able to quantitatively examine the systemic risk of a real economy using this model. In particular, this calibrated model shows that the chance of a systemic run grew from 0% in 2003 to around 25% in early 2007. Second, our model shows that a healthy shadow bank can go bankrupt if a systemic run occurs. This result helps explain why it is difficult to identify Ponzi financial schemes that destabilize the financial system, and also explains why people find it hard to detect systemic risk. Before a systemic run occurs, the model in this paper does not include Ponzi finance schemes, as it is due to the endogenous systemic event itself that all healthy shadow banks suddenly collapse.

Related Literature. Broadly speaking, there are two strands of literature on financial instability. The first starts from the seminal works of [Bernanke and Gertler \(1989\)](#), [Kiyotaki and Moore \(1997\)](#), and [Bernanke et al. \(1999\)](#). These papers point out channels that the credit market could use to amplify the impact of exogenous shocks through adverse feedback loops. The financial instability in this literature takes the form that the financial market magnifies the impact of exogenous aggregate shocks. From this literature, we borrow the important notion that asset volatility could fluctuate endogenously due to frictions within financial markets (i.e., the endogenous risk in our paper). However, financial markets in this literature are not an independent source of instability, which only amplifies the impact of exogenous shocks to the economy. In addition, papers in this literature do not focus on why overheated credit booms expose the economy to the risk of a sudden collapse of the credit market.

[Bryant \(1980\)](#) and [Diamond and Dybvig \(1983\)](#) started the second strand of literature, which studies the self-fulfilling banking panic model. The financial instability in these models is that a bank or a financial market might suddenly break down due to the maturity or liquidity mismatch of their debt structures and the strategic complementarity among creditors. This type of financial instability is the systemic risk featured in this paper. That said, this literature does not consider how an economy could endogenously evolve into a phase during which the risk of a market-wide

panic is high.

[Gertler and Kiyotaki \(2013\)](#), more closely aligned with my present analysis, quantitatively examine the impact of a bank run on the aggregate economy in a dynamic model with financial frictions. The major difference between their paper and our analysis is that they do not attempt to explain how the economy evolves into states where the risk of banking panics is high.² As a result, they do not measure the systemic risk of an economy.

This paper is also related to theoretical studies on shadow banking; these studies include [Gennaioli et al. \(2011\)](#) and [Goodhart et al. \(2012\)](#). In these papers, the distinction between shadow banking and traditional banking is not based on the differences of their institutional details. Our paper particularly focuses on the financial friction associated with off-balance-sheet financing and explains different behaviors of the shadow and traditional banking systems. Also, our explanation of financial crises differs from that provided by [Gennaioli et al. \(2011\)](#) in two aspects. First, we consider a fully rational model, in which financial crises occur on the equilibrium path; in contrast, in [Gennaioli et al. \(2011\)](#), financial crises will not happen unless there are optimistic creditors. Second, there is no difference between shocks that cause financial crises and shocks that do not in our model. Rather, the leverage for off-balance-sheet financing and the potential drop in capital price in the event of a banking panic determines the likelihood of a financial crisis occurring. In [Gennaioli et al. \(2011\)](#), the magnitude of the shock incurring a financial crisis is larger than that of a normal shock. Accordingly, the large size of shadow banking only magnifies the severity of a crisis.

With respect to methodology, this paper follows the emerging literature that investigate economies with financial frictions in a continuous-time setting. This literature includes [Brunnermeier and Sannikov \(2014\)](#) and [He and Krishnamurthy \(2013, 2012a\)](#). The methodology employed in this literature allows for the exact characterization of full equilibrium dynamics. In our model, the tractability allows for explicitly relating the leverage constraint to the endogenous risk. Several papers, such as [Adrian and Boyarchenko \(2012\)](#), [Danielsson et al. \(2012\)](#), and [Phelan \(2012\)](#), also do this by assuming various forms of Value-at-Risk constraints without offering relevant micro-foundations. That said, the leverage constraint in this paper comes from the financial friction that bankers could strategically default on securities generated by off-balance-sheet financing, which is intrinsic to shadow banking. Besides the maximum leverage for off-balance-sheet financing, the microfounded leverage constraint offers an important property—that bankers default if the constraint does not hold—that helps position systemic panic in this paper’s model.

The paper is structured as follows. Section 1 establishes the model setup and defines the equilibrium. In Section 2, we characterize the equilibrium of the baseline model. Section 3 examines the qualitative properties of this model, and Section 4 looks at the quantitative

²For their numerical exercise, they selectively pick values of relevant parameters such that the bank run equilibrium exists right after the economy is hit by a bad shock.

performance of this model, showing that it provides early warning signs of a systemic run.

1 The Model

In this section, we build a continuous-time macro-finance model with both regular banking and shadow banking. More importantly, it incorporates the possibility of a systemic event that the shadow banking system collapses.

1.1 Preferences and Physical Environment

Two types of agents populate in a continuous-time infinite horizon economy: bankers and households. Bankers have logarithmic utility with time discount factor ρ . Households have the Epstein-Zin preference with the time discount rate ρ , the relative risk-aversion coefficient γ , and the elasticity of intertemporal substitution b^{-1} . Both bankers and households consume a single type of perishable final good. Its total stock at time t is denoted by Y_t (i.e., the GDP of the economy). Final goods also serve as the numéraire.

Both bankers and households are equipped with technologies for producing final goods and physical assets. Bankers have final goods production function $y_t = ak_t$, where y_t is the final good output, a the productivity, and k_t the used physical assets. Bankers are more efficient than households in the sense that households' productivity a^h is less than a . However, both types of agents have the same capital good production skill. In particular, they can invert $\Phi(\iota_t)$ units of final goods into ι_t units of physical assets. The holding of physical assets is risky in the sense that the stock of assets will shrink by κ proportion if a macro shock $\{N_t, t \geq 0\}$ hits the economy, which occurs at rate λ . If the investment choice $\{\iota_t, t \geq 0\}$ is the same for all agents, then the law of motion of aggregate capital stock $\{K_t, t \geq 0\}$ is

$$dK_t = K_{t-}(\iota_{t-} - \delta)dt - K_{t-}\kappa dN_t,$$

We next describe the rate of return for bankers' and households' holding physical assets. To do so, We denote the law of motion of the endogenous asset price process $\{q_t, t \geq 0\}$ by

$$dq_t = q_{t-}\mu_{t-}^q dt - q_{t-}((1 - \Gamma_t)\kappa_t^q + \Gamma_t\kappa_t^{q,r})dN_t. \quad (1)$$

where Γ_t indicates the systemic event that might happen after the Poisson shock hits the economy. If a banker spends one dollar on physical assets in the end of a period denoted by $t-$, then she would have $1/q_{t-}$ units. In the next period t , the net output generated by these assets is $(a - \Phi(\iota_{t-}))/q_{t-}$. In addition, she also obtains capital gain $\iota_{t-} - \delta + \mu_{t-}^q$. On

the other hand, if the macro shock hits the economy and the systemic event does not occur, then the banker will have $(1 - \kappa)/q_{t-}$ units of assets left. Moreover, the remaining assets are only worth $q_{t-}(1 - \kappa_t^q)$. Thus, the total loss to the banker's investment is $\kappa + (1 - \kappa)\kappa_t^q$. In sum, the rate of return for holding physical assets is

$$\underbrace{\left[\frac{a - \Phi(\iota_{t-})}{q_{t-}} + \iota_{t-} - \delta + \mu_{t-}^q \right]}_{\equiv R_{t-}} dt - ((1 - \Gamma_t)\kappa_t^Q + \Gamma_t\kappa_t^{Q,r})dN_t,$$

where $\kappa_t^Q \equiv \kappa + (1 - \kappa)\kappa_t^q$ and $\kappa_t^{Q,r} \equiv \kappa + (1 - \kappa)\kappa_t^{q,r}$. Similarly, the rate of return for households to hold assets is

$$\underbrace{\left[\frac{a^h - \Phi(\iota_{t-}^h)}{q_{t-}} + \iota_{t-}^h - \delta + \mu_{t-}^q \right]}_{\equiv R_{t-}^h} dt - ((1 - \Gamma_t)\kappa_t^Q + \Gamma_t\kappa_t^{Q,r})dN_t.$$

1.2 Time Line: Sequential Decisions within a Period

The key insight of this model hinges on the interaction of bankers' sequential movements within a period. In particular, each banker has at most five sequential decisions to make at five stages. To illustrate bankers' decisions more clearly, we split five stages into two groups and introduce them in reverse order. As Figure 2 indicates, if the Poisson shock does not hit the economy, the first group of stages will not be reached in the subgame.

1.2.1 Stage 4 and 5: Regular Banking and Shadow Banking

A banker owns a regular bank as its only equity holder and sponsors a shadow bank as its essential beneficiary. At stage 4, regular banks *i*) return both the principal and interest to their creditors (i.e., households), *ii*) recognize losses to their equity holders if there are any, and *iii*) accumulate physical assets. At stage 5, shadow banks pay back principal and interest, "trade" physical assets with their own regular banks, and issue financial assets that regular banks owned by other bankers invest in.

Regular banks need to pay managerial cost τ per dollar physical assets on their books. However, such costs do not apply to any "safe" financial asset on their balance sheets. More specifically, the total managerial cost paid by a regular bank depends on the maximum number of physical assets that it holds at stage 4 and 5.

Shadow banks, which have zero equity by construction, specialize in warehousing physical assets and do not pay any managerial cost. We name the financial assets that shadow banks

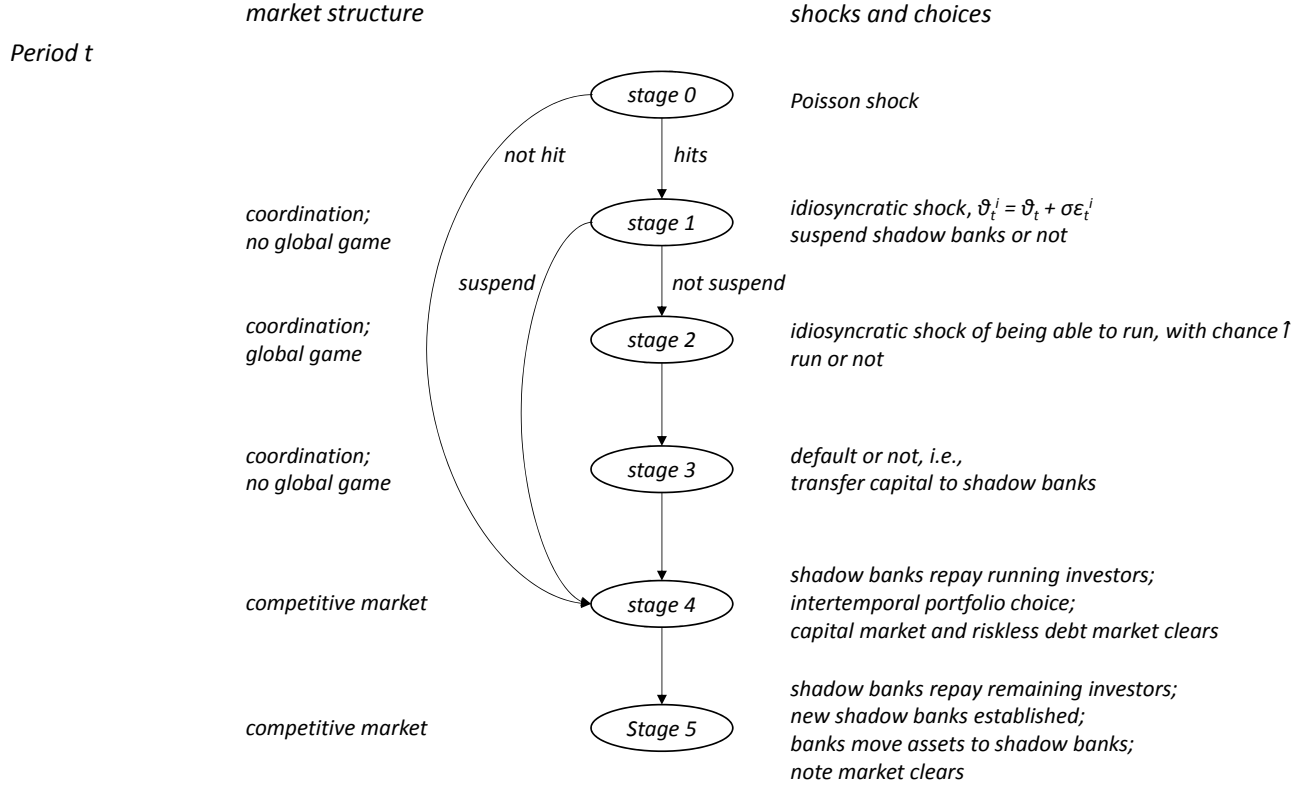


Figure 2: Game Tree

issue as “notes.” The following assumption specifies the structure of the note market.

Assumption 1

- i. A continuum of regular banks can form a union, which offers one-period note contracts $(\bar{s}_t^*, \tilde{r}_t)$ to any shadow bank, whose sponsoring banker is outside of the union.*
- ii. The shadow bank that accepts the contract can borrow up to \bar{s}_t^* times its sponsor’s net worth and pay the principal and interest at rate \tilde{r}_t next period.*
- iii. If a shadow bank fails, then the union would enforce the punishment scheme that its sponsor pays l_0 fraction of her net worth to the union every period in the future. For instance, the fine stream would be $\{l_0 W_s, s \geq t\}$ for a banker given that the process of her net worth is $\{W_s, s \geq t\}$ since she misbehaves in period t .*
- iv. The note market is competitive.*

Although shadow banks are supposed to pay back creditors at stage 5, creditors can—if they run—have their principal back at stage 4.

1.2.2 Stage 1, 2, and 3: Systemic Run and Strategic Default

At stage 1, banker i receives a private signal $\theta_t^i = \theta_t + \sigma\varepsilon_t^i$. Each banker decides whether to suspend her shadow bank, conditional on the Poisson shock's hit. If a banker suspends her shadow bank, then no creditor can have the principal back at stage 4. In equilibrium, all bankers might suspend their shadow banks simultaneously, which will cause the market-wide shadow bank default. The following assumption specifies what happens after such a systemic event.

Assumption 2

- i. If almost all shadow banks default, then the economy falls into a distressed phase where no shadow banking exists.*
- ii. The economy in the distressed phase switches to the normal phase with the intensity π .*
- iii. The capital gain that occurs when the economy switches back to the normal phase is redistributed across all agents, according to their wealth levels.*

Since this systemic event is beyond the control of an individual agent, we consider “safe” financial assets those that guarantee the principal's safety conditional on the systemic event not happening.

At stage 2, each banker receives an idiosyncratic shock about whether she is able to run or not. A banker is able to run with probability \hat{l} .³ This random event is independent across all bankers. At stage 2, bankers, who are able to run, decide whether to let their regular banks run on shadow banks as note investors. If banker i runs, then she pays a utility cost θ_t^i .

At stage 3, the proportion of note investors who run becomes publicly observable, and bankers decide whether to transfer physical assets from their regular banks to their shadow banks. If nearly no bankers suspend their shadow banks at stage 1, then they can renege their early decisions and decide to support their shadow banks at this stage.

1.3 Bankers' and Households' Portfolio Choices

To simplify the analysis of bankers' portfolio choice problem, we conjecture that if almost no banker suspends her shadow bank (i.e., no systemic event occurs), then no regular bank runs at stage 2, and all bankers transfer enough physical assets, if it is necessary, at stage 3. We verify this conjecture in the next section.

Suppose a banker's net worth is W_t , which is also the regular bank's equity value. S_t denotes the value of physical assets on her balance sheet at stage 5, which equals $q_t k_t$ given

³where $\hat{l} < 1 - \kappa_t^{Q,r}$ for all $t \geq 0$

that there are k_t units of physical capital. B_t denotes the value of notes that the regular bank holds. The excess return from the holding of physical assets is

$$S_{t-}(R_{t-} - \tau - r_{t-})dt - ((1 - \Gamma_t)S_{t-}\kappa_t^Q + \Gamma_t S_{t-}\kappa_t^{Q,r})dN_t,$$

where r_t is the risk-free rate. The excess return from holding notes issued by shadow banks (i.e., from lending to shadow banks sponsored by other bankers) is

$$B_{t-}(\tilde{r}_{t-} - r_{t-})dt - \Gamma_t B_{t-}\kappa_t^{Q,r} dN_t,$$

where \tilde{r}_t is the rate of return for holding notes.

The increase in the banker's net worth is also due to the return from sponsoring the shadow bank. As the beneficiary of the shadow bank, the banker then obtains all excess return, that is

$$S_{t-}^*(R_{t-} - \tilde{r}_{t-})dt - (1 - \Gamma_t)S_{t-}^*\kappa_t^Q dN_t,$$

where S_t^* is the value of notes issued by the shadow bank. The loss comes from the fact that the banker transfers physical assets to cover the loss of the shadow bank at stage 3, when a Poisson shock hits the economy. Consequently, such a loss will be recognized as the loss to the regular bank's equity value.

In sum, the law of motion for the banker's net worth W_t is

$$\begin{aligned} dW_t = & (W_{t-}r_{t-} + S_{t-}(R_{t-} - \tau - r_{t-}) + B_{t-}(\tilde{r}_{t-} - \hat{r}_{t-}) + S_{t-}^*(R_{t-} - \tilde{r}_{t-}))dt \\ & - ((1 - \Gamma_t)(S_{t-} + S_{t-}^*)\kappa_{t-}^Q + \Gamma_t(S_{t-} + B_{t-})\kappa_t^{Q,r})dN_t - c_{t-}dt. \end{aligned} \quad (2)$$

Taking $\{q_t, r_t, \tilde{r}_t, \bar{s}_t^*, t \geq 0\}$ as given, the banker's problem is to choose $\{c_t, S_t, B_t, S_t^*, \iota_t, t \geq 0\}$ and maximize

$$E \left[\int_0^\infty e^{-\rho t} \ln(c_t) dt \right]$$

subject to the dynamic budget constraint (2) and the leverage constraint $S_t^* \leq \bar{s}_t^* W_t$.

Households split their wealth between physical assets and riskless debts issued by regular banks. With S_t^h as the value of physical assets that household h has, the wealth W_t^h of the household then evolves according to

$$dW_t^h = W_{t-}^h r_{t-} dt + S_{t-}^h (R_{t-}^h - r_{t-}) dt - ((1 - \Gamma_t)S_{t-}^h \kappa_t^Q + \Gamma_t S_{t-}^h \kappa_t^{Q,r}) dN_t - c_{t-}^h dt. \quad (3)$$

By choosing $\{c_t^h, S_t^h, t \geq 0\}$ a household maximizes

$$U_0^h = E_0 \left[\int_0^\infty f [c_s^h, U_s^h] ds \right],$$

where

$$f [c^h, U^h] = \frac{1}{1-b} \left\{ \frac{\rho (c^h)^{1-b}}{((1-\gamma) U^h)^{\frac{\gamma-b}{1-\gamma}}} - \rho (1-\gamma) U^h \right\}$$

and

$$U_t^h = E_t \left[\int_t^\infty f [c_s^h, U_s^h] ds \right], \text{ for } t > 0,$$

subject to the dynamic budget constraint (3).

1.4 Equilibrium

We make the following assumption to guarantee that the wealth share of bankers does not become large enough to undo all financial frictions:

Assumption 3 *Each banker exits the economy independently with intensity ξ .*

$\mathbf{I} = [0, 1]$ and $\mathbf{J} = (1, 2]$ denote sets of bankers and households, respectively. Individual bankers and households are indexed by $i \in \mathbf{I}$ and $j \in \mathbf{J}$.

Definition 1 *Given any initial endowments of capital goods $\{k_0^i, k_0^j; i \in \mathbf{I}, j \in \mathbf{J}\}$ such that*

$$\int_0^1 k_0^i di + \int_1^2 k_0^j dj = K_0,$$

the equilibrium is defined by a set of stochastic processes adapted to the filtration generated by $\{N_t, \theta_t, \varepsilon_t^i, t \geq 0, i \in \mathbf{I}\}$: capital price $\{q_t, t \geq 0\}$, risk-free rate $\{r_t, t \geq 0\}$, the rate return of notes $\{\tilde{r}_t, t \geq 0\}$, the likelihood of the systemic event $\{\varrho_t, t \geq 0\}$, wealth $\{W_t^i, W_t^j, t \geq 0\}$, capital holdings $\{k_t^i, k_t^j, t \geq 0\}$, investment decisions $\{i_t^i, t \geq 0\}$, holdings of financial instruments $\{B_t^i, t \geq 0\}$, maximum leverage of shadow banking $\{\bar{s}_t^, t \geq 0\}$, suspension decisions $\{\gamma_t^i, t \geq 0\}$, and consumption $\{c_t^i, c_t^j, t \geq 0\}$ of individual banker $i \in \mathbf{I}$ and individual household $j \in \mathbf{J}$; such that*

1. $\{W_0^i, W_0^j\}$ satisfy $W_0^i = q_0 k_0^i$ and $W_0^j = q_0 k_0^j$, for $i \in \mathbf{I}$ and $j \in \mathbf{J}$;
2. bankers solve their problems given $\{q_t, t \geq 0\}$, $\{r_t, t \geq 0\}$, $\{\tilde{r}_t, t \geq 0\}$, $\{\varrho_t, t \geq 0\}$, and $\{\bar{s}_t^*, t \geq 0\}$;
3. households solve their problems given $\{q_t, t \geq 0\}$ and $\{r_t, t \geq 0\}$;

4. markets for consumption goods and capital goods clear for all $t \geq 0$

$$\begin{aligned} \int_0^1 c_t^i di + \int_1^2 c_t^j dj &= \int_0^1 (a - g(\iota_t^i)) k_t^i di + \int_1^2 (a^h - g(\iota_t^j)) k_t^j dj, \\ \int_0^1 k_t^i di + \int_1^2 k_t^j dj &= K_t, \\ \text{where } dK &= \left(\int_0^2 (\iota_t^i - \delta) k_t^i di \right) dt - \kappa K_{t-} dN_t; \end{aligned} \quad (4)$$

5. Note market clears for all $t \geq 0$

$$\int_0^1 S_t^{*,i} di = \int_0^1 B_t^i di.$$

Given this definition, the debt market automatically clears by Walras' Law given this definition. Later, we show that bankers' decisions to run play a role off the equilibrium path.

2 Systemic Risk and Banking Panic

In this section, we endogenize the systemic event that leads to the shadow banking system shutting down and characterize the likelihood of the systemic event $\{\varrho_t, t \geq 0\}$.

2.1 Characterizing Portfolio Choices

Bankers have both standard portfolio choices and strategic choices to make. To characterize their decisions, we begin our analyses with bankers' portfolio choice problem, given the conjecture that no banker defaults in the absence of the systemic event. By using bankers' continuation values, we will verify our conjecture by examining their strategic decisions.

2.1.1 Bankers' Portfolio Choices

ι_t denotes a banker's investment at time t . The expression of R_t implies that optimal level of ι_t maximizes

$$\frac{-\iota_t - 0.5\tau(\iota_t - \delta)^2}{q_t} + \iota_t.$$

The first-order condition yields that the optimal investment rate is a function of the capital price q_t

$$\iota(q_t) = \delta + \frac{q_t - 1}{\tau}. \quad (5)$$

Households have the same investment function $\iota(\cdot)$ since they have the same technology.

We apply the stochastic control approach to solve for the bankers' optimal consumption and portfolio choices, based on the conjecture that no banker defaults. The following proposition summarizes analytical results:

Proposition 1 *A banker's optimal consumption $\{c_t, t \geq 0\}$ and optimal portfolio weights $\{s_t, b_t, s_t^*, t \geq 0\}$ satisfy*

$$c_t = \rho W_t, \quad (6)$$

$$\max\{(s_{t-} + s_{t-}^*)\kappa_t^Q, (s_{t-} + b_{t-})\kappa_t^{Q,r}\} < 1, \quad (7)$$

$$R_{t-} - \tau - r_{t-} \leq \frac{\lambda(1 - \varrho_t)\kappa_t^Q}{1 - (s_{t-} + s_{t-}^*)\kappa_t^Q} + \frac{\lambda\varrho_t\kappa_t^{Q,r}}{1 - (s_{t-} + b_{t-})\kappa_t^{Q,r}}, = \text{if } s_{t-} > 1, \quad (8)$$

$$\tilde{r}_{t-} - r_{t-} \leq \frac{\lambda\varrho_t\kappa_t^{Q,r}}{1 - (s_{t-} + b_{t-})\kappa_t^{Q,r}}, = \text{if } b_{t-} > 0, \quad (9)$$

$$R_{t-} - \tilde{r}_{t-} \geq \frac{\lambda(1 - \varrho_t)\kappa_t^Q}{1 - (s_{t-} + s_{t-}^*)\kappa_t^Q}, = \text{if } s_{t-}^* < \bar{s}_{t-}^*, \quad (10)$$

where $s_t = S_t/W_t$ and $s_t^* = S_t^*/W_t$.

Proof. See Appendix. ■

Equation (7) guarantees that the banker's net worth is positive after a bad shock to avoid negative infinite utility. Equation (8) – (10) are standard asset pricing equations.

Notice that the portfolio weight on notes b is indeterminate given equation (9) and other first-order conditions. To tackle this problem, we make the following assumption:⁴

Assumption 4 *If bankers are indifferent about holding notes or not, then their portfolio weights on notes are the same.*

2.1.2 Households' Portfolio Choice

In solving for households' optimal portfolio and consumption strategies, we also apply the stochastic control approach. For the household with net worth W_t^h , her continuation value function is defined as

$$V_t^h = \max_{\{c_u^h, S_u^h, u \geq t\}} U_t^h.$$

⁴This assumption is only necessary for the simple financial regulation setup in this model. The simple setup offers a quantitative advantage because there is only one parameter associated with the financial regulation that needs to be calibrated.

We conjecture that the continuation value takes the functional form

$$V_t^h = V(\zeta_t, W_t^h) \equiv \frac{(\zeta_t W_t^h)^{1-\gamma}}{1-\gamma},$$

where ζ_t follows

$$d\zeta_t = \zeta_t \mu_{t-}^{\zeta} dt - \zeta_t \left((1 - \Gamma_t) \kappa_t^{\zeta} + \Gamma_t \kappa_t^{\zeta, r} \right) dN_t.$$

As in [Di Tella \(2012\)](#), ζ_t is interpreted as the continuation value multiplier of households' net worth since the function $V(\zeta, W^h)$ is homogeneous of degree $1 - \gamma$ with respect to W^h . Given this conjecture, the Hamilton-Jacobi-Bellman equation of the household's dynamic programming problem is

$$0 = \max_{c^h, S^h} \left\{ f(c_t^h, V_t^h) + \mathcal{D}^{c^h, S^h} V(\zeta_t, W_t^h) \right\}, \quad (11)$$

where

$$\begin{aligned} \mathcal{D}^{c^h, S^h} V(\zeta_t, W_t^h) &= (W_{t-}^h r_{t-} + S_{t-}^h (R_{t-}^h - r_{t-}) - c_{t-}^h) \zeta_{t-}^{1-\gamma} (W_{t-}^h)^{-\gamma} + \mu_{t-}^{\zeta} (\zeta_{t-} W_{t-}^h)^{1-\gamma} \\ &+ \frac{\lambda(1 - \varrho_t) \left((\zeta_{t-} - \zeta_{t-} \kappa_t^{\zeta}) (W_{t-}^h - S_{t-}^h \kappa_t^Q) \right)^{1-\gamma}}{1 - \gamma} \\ &+ \frac{\lambda \varrho_t \left((\zeta_{t-} - \zeta_{t-} \kappa_t^{\zeta, r}) (W_{t-}^h - S_{t-}^h \kappa_t^{Q, r}) \right)^{1-\gamma}}{1 - \gamma} - \frac{\lambda (\zeta_{t-} W_{t-}^h)^{1-\gamma}}{1 - \gamma}. \end{aligned}$$

We summarize the key results of the problem in the following proposition:

Proposition 2 *Each household's optimal consumption weight $\{\hat{c}_t^h, t \geq 0\}$, optimal portfolio weight $\{s_t^h, t \geq 0\}$, and the process $\{\zeta_t, t \geq 0\}$ satisfy*

$$(\hat{c}_t^h)^b = \frac{\rho}{\zeta_t^{1-b}}, \quad (12)$$

$$R_{t-}^h - r_{t-} \leq \frac{\lambda(1 - \Gamma_t) \kappa_t^Q}{(1 - s_{t-}^h \kappa_t^Q)^\gamma} (1 - \kappa_t^{\zeta})^{1-\gamma} + \frac{\lambda \Gamma_t \kappa_t^{Q, r}}{(1 - s_{t-}^h \kappa_t^{Q, r})^\gamma} (1 - \kappa_t^{\zeta, r})^{1-\gamma}, \quad = \text{ if } s_{t-}^h > 0, \quad (13)$$

$$\begin{aligned} 0 &= \frac{\rho (\hat{c}_{t-}^h)^{1-b}}{(1-b) \zeta_{t-}^{1-b}} - \frac{\rho}{1-b} + r_{t-} + s_{t-}^h (R_{t-}^h - r_{t-}) - \hat{c}_{t-}^h + \mu_{t-}^{\zeta} \\ &+ \lambda \left(\frac{\left((1 - \varrho_t) \left((1 - \kappa_t^{\zeta}) (1 - s_{t-}^h \kappa_t^Q) \right) \right)^{1-\gamma}}{1 - \gamma} + \frac{\varrho_t \left((1 - \kappa_t^{\zeta, r}) (1 - s_{t-}^h \kappa_t^{Q, r}) \right)^{1-\gamma}}{1 - \gamma} - \frac{1}{1 - \gamma} \right), \end{aligned} \quad (14)$$

where $\hat{c}_t^h = c_t^h / W_t^h$ and $s_t^h = S_t^h / W_t^h$.

Proof. See Appendix. ■

The first-order conditions of households have similar interpretations except that the continuation value multiplier ζ_t affects households' consumption and investment decisions, as shown by equation (12) and (13). By plugging the household's optimal choices into the HJB equation (11), we can derive equation (14), which specifies the condition that $\{\zeta_t, t \geq 0\}$ should satisfy.

2.2 Enforceability Constraint and Inevitable Systemic Risk

By analyzing the the note market's equilibrium, we characterize the maximum leverage of shadow banking and explain why the financial market cannot escape from the systemic risk by itself.

2.2.1 Maximum Leverage of Shadow Banking

Bankers assume the maximum leverage of shadow banking $\{\bar{s}_t^*, t \geq 0\}$ when they solve for optimal portfolio choices at stage 4 and 5 of each period. We next characterize $\{\bar{s}_t^*, t \geq 0\}$ based on the strategic default choices of bankers at stage 3 and the market clearing condition of the note market at stage 5.

Thanks to the logarithm preference, the expected lifetime utility of a banker with net worth W_t at time t is $\ln(W_t) / \rho + h_t$, which will be referred to as the banker's continuation value. The following lemma characterizes $\{h_t, t \geq 0\}$.

Lemma 1 *The law of motion of $\{h_t, t \geq 0\}$ satisfies*

$$\begin{aligned} \rho h_{t-} &= h_{t-} \mu_{t-}^h + \xi \left(\frac{r - \rho}{\rho} - h_{t-} \right) + \log(\rho) \\ &+ \frac{1}{\rho} (r_{t-} + s_{t-} (R_{t-} - \tau - r_{t-}) + b_{t-} (\tilde{r}_{t-} - r_{t-}) + s_{t-}^* (R_{t-} - \tilde{r}_{t-}) - \rho) \\ &+ \lambda \left(\frac{1}{\rho} \ln (1 - (1 + s_{t-} + s_{t-}^*) \kappa_t^Q) + h_{t-} (1 - \kappa_t^h) - h_{t-} \right). \end{aligned}$$

Furthermore, if a banker defaults at time t , her continuation value is

$$\frac{\ln(W_t)}{\rho} + h_t - \frac{l_0}{\rho(\rho + \xi)}.$$

Proof. See Appendix ■

Given the continuation value that a banker would have if she defaults, we consider the default decision of the banker at stage 3 conditional on the hit of a Poisson shock. Without loss of generality, we only focus on the case that the systemic event does not happen. If the banker does not default, her continuation value will be

$$\frac{\ln\left(W_{t-}(1-(s_{t-}+s_{t-}^*)\kappa_t^Q)\right)}{\rho}+h_t.$$

If she defaults instead, the banker does not bear the loss $s_{t-}^*W_{t-}\kappa_t^Q$. The downside is that she will receive the punishment described in Assumption 1. Hence, the continuation value of the banker would be

$$\frac{\ln\left(W_{t-}(1-s_{t-}\kappa_t^Q)\right)}{\rho}+h_t-\frac{l_0}{\rho(\rho+\xi)}.$$

Thus, the banker defaults if

$$\frac{\ln\left(W_{t-}(1-(s_{t-}+s_{t-}^*)\kappa_t^Q)\right)}{\rho}<\frac{\ln\left(W_{t-}(1-s_{t-}\kappa_t^Q)\right)}{\rho}-\frac{l_0}{\rho(\rho+\xi)}.$$

Proposition 3 *The note contract in equilibrium across the time $\{(\bar{s}_t^*, \tilde{r}_t), t \geq 0\}$ can be characterized by*

$$\frac{l_0}{\rho+\xi}=\ln\left(1+\frac{\bar{s}_{t-}^*\kappa_t^Q}{1-(s_{t-}+\bar{s}_{t-}^*)\kappa_t^Q}\right). \quad (15)$$

$$\tilde{r}_{t-}=r_{t-}+\frac{\lambda\varrho_t\kappa_t^{Q,r}}{1-(s_{t-}+b_{t-})\kappa_t^{Q,r}} \quad (16)$$

Proof. Assume that ϱ_t is zero. Suppose that (15) does not hold at time $t-$, and the left hand side of (15) is strictly less than its right hand side; individual bankers would then default at stage 3 if the Poisson shock hits the economy at time t . Thus, notes become risky assets, and regular banks have to pay managerial cost τ to hold them. The managerial cost will be transmitted to the required rate of return for holding notes, which neutralizes any advantage of shadow banking. Thus, there would be no demand for any note contract. However, this cannot be true in equilibrium because unions formed by regular banks could be strictly better off by lowering \bar{s}_t^* such that it is risk-free to hold notes and earn a positive excessive return $\tilde{r}_t - r_t$ without taking any risk.

If the left hand side of (15) is strictly larger, then unions can always increase their profits by raising \bar{s}_t^* up to the level that equation (15) exactly holds. Thus, this cannot be true,

either.

Notice that if bankers want to deviate at time $t-$ and choose the leverage of regular banking larger than s_{t-} as specified by (8), then note investors at stage 5 can easily refuse to lend to them.

Next, consider the case where the systemic event occurs with some positive probability. In this case, although the systemic event leads to any individual banker's default decision because $\kappa_t^{Q,r} \geq \kappa_t^Q$, the holding of notes is not subject to managerial costs.

Due to the perfect competition among unions, the excessive rate of return $\tilde{r}_{t-} - r_{t-}$ has to equal the expected loss $\lambda_{\rho_t} \kappa_t^{Q,r}$ factored by the marginal impact on bankers' continuation value $1/(1 - (s_{t-} + b_{t-})\kappa_t^{Q,r})$. ■

Proposition (3) implies that when a bad shock hits the economy, any individual banker finds it optimal to default if all other bankers do so because the economy switches to a distressed phase, and the asset price declines by $\kappa_t^{q,r} (> \kappa_t^q)$.

Corollary 1 *The systemic event that all shadow banks default simultaneously is an equilibrium outcome.*

2.2.2 Systemic Risk as the Tragedy of the Commons

We will elaborate how bankers' optimal decisions and the note market clearing lead to the systemic risk. We consider a scenario in which all unions of regular banks offer the same systemic-risk-proof note contract $\{(\tilde{r}_{t,pf}, \bar{s}_{t,pf}^*), t \geq 0\}$, where $\bar{s}_{t,pf}^*$ satisfies

$$\frac{l_0}{\rho + \xi} = \ln \left[1 + \frac{\bar{s}_{t-,pf}^* \kappa_t^{Q,r}}{1 - (s_{t-} + \bar{s}_{t-,pf}^*) \kappa_t^{Q,r}} \right]. \quad (17)$$

Equation (17) is such a tight enforceability constraint that individual bankers would not default even, if all other bankers do so when a negative shock comes. Thus, no systemic event occurs in equilibrium if equation (17) is true.

However, Proposition 3 implies that the hypothetical scenario is unstable since a union can always propose an alternative contract to zero measure of bankers, such that they could raise more credit for paying a slightly higher interest rate. In particular, the alternative contract $(\tilde{r}'_t, \bar{s}'_t)$ would be such that \tilde{r}'_t is slightly higher than \tilde{r}_{t-} and \bar{s}'_t satisfies

$$\frac{l_0}{\rho + \xi} = \ln \left(1 + \frac{\bar{s}'_t \kappa_t^Q}{1 - (s'_{t-} + \bar{s}'_t) \kappa_t^Q} \right). \quad (18)$$

Since $\kappa_t^{Q,r} > \kappa_t^Q$ implies $\bar{s}'_t > \bar{s}_t^*$, the alternative contract provides more credit. As a result,

bankers would accept the new contract if \tilde{r}'_t is slightly higher than \tilde{r}_t . For note investors (i.e., regular banks), the alternative contract offers risk-free notes because *i*) the dynamics of the economy do not change if only zero measure of bankers accept the contract; *ii*) no systemic event would happen since equation (17) still holds; and *iii*) the enforceability constraint (18) guarantees that bankers who accept the alternative contract would not default if a systemic event does not happen. Therefore, it is profitable for the union of regular banks to offer the new contract. However, since bankers think in the same way, they would deviate from the original contract $(\tilde{r}_{t,pf}, \bar{s}_{t,pf}^*)$, which shows how the systemic risk is caused by bankers' optimal decisions and the note market clearing. Generally speaking, the optimal decisions of individual bankers fail to internalize the effect that if everyone borrows or lends aggressively, then the economy faces the systemic risk that the shadow banking system could collapse suddenly.

The inevitability of the systemic risk is the tragedy of the commons. If the tight enforceability constraint (17) holds in an economy, then no systemic event occurs, which is a common resource. While individual bankers could deplete this common resource and raise more credit, the common resource would be exhausted eventually, and the economy would face systemic risk if all bankers exploited the common resource to their best.

2.3 Systemic Run

The market-wide run on shadow banks triggers the systemic event that all shadow banks default simultaneously. In this section, we use the global game method to show that a systemic run is more likely to occur when creditors have larger exposure to shadow banking.

2.3.1 Default Decisions at Stage 3 in the Event of Banking Panic

Bankers at stage 3 can perfectly foresee the price of physical assets and the note contract because there is no uncertainty from stage 3 to stage 5, and all markets are competitive. Therefore, bankers can make their strategic choices at stage 3 based on their continuation values in the beginning of the next period.

The panic of shadow banks' creditors causes the default of a banker at stage 3; we defer the full analysis to the proof of Proposition 4. If shadow banks' creditors run at stage 2 because of a Poisson shock hit, then shadow banks have to sell physical assets to repay their creditors at stage 4. Accordingly, regular banks absorb these assets and expand their balance sheets at stage 4. Recall that the balance sheet expansion is expensive for regular banks because of managerial costs. Thus, if shadow banks are forced to liquidate too many physical assets by creditors who run, then regular banks will be very reluctant to purchase

these assets. As a result, the asset price at stage 4 would be lower than it would be if no creditor runs. Bankers at stage 3 could perfectly predict how much the capital price would drop at stage 4, knowing how many note investors run at stage 2. A large drop in capital price implies a large cost of bailing out shadow banks. If the cost is too large, then bankers would default at stage 3. In sum, bankers will default at stage 3 if they observe that there are too many investors who run on shadow banks.

Proposition 4 derives the threshold \bar{l}_t such that all bankers default at stage 3 if the proportion of running investors is larger than \bar{l}_t , where $\bar{l}_t \equiv 1 - \bar{s}_t^*(1 - (s_{t-} + \bar{s}_{t-}^*)\kappa_t^Q) / \bar{s}_{t-}^*$.

2.3.2 Global Games at Stage 2

At stage 2, creditors of each shadow bank play a standard global game. If creditor i runs, then she will suffer the utility loss θ_t and the benefit is to secure the investment principal. If a creditor does not run, then the creditor risks losing a proportion of the principal when the systemic event happens. There is strategic complementarity among creditors of a shadow bank because the loss to creditors who do not run increases with respect to the number of creditors who run and have their investments back at stage 4. Hence, the loss of a failing shadow bank is borne by creditors who fail to run and wait for its liquidation at stage 5.

Global games across all shadow banks are interconnected because the failure of a shadow bank is the choice of its sponsor. A banker's decision to default relies on the market clearing price of physical assets at stage 4, which in turn is affected by the proportion of running investors in the entire economy at stage 2, according to the analysis in Section 2.3.1.

Next, we characterize a banker's payoff function with respect to her run decision at stage 2. Notice that each regular bank holds notes issued by a continuum of shadow banks. The payoff to the banker depends on decisions made by creditors of all relevant shadow banks. To simplify this analysis, we focus on the symmetric equilibrium in which all bankers use the same switching strategy. Conveniently, a symmetric equilibrium analysis yields the following three variables that are equal: The proportion of creditors who run against a single shadow bank, the proportion of notes that a shadow bank has to pay back early at stage 4, and the proportion of creditors who run in the economy. Hereafter, l_t denotes the fraction of run creditors among those who are able to do so. To explain the payoff function, suppose the proportion of run creditors $l_t \hat{l}_t$ is larger than \bar{l}_t and thus all bankers default at stage 3. Hence, the capital price declines drop by $\kappa_{t-}^{q,r}$ at stage 4 and 5. The total loss to a shadow bank is $S_{t-}^* \kappa_t^{q,r}$ given its size S_{t-}^* . Since $l_t \hat{l}_t$ fraction of notes would be redeemed in full at stage 4, the loss per dollar note that has not been redeemed is $\kappa_t^{q,r} / (1 - l_t \hat{l}_t)$. Thus, if a banker's regular bank does not run at stage 2, then her net worth at the end of the

period would be $W_{t-}(1 - (s_{t-}\kappa_t^{Q,r} + b_{t-}\kappa_t^{Q,r} / (1 - l_t\hat{l}_t)))$; otherwise, her net worth would be $W_{t-}(1 - s_{t-}\kappa_t^{Q,r})$.

In the second case that $l_t\hat{l}_t < \bar{l}_t$, no banker defaults, and all creditors of shadow banks receive their investment back in full. Thus, by plugging the note investor's net worth into her continuation value function $\ln(W_t)/\rho + h_t$, we derive the payoff function for note investor i with respect to her run decision, which is

$$\begin{cases} -\frac{1}{\rho} \ln \left(1 - \frac{b_{t-}\kappa_t^{Q,r}}{(1-l_t\hat{l}_t)(1-s_{t-}\kappa_t^{Q,r})} \right) - \theta_t^i, & \text{if } l_t\hat{l}_t \geq \bar{l}_t; \\ -\theta_t^i, & \text{otherwise.} \end{cases} \quad (19)$$

The payoff function increases weakly in l_t , which satisfies the property of global strategic complementarities. Additionally, the payoff function also increases weakly in both b_{t-} and $\kappa_t^{Q,r}$, where b_{t-} is bankers' portfolio weight on notes and $\kappa_t^{Q,r}$ is the fraction by which the asset value of a shadow bank drops, conditional with respect to the systemic event. Both terms affect an investor's exposure to the systemic risk.

Two equilibria exist for the subgame starting at stage 2. One equilibrium is about the banking panic, such creditors run on all shadow banks, and bankers default on notes in expectation of shadow banks' early liquidation at stage 4. Thus, we associate the systemic event with the systemic run that creditors of shadow banks initiate at stage 2. In the Appendix, we solve for the multiple equilibria problem at stage 2 by applying the equilibrium selection mechanism developed in the global game literature, see [Morris and Shin \(2002\)](#).

2.4 Subgame Perfect Equilibrium

The following proposition defines a subgame perfect equilibrium, which endogenizes the probability that the systemic event would happen:

Proposition 4 *If σ is arbitrarily small, then the model specified in section 1 permits a subgame perfect equilibrium for games that start at any period t when the economy is in the normal phase, where shadow banking is active. In the equilibrium,*

at stage 1, bankers suspend their shadow banks if their private signals are lower than θ_t^ ; otherwise, they do not suspend;*

at stage 2, bankers ask their regular banks to run if their private signals are lower than θ_t^ ; otherwise, they do not;*

at stage 3, bankers default if $l_t\hat{l}_t > \bar{l}_t$; otherwise, they do not;

at stage 4, bankers and households solve their intertemporal portfolio choice problems;

at stage 5, unions of regular banks offer the same note contract $(\tilde{r}_t, \bar{s}_t^*)$, where l_t is the proportion of run investors among those who are able to do so, θ_t^* satisfies

$$\rho\theta_t^* = - \int_{\bar{l}_t}^1 \ln \left(1 - \frac{b_{t-} \kappa_{t-}^{Q,r}}{(1 - \hat{l}) (1 - s_{t-} \kappa_{t-}^{Q,r})} \right) dl, \quad (20)$$

\bar{l}_t is defined by

$$\bar{l}_t = 1 - \frac{\bar{s}_t^*}{\bar{s}_{t-}^*} \left(1 - (s_{t-} + \bar{s}_{t-}^*) \kappa_{t-}^Q \right), \quad (21)$$

\tilde{r}_t satisfies equation (16), and \bar{s}_t^* is such that equation (15) holds.

The proof is deferred to the Appendix. In equilibrium, the systemic event occurs if $\theta_t < \theta_t^*$. Thus,

$$\varrho_t = E_{t-} [\mathbf{1}_{(\theta_t < \theta_t^*)}]. \quad (22)$$

2.5 Markov Equilibrium

This model has the property of scale-invariance with respect to K_0 . This property implies that the equilibrium can be described by state variables normalized by K_0 . Since all endogenous variables such as $\{q_t, t \geq 0\}$ are identical in economies, which are the same up to scale K_0 , we use the reciprocal of the total wealth in an economy as the scaling factor.

In this paper, we focus on a Markov equilibrium with a single state variable ω , which denotes bankers' wealth share in the economy $\left(\int_0^1 W_t^i di / q_t K_t \right)$. Later in this paper, we verify the existence of the Markov equilibrium.

Lemma 2 *The law of motion of ω_t in the Markov equilibrium is*

$$d\omega_t = \omega_{t-} \mu_{t-}^\omega dt - \omega_{t-} \kappa_{t-}^\omega dN_t, \quad (23)$$

$$\text{where } \mu_{t-}^\omega \equiv r_{t-} + s_t (R_{t-} - \tau - r_{t-}) + s_{t-}^* (R_{t-} - r_{t-}) - \mu_{t-}^q - \mu_{t-}^K - \rho - \xi, \quad (24)$$

$$\text{and } \kappa_{t-}^\omega \equiv \frac{(s_{t-} + s_{t-}^* - 1) \kappa_{t-}^Q}{1 - \kappa_{t-}^Q}. \quad (25)$$

Proof. See Appendix. ■

In the Markov equilibrium, all stochastic processes of endogenous variables can be expressed as functions of ω_t including q_t . Hence:

$$\mu_t^q = \frac{q'(\omega_t)}{q(\omega_t)} \omega_t \mu_t^\omega, \quad (26)$$

$$\kappa_t^q = \frac{q(\omega_{t-}) - q(\omega_{t-}(1 - \kappa_t^\omega))}{q(\omega_{t-})}, \quad (27)$$

$$\mu_t^\zeta = \frac{\zeta'(\omega_t)}{\zeta(\omega_t)} \omega_t \mu_t^\omega, \quad (28)$$

$$\kappa_t^\zeta = \frac{\zeta(\omega_{t-}) - \zeta(\omega_{t-}(1 - \kappa_t^\omega))}{\zeta(\omega_{t-})}. \quad (29)$$

from Ito's Lemma.

In the Markov equilibrium, the consumption good market clearing condition (4) can be simplified as

$$a\psi + \underline{a}(1 - \psi) - g(\iota) = \left(\rho\omega + \rho^{\frac{1}{b}} \zeta^{\frac{b-1}{b}} (1 - \omega) \right) q \quad (30)$$

based on the optimal consumption choices of bankers and households, where ψ is the portion of capital held by bankers.

The following proposition establishes the existence of the Markov equilibrium and characterizes its properties with a system of delay differential equations and their boundary conditions.

Proposition 5 *The Markov equilibrium can be fully characterized by $q[\omega]$ and $\zeta[\omega]$, both of which are defined over $(0, \bar{\omega})$. In addition, $q[\omega]$ and $\zeta[\omega]$ solve the system of delay differential equations implicitly defined by (5) – (16) and (20) – (30) with boundary conditions*

$$\begin{aligned} \mu^q[\bar{\omega}] &= \mu^\zeta[\bar{\omega}] = \mu^\omega[\bar{\omega}] = 0, \\ \lim_{\omega \rightarrow 0} q[\omega] &= \underline{q} \text{ and } \lim_{\omega \rightarrow 0} \zeta[\omega] = \underline{\zeta}, \end{aligned}$$

where \underline{q} and $\underline{\zeta}$ satisfy

$$\frac{\rho}{1-b} = \frac{\rho \underline{\hat{c}}^{1-b}}{(1-b) \underline{\zeta}^{1-b}} + \frac{\underline{a} - \delta}{\underline{q}} + \frac{(\underline{q} - 1)^2}{2\tau \underline{q}} - \underline{\hat{c}} + \lambda \left(\frac{(1 - \kappa)^{1-\gamma}}{1 - \gamma} - \frac{1}{1 - \gamma} \right), \quad (31)$$

$$\underline{a} - \delta - \frac{\underline{q}^2 - 1}{2\tau} = \rho^{\frac{1}{b}} \underline{\zeta}^{\frac{b-1}{b}} \underline{q}. \quad (32)$$

Proof. See Appendix for the proof and associated algorithm for solving the system of differential equations. ■

3 Qualitative Results

In this section, we highlight several qualitative properties of this model. To do so, we computed a particular example numerically with the parameter choices shown by Table 1 in Section 4.

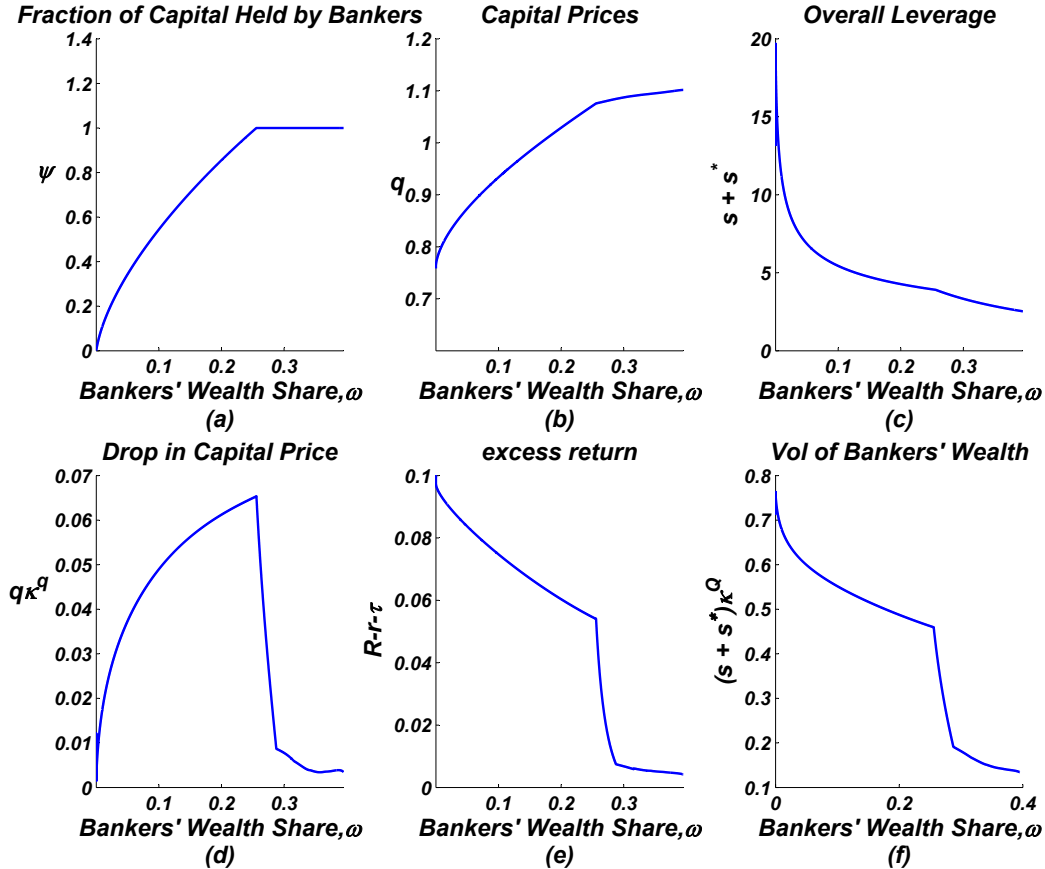


Figure 3: ψ , q , μ^q , $q\kappa^q$, $\phi (= b + s)$, and s^* as functions of the state variable ω (i.e., bankers' wealth share) in equilibrium. For parameter values, see the beginning of Section 3.

3.1 Capital Misallocation and Endogenous Risk

To facilitate further analyses, we briefly mention several properties of this model that are standard in the continuous-time macro-finance literature, such as Brunnermeier and Sanikov (2014) and He and Krishnamurthy (2013). We only focus on the phase where bankers are able to raise funds through shadow banking.

Due to the no equity financing setup, there is capital misallocation when bankers' wealth share is low in the economy (Panel a in Figure 3). As a result, as bankers own more wealth of

the economy, both aggregate productivity and the capital price increase (Panel b in Figure 3).

As is the case with standard models that have financial frictions, no equity financing friction gives rise to the endogenous risk in this model. If bad shocks hit the asset side of their balance sheets, then bankers' net worth declines substantially due to the leverage effect. As the result of their risk management, all banks sell physical assets on the market simultaneously. The asset price drops accordingly, which in turn hurts bankers' net worth further and triggers yet another round of loss spiral. Since the overall leverage that bankers can take is mechanically low when their wealth share is large (Panel c in Figure 3), the impact of the loss spiral is less significant. On the other hand, when bankers seize only a small fraction of physical assets in the economy, the endogenous risk is small, due to the low selling pressure. Panel d in Figure 3 shows the hump-shaped property of the endogenous risk.

As in other continuous-time macro-finance papers, the risk premium and the volatility of productive agents' net worth is counter-cyclical in this model (Panels e and f in Figure 3).

3.2 Threshold of Triggering a Systemic Run: θ^* .

The value of the threshold variable θ^* depends on relevant endogenous variables $\kappa^{Q,r}$, b , and \bar{l} . Figure 4 shows that the driving force of high θ^* in economic booms is creditors' large exposure to shadow banking. Further, the decline in the threshold \bar{l} also contributes to high θ^* . Given the note market clearing condition $b = s^*$, it is shadow banking's high leverage that causes the increase in the systemic risk.

θ^* is rather low when the state is between 0.25 and 0.3 for this example. $\kappa^{Q,r}$ has two kinks, which is similar to $q\kappa^q$. The second factor is that the threshold \bar{l} rises sharply between the two kinks, which implies that it requires a large fraction of run creditors to trigger shadow banks' default. The increase in \bar{l} is due to the rise of the leverage for shadow banking between two kinks. Hence, if the economy is around a state between two kinks, then a bad shock leads to a substantial drop in the leverage for shadow banking, which results in a large threshold \bar{l} according to equation (21).

3.3 Dynamics of the State Variable

Panel a in Figure 5 shows that the economy typically stays in states where the systemic risk is low (i.e., between $\omega = 0.25$ and $\omega = 0.3$). However, the economy occasionally enters a high systemic risk regime due to shadow banking's expansion, as shown by Panel b in Figure

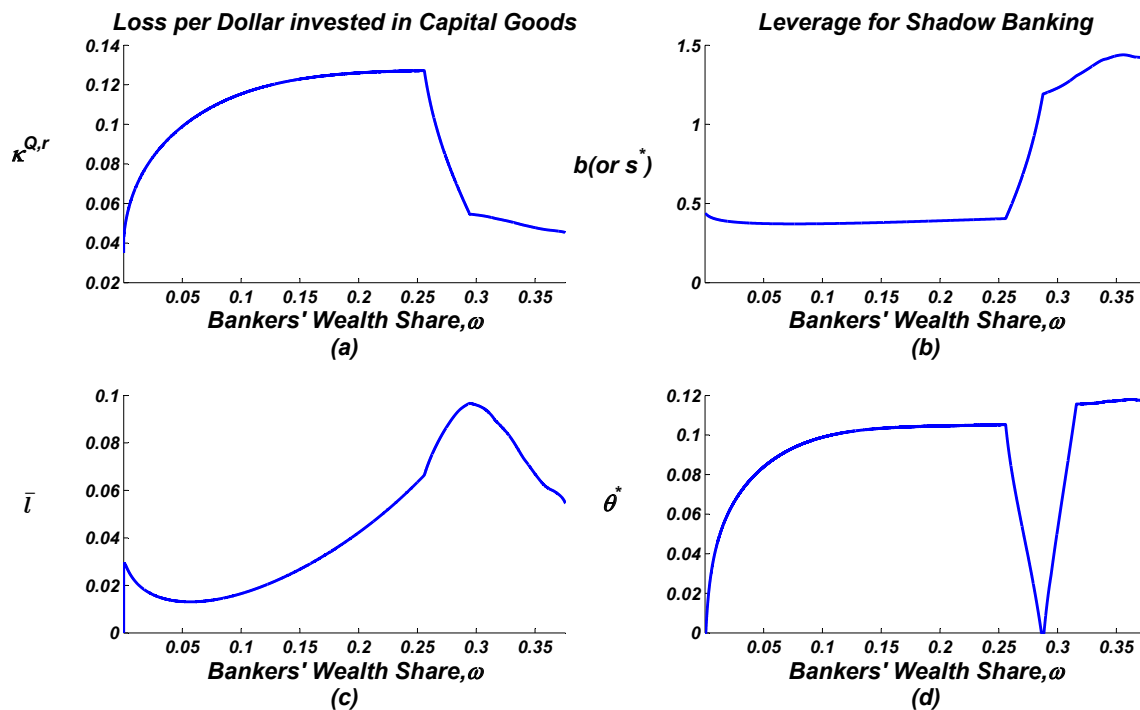


Figure 4: This figure shows the $\kappa^{Q,r}$, $b(s^*)$, \bar{l} , and θ^* as functions of the state variable ω . For parameter values, see Table 1 in Section 4.

5. If the economy is indeed in the high systemic risk regime, the decline in the state variable ω due to the systemic event is much larger than its drop in the absence of it (see Panel c of Figure 5). This is mainly due to the fact that the asset price drops substantially as the economy drops from the normal phase to the distressed phase given the systemic event (Panel d in Figure 5). Notice that the drop of ω leads to a further decrease in capital price, which in turn hurts bankers' net worth. This adverse feedback loop is drastic, as the asset price is lower in the distressed phase than it is in the normal one.

3.4 Heterogeneity of Credit Markets.

Panel b in Figure 5 illustrates the different behaviors of regular banking and shadow banking. In states where bankers' wealth share is large, the growth of shadow banking is accompanied by the decline of regular banking.

As Figure 5 shows, this model tells a particular story about the crisis of a systemic run. In economic upturns, financial institutions—especially shadow banks—accumulate lots of physical assets, and the economy grows gradually if it is not hit by any bad shock. Meanwhile, asset volatility goes down. This declining volatility then accelerates the expansion of shadow banking (i.e., the substitute of regular banking). However, if the economy is overheated,

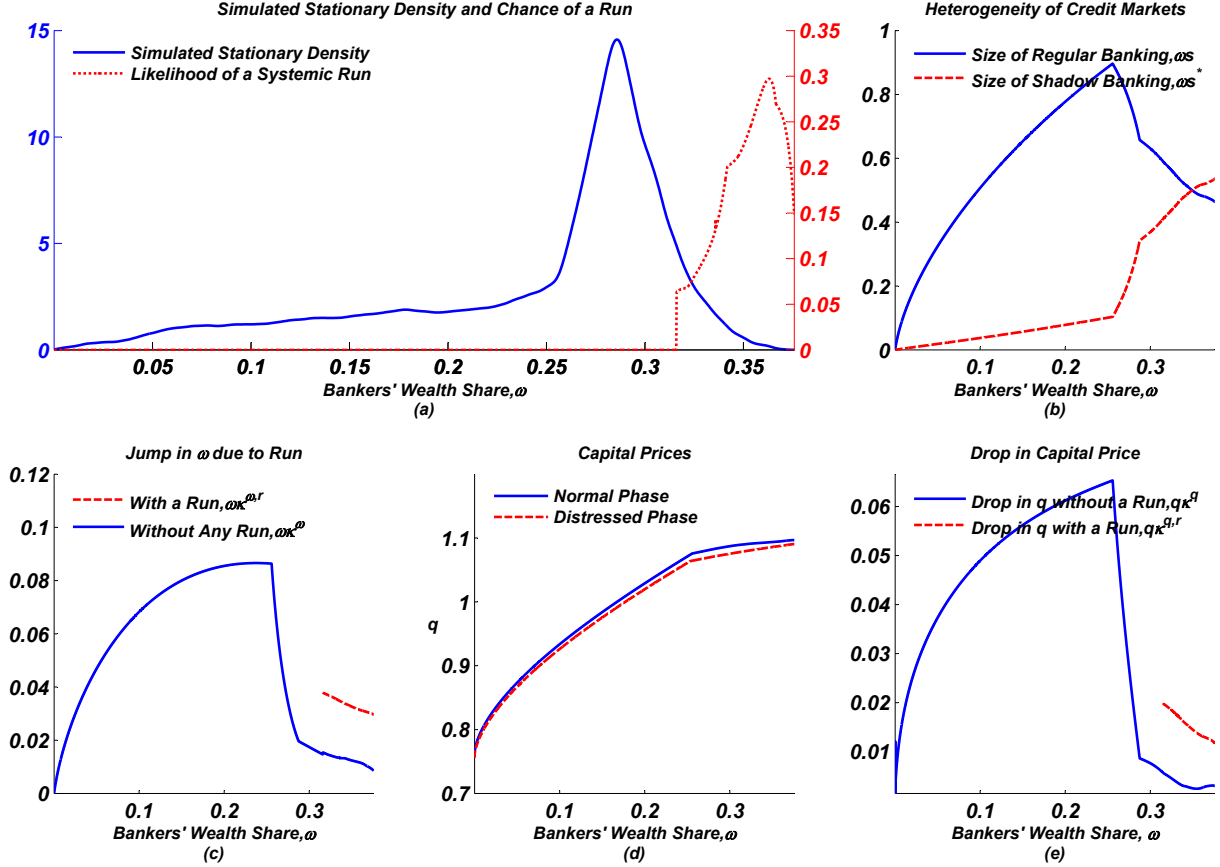


Figure 5: Panel (a) shows the estimated density of simulated stationary distribution (solid line) and the likelihood that a systemic run would occur, given a Poisson shock (dashed line); Panel (b) displays riskless debt outstanding (solid line) and note outstanding (dashed line); Panel (c) reports the drop size of the state variable ω if a systemic run does not happen (solid line) and the drop size if a systemic run does occur (dashed line); Panel (d) shows the capital price in the normal phase (solid line) and capital price in the distressed phase (dashed line); Panel (e) reports the drop size in capital price if a systemic run occurs (solid line) and the drop size if a systemic run does not happen. For parameter values, see Table 1 in Section 4.

then the credit boom may go wrong. Specifically, the improper expansion of shadow banking exposes the economy to the risk that the shadow banking system may collapse suddenly.

The instability of shadow banking has two features. First, a normal negative shock could cause a self-fulfilling panic, thereby leading to a credit market breakdown. Second, the collapse of shadow banking is preceded by its expansion in a tranquil period (see Panels a and b of Figure 5).

Market Illiquidity and Credit Risk. There is liquidity mismatch in shadow banks' balance sheets (i.e., bankers' hidden balance sheets). Notes are liquid because they are short-term and investors could stop rolling over them at any time they like. Physical assets

are less liquid because it is costly to convert them into consumption goods, and the only option left is to sell them in the market. Although the market for capital goods itself is fully liquid, shadow banks have to sell at a very low price during a market-wide panic because all shadow banks dump physical assets at the same time, and regular banks find it very costly to hold too many physical assets on their balance sheets. As Panel e of Figure 5 shows, the capital price drops a lot when the economy enters the distressed phase. This market illiquidity worsens the credit risk of each shadow bank, which in turn intensifies the banking panic.

3.5 Endogenous Risk and Systemic Risk

Panels a and e of Figure 5 show that the risk of a systemic run elevates as endogenous risk declines. The leverage behavior of shadow banking explains why systemic risk and endogenous risk move in opposite directions. The maximum leverage of shadow banking goes up when the endogenous risk goes down. Thus, as the endogenous risk declines, creditors of shadow banks are increasingly exposed to systemic risk, which in turn increases their incentives to run, as well as increases the likelihood of the banking panic.

The systemic risk of banking panics shares some features with endogenous risk. First, systemic risk is also endogenous. Second, when systemic risk materializes, it also involves the adverse feedback loop and the margin spiral related to reintermediation, the process in which shadow banks fire sell physical assets. However, the impact of an exogenous shock is more profound quantitatively when the systemic risk comes on stage.

There are some qualitative differences between the two types of risks in this model. In contrast to situations that involve endogenous risk, shadow banks fail massively upon realizing systemic risk and the note market closes for a certain period afterwards, which explains why the systemic risk causes more severe consequences. On the other hand, the materialization of systemic risk is much less frequent than that experienced by the endogenous risk; the former shows up only occasionally, while the latter unfolds every time when a negative shock hits. Hence, systemic risk is not as frequently observed as endogenous risk is. Moreover, systemic risk hides in the economy in the sense that its outbreak is preceded by a quiet process, during which time both the endogenous risk declines and the leverage for shadow banking increases smoothly. This observation implies that it might not be possible to gauge the systemic risk precisely, for there may be a misinterpretation of the connection between endogenous risk observed everyday and the systemic risk that the economy would face occasionally.

3.6 Minsky Moments

The qualitative prediction of this model is consistent with Minsky’s Financial Instability Hypothesis summarized by [Minsky \(1992\)](#). In this model, the economy is not always in states where the likelihood of the systemic run is positive, which reflects the first theorem of Minsky’s hypothesis: that an economy has both stable financing regimes and unstable financing regimes. The economy enters unstable financing regimes in this model because of the expansion of the shadow banking sector and the increase in creditors’ exposure to the risk of a systemic run. The blossoming of the shadow banking sector, in turn, is the consequence of either prolonged economic growth or extended periods that did not include any bad shock. The fact that long-term economic growth can endanger the financial system is consonant with the second theorem of Minsky’s hypothesis: that the economy shifts from stable regimes to unstable regimes after a long period of economic prosperity.

The Minsky moment in this model is observed when the systemic run on the shadow banking sector occurs. Two factors contribute to the economy-wide failure of shadow banks: the endogenous decline in asset prices, as well as bankers’ refusal to offer protection to shadow banks in trouble. These two factors not only reinforce each other but also turn the business of shadow banking into Ponzi finance, as defined by [Minsky \(1992\)](#). The lack of support from their sponsors and the decline in asset prices make the asset value of shadow banks short of their liability value. Moreover, the systemic run on shadow banks disallows these banks to rollover their debts, which further supports the presence of the Ponzi financing in shadow banking.

Different from Minsky’s original hypothesis, shadow banking shifts to Ponzi financing in this model only when creditors initiate a market-wide run on shadow banks. This property shows that it is self-fulfilling to market, whether or not a financing business can be classified as Ponzi finance in this model. This result highlights the intrinsic instability of the financial market as well as the difficulty of detecting Ponzi finance, which destabilizes the economy.

4 Quantitative Exercises

In this section, we calibrate the model and compare the quantitative performance of this study with that of [He and Krishnamurthy \(2012b\)](#) (hereafter HK), who calibrate a continuous-time macro-finance model and use it to match several empirical facts. In addition, we use this model to assess the likelihood that a systemic run might occur before the Great Recession.

4.1 Calibration

Table 1: Parameters

	PARAMETER	CHOICE	TARGET
<i>Panel: Preference</i>			
ρ	Time discount rate	0.04	Literature
γ	Risk aversion	2	Literature
$1/b$	EIS	2	Literature
<i>Panel: Technology</i>			
a	Productivity of Bankers	0.21	Investment Rate
\underline{a}	Productivity of Households	0.1	Price-to-dividend ratio
δ	Depreciation rate	0.1	Literature
τ	Capital adjustment cost	3	Literature
ξ	Bankers exit intensity	0.15	Steady state
<i>Panel: Shocks</i>			
λ	Poisson shock intensity	1	Literature
κ	Poisson shock magnitude	0.035	Consumption volatility
<i>Panel: Frictions</i>			
τ	Managerial cost	0.05	Leverage
l_0	Leverage constraint	1.45	Securitization

The choices of preference parameters ρ , γ , and b and technology parameters δ and τ are consistent with the literature. For instance, [Gertler and Kiyotaki \(2013\)](#) and [Gertler and Kiyotaki \(2010\)](#) use quarterly discount factor $\beta = 0.99 (\approx \exp(-0.25 \times 0.04))$ and [He and Krishnamurthy \(2013\)](#) choose $\rho = 4\%$; for γ , [Gertler et al. \(2012\)](#) and [He and Krishnamurthy \(2013\)](#) pick the same value; [Gruber \(2006\)](#) estimates that the elasticity of intertemporal substitution is 2 using non-durable consumption data; for δ , HK, [Gertler and Kiyotaki \(2010\)](#), and [Gertler et al. \(2012\)](#) choose 10% depreciation rate; and $\tau = 3$ follows [He and Krishnamurthy \(2012b\)](#).

We choose $a = 0.21$ to match the average investment-to-capital ratio, which is 11% from 1973 to 2010 in data according to HK, and it is 11.63% in this paper’s simulated model. The productivity parameter of households a^h affects the boundary conditions as the state variable $\omega \rightarrow 0$. We set $a^h = 0.1$ to target the lowest price dividend ratio 10 in the past 100 years in the US, as in [Muir \(2013\)](#). In this model, $a^h = 0.1$ yields the lowest price-dividend ratio 9.88. We adjust the banker exit rate ξ so that banks’ leverage is reasonable around the stochastic steady state (i.e. the state in which $\mu^\omega - \lambda\kappa^\omega = 0$). In the simulation, the leverage around the stochastic steady state is 3.83.

There are two parameters to choose for the Poisson shock. The intensity λ is set to be 1 as in [Brunnermeier and Sannikov \(2013\)](#). Also, $\kappa = 0.035$ is set to match the volatility of

consumption and the investment growth rate during the tranquil period, which is discussed in detail later.

For the managerial cost, τ , its value affects the leverage of bankers in the model. A high τ leads to low leverage chosen by regular banks. For the purposes of this study, the target is the same as it is in [Gertler and Kiyotaki \(2010\)](#): banks' leverage being 4. On the other hand, the leverage for shadow banking financing is affected by the parameter l_0 . The value of l_0 is selected such that the likelihood of a systemic run is most likely when the size of shadow banking is comparable to that of regular banking, which is used to finance physical assets.

Further, $\pi = 1/3.4$ is set so that the economy would be in the distressed phase for 3.4 years on average, which is the average year duration of downturn found by [Reinhart and Rogoff \(2009\)](#). Given this average duration, the upper and lower bounds of θ are adjusted such that the unconditional probability of the economy being in the distressed phase is close to 7%.

Finally, \hat{l} involves two regularity problems. The first has to do with the global complementarity property, which can be guaranteed by $\hat{l} < 1 - \kappa^{Q,r}$. The second is also associated with the payoff function that investors have with respect to the run decision. When s is very large, investors could also have a very strong incentive to run, although their exposure to the failure of notes itself is low. This is because the substantial net worth loss due to high s considerably increases the marginal benefit of extra money that a banker could have. Hence, even when investors' exposure to the risk that notes might fail is not large in absolute terms, the marginal benefit of recovering their investments in notes could still be very high. Therefore, the incentive to run could also be very high. This channel looks less sensible in the real world. We attempt to dampen the impact of this channel by choosing a relatively low value of \hat{l} .

4.2 Nonlinearity and Conditional Moments

To assess the quantitative performance of our model, we compare the nonlinear results of HK and what they find in the data with the results of this study. The nonlinearity in the model refers to the fact that moments of endogenous variables, such as the volatility of equity growth rate, conditional on that the banking sector is under-capitalized is significantly different than those moments conditional on that the banking sector is well-capitalized. See HK for the detail of the data used in that paper and how the data are processed.

For each simulation, we run the model for 5000 years and only record the data for the last 3000 years. We then simulate the model 2000 times and report the means of moments

for simulations. Then, we report these results as well as those from HK in Table 2. Following HK, we also classify periods with the top one-third of Sharpe ratio realizations as “financially distressed” periods and others as “financially tranquil” periods.⁵ The Sharpe ratio is defined as the ratio of the excess rate of return from holding capital over its expected loss (i.e., $SR = (R - r - \tau - \lambda \kappa^Q) / (\lambda \kappa^Q)$).

Table 2: Conditional Moments (in percentage)

This table displays standard deviations and covariances for bankers’ net worth (Eq), investment growth (I), and consumption growth (C), and their counterparts in the data and in He and Krishnamurthy (2012b). As in He and Krishnamurthy (2012b), growth rates are defined as annual changes in log value from t to $t + 1$. Numbers under column “Data” and “HK” are from the “Data” and “Baseline” column in Table 4 in He and Krishnamurthy (2012b). Numbers in the “Model” column are calculated based on the simulation of the full model, whose details are described in the text. The “Financially Distressed Periods” are classified as periods with the highest one-third Sharpe ratio realizations.

<i>Financially Tranquil Periods</i>							
	vol(Eq)	vol(I)	vol(C)	vol(SR)	cov(Eq,I)	cov(Eq,C)	cov(Eq,SR)
Data	17.54	6.61	1.28	12.72	0.07	0.03	-0.14
HK	6.4	4.8	2.4	8.7	0.3	0.1	-0.2
Model	14.37	5.86	1.53	12.22	0.43	-0.05	-0.18
<i>Financially Distressed Periods</i>							
	vol(Eq)	vol(I)	vol(C)	vol(SR)	cov(Eq,I)	cov(Eq,C)	cov(Eq,SR)
Data	31.48	8.05	1.71	60.14	1.31	0.25	-6.81
HK	31.2	5.4	1.8	71.1	0.9	0.0	-13.0
Model	57.78	19.35	4.75	20	5.46	0.32	-3.84

As indicated in Table 2, the performance of this model is slightly better than that of HK in terms of volatility terms for the “financially tranquil” periods. For the “financially distressed” periods, the basic nonlinearity property is well preserved. For instance, the volatility of consumption growth in this model is higher in the tranquil periods, which is true in the data but not true in HK. However, the volatility terms of equity growth, investment growth, and consumption growth in this model are larger than their counterparts for this data and HK, mainly due to an assumption in this model’s setup: that households also could hold physical assets, although they are less productive. However, households in HK are unproductive. Since bankers sell capital to households in the “financially distressed” periods in this model, the aggregate productivity and, thus, total output are more volatile in those periods. Therefore, the growth rates of equity, investment, and consumption are more volatile in “financially distressed” periods. On the other hand, HK match the volatility of

⁵In He and Krishnamurthy (2012b), they are named as “distress” and “non-distress” periods. To avoid the confusion with the distressed phase in this paper, we rename them as “financially distressed” and “financially tranquil” periods.

the Sharpe ratio in the data for the “financially distressed”, since they calibrate their model such that its highest Sharpe ratio is 15 times the average level, which is consistent with the finding of [Gilchrist and Zakrajšek \(2012\)](#).

4.3 Data

We use data that associated with endogenous variables in the model to estimate the underlying state variable of the economy. Specifically, we look at endogenous variables that represent three aspects of the economy: price, quantity, and volatility.

We assess the systemic risk of the Great Recession based on the data of the Sharpe ratio, securitization, and the volatility of the stock market. Following HK, we compute the monthly Sharpe ratio based on the excess bond premium of [Gilchrist and Zakrajšek \(2012\)](#). The securitization data are drawn from the “Flow of Funds Accounts of the United States.” Appendix B contains the construction of the ratio of securitization. We use the Chicago Board Options Exchange Volatility Index (VIX) as the measure of the stock market volatility.

Upper panels of Figure 6 summarize the movements of the three variables based on this data. Three plots on the top show that both the Sharpe ratio and VIX index were around their historically low level in early 2007, and the ratio of securitization was well above its trend around the same time. Finally, we use the detrended data about the ratio of securitization to assess the systemic risk.

4.4 Assessment of the Systemic Risk

We first define endogenous variables that can be linked to available data. The Sharpe ratio is denoted by SR . The volatility of the stock market (Vol) is defined as the volatility of bankers’ net worth (i.e., $Vol = (s + s^*) \kappa^Q$). The ratio of securitization (RA) has a natural definition (i.e., $RA = s^*/(s + s^*)$). Figure 7 shows the behaviors of these endogenous variables in the equilibrium. It is clear that when the systemic risk is high, the Sharpe ratio and stock market volatility tend to be lower than their average level, and the ratio of securitization is higher than its average level.

To find the underlying state of an economy at a given time, we search for a state that minimizes the distance between endogenous variables in the model and their counterparts in the data. Since the range of some variables is different from their range in the data, such as the Sharpe ratio, we first standardize variables and then calculate the distance. For instance, given (SR_t, RA_t, Vol_t) at time t from the data, to find the corresponding ω_t , we solve the

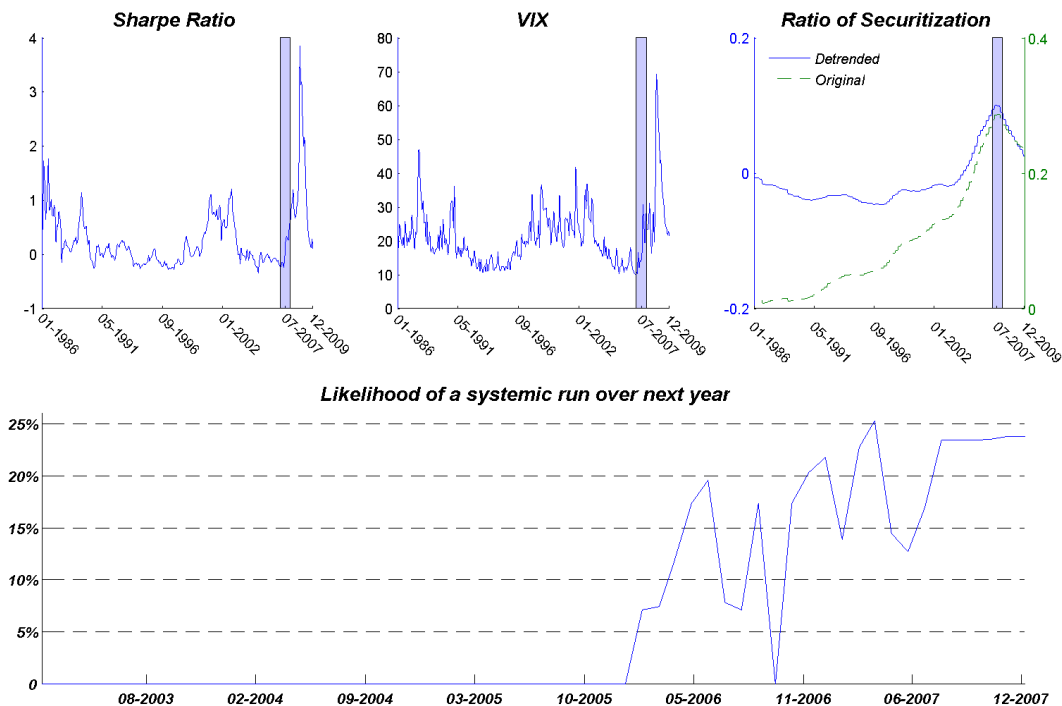


Figure 6: In this figure, the upper panels from left to right show monthly Sharpe ratio, the monthly Chicago Board Options Exchange Volatility Index, and the quarterly ratio of securitization done by non-agency Asset-Backed Security issuers over aggregate originated loans from January, 1986 to December, 2009. The lower panel presents the estimated probability that a systemic run would happen over the next year from January, 2003 to January, 2007. We first estimate the underlying state of the economy at a given by using the method described in the current section. Given that the economy starts from the estimated state, we simulate the model for one year 10000 times and report the frequency of the simulation for which a systemic run happens within one year. *Source:* See Appendix B

following problem:

$$\max_{\omega} : \left(\widehat{SR}[\omega] - \widehat{SR}_t \right)^2 + \left(\widehat{RA}[\omega] - \widehat{RA}_t \right)^2 + \left(\widehat{Vol}[\omega] - \widehat{Vol}_t \right)^2,$$

where \hat{x} is the standardized x . In doing so, we follow the approach used in [Eisfeldt and Muir \(2012\)](#) and [Muir \(2013\)](#).

To show that this model is able to provide early warning signs, we only use data available up to the end of 2007 for the standardization of variables. Given the corresponding state of an economy, we simulate the model and calculate the probability that a systemic run occurs in the economy over the next few years. The lower panel of Figure 6 shows that the risk of a systemic run gradually increased from 0 percent in 2003 to 25 percent in 2007.

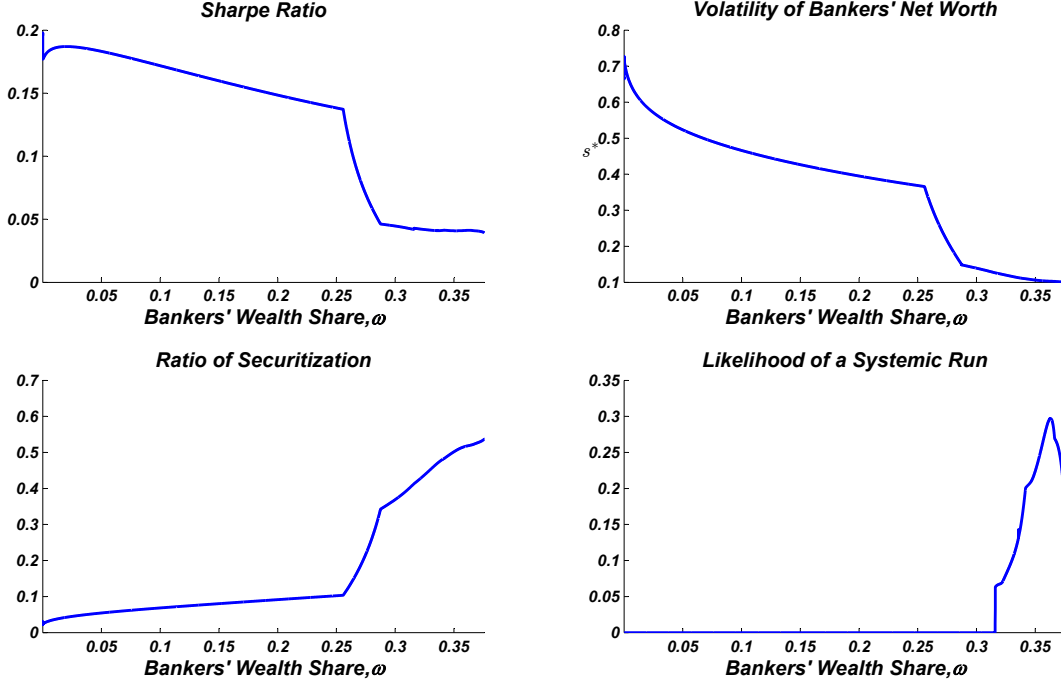


Figure 7: This figure shows the Sharpe ratio (the upper left panel), the volatility of the stock market return (the upper right panel), the ratio of securitization (the lower left panel), and the likelihood of a systemic run (conditional on a bad capital quality shock, as depicted in the lower right panel) as functions of the state variable ω in Equilibrium R . The Sharpe ratio is defined as $(R - r\tau - \lambda\kappa^Q)/(\lambda\kappa^Q)$, the ratio of securitization as $s^*/(s + s^*)$, and the volatility of stock market return as $(s + s^*)\kappa^Q$. For parameter values, see Table 1 in Section 4.

5 Final Remarks

The idea that the instability of credit causes financial crises can be traced back to nineteenth-century economists such as John Stuart Mill and Alfred Marshall (Hansen, 1951). Oft-cited modern works on this idea include Minsky (1986) and Kindleberger (2000). In addition, recent empirical works based on long-run historical data sets for developed countries, such as Schularick and Taylor (2012) and Jordà et al. (2011), show that the predictive power of a credit boom for a financial crisis is statistically significant. Nevertheless, the importance of the instability of credit for financial stability was undervalued from time to time, reflecting Charles Kindleberger’s quote in this paper’s introduction. In short, every time that we tame the credit of a new form that caused the last financial crisis, we tend to think that theories on the instability of credit become obsolete.

This current model could be extended in several directions. For instance, one could investigate the extent to which a risk of systemic run within the shadow banking sector can be transmitted into the traditional banking sector. Also, one could incorporate the labor

market into our model and quantify the impact of a systemic run on shadow banking in the real sectors.

The policy implication of this model is straightforward. According to this model, the government should take active actions to slow down the growth of the shadow banking system and perhaps slow down the economic growth as well if the shadow banking system is overheated, for the risk of systemic run is very high. On the other hand, the government is reluctant to sacrifice economic growth for the sake of financial stability. This model then offers a one-to-one relationship between economic growth and systemic risk such that the government can find an ideal combination based on its specific objectives.

References

- Adrian, Tobias and Nina Boyarchenko (2012), “Intermediary leverage cycles and financial stability.” Technical report, Federal Reserve Bank of New York Staff Report No.567.
- Bernanke, Ben and Mark Gertler (1989), “Agency costs, net worth, and business fluctuations.” *The American Economic Review*, 14–31.
- Bernanke, Ben S., Mark Gertler, and Simon Gilchrist (1999), “Chapter 21 the financial accelerator in a quantitative business cycle framework.” volume 1, Part C of *Handbook of Macroeconomics*, 1341 – 1393, Elsevier, URL <http://www.sciencedirect.com/science/article/pii/S157400489910034X>.
- Brunnermeier, Markus K and Yuliy Sannikov (2013), “The i theory of money.” *Unpublished working paper, Princeton University*.
- Brunnermeier, Markus K and Yuliy Sannikov (2014), “A macroeconomic model with a financial sector.” *The American Economic Review*, 104, 379–421.
- Bryant, John (1980), “A model of reserves, bank runs, and deposit insurance.” *Journal of Banking & Finance*, 4, 335–344.
- Campbell, John Y and John H Cochrane (1999), “Vby force of habit: A consumption based explanation of aggregate stock market behaviorv.” *Journal of Political Economy*, 107, 205–251.
- Danielsson, Jon, Hyun Song Shin, and Jean-Pierre Zigrand (2012), “Procyclical leverage and endogenous risk.” Technical report, Unpublished Working Paper, Princeton University.

- Di Tella, Sebastian (2012), “Uncertainty shocks and balance sheet recessions.” Technical report.
- Diamond, Douglas W and Philip H Dybvig (1983), “Bank runs, deposit insurance, and liquidity.” *The Journal of Political Economy*, 401–419.
- Eisfeldt, Andrea and Tyler Muir (2012), “Aggregate issuance and savings waves.” *manuscript, UCLA*.
- Gennaioli, Nicola, Andrei Shleifer, and Robert W. Vishny (2011), “A model of shadow banking.” Working Paper 17115, National Bureau of Economic Research, URL <http://www.nber.org/papers/w17115>.
- Gertler, Mark and Nobuhiro Kiyotaki (2010), “Chapter 11 - financial intermediation and credit policy in business cycle analysis.” volume 3 of *Handbook of Monetary Economics*, 547 – 599, Elsevier, URL <http://www.sciencedirect.com/science/article/pii/B9780444532381000119>.
- Gertler, Mark and Nobuhiro Kiyotaki (2013), “Banking, liquidity and bank runs in an infinite horizon economy.” *Unpublished Working Paper, Princeton University*.
- Gertler, Mark, Nobuhiro Kiyotaki, and Albert Queralto (2012), “Financial crises, bank risk exposure and government financial policy.” *Journal of Monetary Economics*.
- Gilchrist, Simon and Egon Zakrajšek (2012), “Credit spreads and business cycle fluctuations.” *American Economic Review*, 102, 1692–1720, URL <http://www.aeaweb.org/articles.php?doi=10.1257/aer.102.4.1692>.
- Goodhart, Charles A.E., Anil K Kashyap, Dimitrios P. Tsomocos, and Alexandros P. Var-doulakis (2012), “Financial regulation in general equilibrium.” Working Paper 17909, National Bureau of Economic Research, URL <http://www.nber.org/papers/w17909>.
- Gruber, Jonathan (2006), “A tax-based estimate of the elasticity of intertemporal substitution.” Technical report, National Bureau of Economic Research.
- Hansen, A.H. (1951), *Business Cycles and National Income*. Norton, URL <http://books.google.com/books?id=TuFdAAAAIAAJ>.
- He, Z. and A. Krishnamurthy (2012a), “A model of capital and crises.” *The Review of Economic Studies*, 79, 735–777.

- He, Zhiguo and Arvind Krishnamurthy (2012b), “A macroeconomic framework for quantifying systemic risk.” *Fama-Miller Working Paper*, 12–37.
- He, Zhiguo and Arvind Krishnamurthy (2013), “Intermediary asset pricing.” *American Economic Review*, 103, 732–70, URL <http://www.aeaweb.org/articles.php?doi=10.1257/aer.103.2.732>.
- Jordà, Òscar, Moritz HP Schularick, and Alan M Taylor (2011), “When credit bites back: Leverage, business cycles, and crises.” Technical report, National Bureau of Economic Research.
- Kindleberger, C.P. (2000), *Manias, Panics, and Crashes: a History of Financial Crises*. Wiley investment classics, Wiley, URL <http://books.google.com/books?id=fHx-kko1GaYC>.
- Kiyotaki, Nobuhiro and John Moore (1997), “Credit cycles.” *Journal of Political Economy*, 105, pp. 211–248, URL <http://www.jstor.org/stable/10.1086/262072>.
- Loutskina, Elena (2011), “The role of securitization in bank liquidity and funding management.” *Journal of Financial Economics*, 100, 663 – 684, URL <http://www.sciencedirect.com/science/article/pii/S0304405X11000389>.
- McCulley, Paul (2009), “The shadow banking system and hyman minsky’s economic journey.”
- Minsky, H.P. (1986), *Stabilizing an Unstable Economy*. Number v. 1 in A Twentieth Century Fund report, Yale University Press, URL <http://books.google.com/books?id=XT19QgAACAAJ>.
- Minsky, Hyman P (1992), “The financial instability hypothesis.” *The Jerome Levy Economics Institute Working Paper*.
- Morris, Stephen and Hyun Song Shin (2002), “Global games: Theory and applications, advances in economic theory and econometrics: Proceedings of the eighth world congress of the econometric society (m. dewatripont, l. hansen, and s. turnovsky, eds.)”
- Muir, Tyler (2013), “Financial crises, risk premia, and the term structure of risky assets.” Technical report.
- Phelan, Gregory (2012), “Financial intermediation, leverage, and macroeconomic instability.” Working paper, Yale University.

Pozsar, Z., T. Adrian, A. Ashcraft, and H. Boesky (2010), “Shadow banking.” Technical report, Federal Reserve Bank of New York Staff Report No.458.

Reinhart, Carmen M and Kenneth S Rogoff (2009), “The aftermath of financial crises.” Technical report, National Bureau of Economic Research.

Schularick, Moritz and Alan M. Taylor (2012), “Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008.” *American Economic Review*, 102, 1029–61, URL <http://www.aeaweb.org/articles.php?doi=10.1257/aer.102.2.1029>.

Appendix

A Proofs

Proof of Proposition 1. We define the continuation value of a banker by

$$J(W_t, \omega_t) = \max_{\{c_u, s_u, b_u, s_u^*, t_u, u \geq 0\}} E \left[\int_t^\infty e^{-\rho u} \ln(c_u) du \right].$$

T denotes the time when the banker retires. If W_T is positive, her continuation value is

$$(\ln(W_T) + r - \rho) / \rho;$$

otherwise, the continuation value would be negative infinity. Given that a banker will retire almost surely, the strategy, which yields negative net worth upon retirement, would be less optimal than the strategy of always holding risk-free debts before retirement. Thus, W_T is positive almost surely.

Next, we show that W_t is positive almost surely. Suppose $t < T$ and there is no bad shock between t and T , then

$$W_T = W_t \exp \left[\int_t^T (r_u + s_u(R_u - \tau - r_u) + b_u(\tilde{r}_u - r_u) + s_u^*(R_u - \tilde{r}_u) - \rho_u) du \right]$$

given any consumption and portfolio choices $\{c_u, s_u, b_u, s_u^*, t \leq u \leq T\}$. Since it has positive probability that there is no bad shock between t and T , W_t must be positive almost surely. Then, we show that (7).

We proceed to show (8), (9), and (10). Conjecture that $J(W, \omega) = \ln(W) / \rho + h(\omega)$

satisfies the following Hamilton-Jacobi-Bellman (HJB) equation

$$\rho J(W, \omega) = \max_{c, s, b, s^*} \left\{ \begin{aligned} & \ln(c) + W(r + s(R - \tau - r) + s^*(R - \tilde{r}) + b(\tilde{r} - r) - c) J_W + \omega \mu^\omega J_\omega \\ & + \lambda (J(W(1 - (s + s^*)\kappa^Q), \omega - \omega\kappa^\omega) - J(W, \omega)) \\ & + \xi((\ln(W) + r - \rho)/\rho - J(W, \omega)) \end{aligned} \right\}.$$

(8), (9), and (10) are just first-order conditions with respect to s , b , and s^* . Furthermore, $h[\omega]$ has to satisfy

$$\begin{aligned} \rho h(\omega) &= \ln(\rho) + \frac{1}{\rho} (r + s(R - \tau - r) + s^*(R - \tilde{r}) + b(\tilde{r} - r) - \rho) + \omega \mu^\omega h'[\omega] \\ &+ \lambda \left(\frac{1}{\rho} \ln(1 - (s + s^*)\kappa^Q) + h(\omega(1 - \kappa^\omega)) - h(\omega) \right) \\ &+ \xi((r - \rho)/\rho - h(\omega)). \end{aligned} \quad (33)$$

To complete the argument, the transversality condition

$$\lim_{t \rightarrow \infty} E[\exp(-\rho t) J(W_t, \omega_t)] = 0.$$

also must hold ■

Proof of Proposition 2. Combining with the transversality condition

$$\lim_{t \rightarrow \infty} E[\exp(-\rho t) V(W_t^h, \zeta_t)] = 0,$$

the HJB equation with respect to $V(W^h, \zeta)$ is sufficient to yield a household's optimal choices at each state ω . Plugging $V(\zeta_t, W_t^h) \equiv (\zeta_t W_t^h)^{1-\gamma} / (1-\gamma)$ into (11), we have

$$0 = \max_{\underline{c}, \underline{s}} \left\{ \begin{aligned} & \frac{1}{1-b} \left\{ \frac{\rho(c^h)^{1-b}}{(\zeta W^h)^{\gamma-b}} - \rho (\zeta W^h)^{1-\gamma} \right\} \\ & + (r + s^h(R^h - r) - c^h) \zeta^{1-\gamma} (W^h)^{-\gamma} + \mu^\zeta \zeta^{1-\gamma} (W^h)^{1-\gamma} \\ & + \lambda \left(\frac{(\zeta(1-\kappa^\zeta)W^h(1-s^h\kappa^Q))^{1-\gamma}}{1-\gamma} - \frac{(\zeta W^h)^{1-\gamma}}{1-\gamma} \right) \end{aligned} \right\},$$

where To take the first order condition with respect to \underline{c} and \underline{s} , we have (12) and (13). After we plug \underline{c} back to above equation, we derive (14). ■

Proof of Lemma 1. Equation (33) is equivalent to the equation that $\{h_t, t \geq 0\}$ must satisfy taking into account the Ito's Lemma.

Given the aggregate state variable ω , consider a banker with net worth W , who has just defaulted. Given that she has to pay l fraction of her net worth as fines at each period, her

HJB equation is

$$\rho J^a(W, \omega) = \max \left\{ \begin{aligned} & \ln(c) + (Wr + S(R - \tau - r) + S^*(R - \tilde{r}) + B(\tilde{r} - r) - c - l_0W) J_W^a \\ & + \omega \mu^\omega J_\omega^a + \lambda (J^a(W - (S + S^*) \kappa^Q, \omega - \omega \kappa^\omega) - J^a(W, \omega)) \\ & + \xi ((\ln(W) + r - \rho) / \rho - J^a(W, \omega)) \end{aligned} \right\}.$$

Corresponding $h^a[\omega]$ is the solution of

$$\begin{aligned} \rho h^a(\omega) &= \ln(\rho) + \frac{1}{\rho} (r + s(R - \tau - r) + s^*(R - \tilde{r}) + b(\tilde{r} - r) - \rho - l) + \omega \mu^\omega h^a[\omega] \\ &+ \lambda \left(\frac{1}{\rho} \ln(1 - (s + s^*) \kappa^Q) + h^a(\omega(1 - \kappa^\omega)) - h^a[\omega] \right) \\ &+ \xi \left(\frac{r - \rho}{\rho} - h^a(\omega) \right). \end{aligned}$$

Hence, $H(\omega)$, which is defined as $h(\omega) - h^a(\omega)$ satisfies

$$\rho H(\omega) = \frac{l_0}{\rho} + \omega \mu^\omega H'(\omega) - \xi H(\omega),$$

which has a solution that

$$H(\omega) = \frac{l_0}{\rho(\rho + \xi)}.$$

■

Proof of Proposition 4. We will prove the proposition by using backward induction.

First, we conjecture that, if the note market opens at stage 5, then the note contract in equilibrium at stage 5 is $(\tilde{r}_t^*, \bar{s}_t^*)$. In addition, we also conjecture that the proportion of run creditors $l_t \hat{l}$ on a single shadow bank is the same as the proportion of notes redeemed by them, and that the proportion of run creditors across different shadow banks is the same. We will verify these conjectures later.

Since there is no uncertainty between stage 4 and 5, bankers solve their intertemporal portfolio choice problems at stage 4 in expectation of the note contract they would have at stage 5. Households also solve their own problems at stage 4. All markets clear at stage 4 except the note market. Hereafter, we only focus on the case that a Poisson shock hits the economy after stage 2. Given the market clearing capital price and bankers' early decisions, we can fix bankers' continuation value at the beginning of next period.

If all shadow banks fail at stage 5, which can be expected at stage 4, capital price drop by $\kappa_t^{q,r}$ fraction. Consider a banker, whose net worth at the beginning of period t is W_{t-} . If the banker runs at stage 3, her net worth would be $W_{t-}(1 - s_{t-} \kappa_t^{Q,r})$; otherwise, her remaining

net worth would be $W_{t-}(1 - s_{t-}\kappa_t^{Q,r} - b_{t-}\kappa_t^{Q,r} / (1 - l_t\hat{l}))$.

Suppose that no shadow bank fails at stage 5. Then, the note market opens at stage 5. Now, let's look at how each bank's balance sheet changes from stage 4 to 5. Consider a banker, whose net worth and portfolio weights at the beginning of this period are W_{t-} and (s_{t-}, s_{t-}^*) . Given the banker's portfolio in the end of stage 5, (s_t, s_t^*) , the value of physical assets that she holds on her balance sheet at stage 4 is $W_t(s_t + s_t^*) - (1 - l_t\hat{l})W_{t-}s_{t-}^*$, where $(1 - l_t\hat{l})W_{t-}s_{t-}^*$ is the value of physical assets on the balance sheet of her shadow bank at stage 4. We can rearrange $W_t(s_t + s_t^*) - (1 - l_t\hat{l})W_{t-}s_{t-}^*$ as $W_t s_t + (l_t\hat{l} - \bar{l}_t)W_{t-}s_{t-}^*$.

If $l_t\hat{l} < \bar{l}_t$, the managerial cost that bankers pay does not exceed the level in the normal case where there is no systemic run. Hence, the capital price drops by κ_{t-}^q fraction as what it is in the normal case. Then, the banker's net worth in the beginning of next period is $W_{t-}(1 - s_{t-}\kappa_t^Q - s_{t-}^*\kappa_t^Q)$

If $l_t\hat{l} > \bar{l}_t$, all regular banks have to pay higher managerial costs than they do in the normal case. The general equilibrium effect is that they would ask for a higher rate of return for holding physical capital and the capital price has to drop by more than κ_t^q in order to clear the capital market. $\kappa_t^{q'}$ denotes the fraction by which the capital price drops. Then, the banker's net worth at the beginning of next period is $W_{t-} \left(1 - s_{t-}\kappa_t^{Q'} - s_{t-}^*\kappa_t^{Q'}\right)$.

Now, we are ready to discuss bankers' strategic choices at stage 3. If $l_t\hat{l} > \bar{l}_t$, it is impossible to have an equilibrium in which no banker defaults at stage 3 because a banker's continuation value with default $\ln(W_{t-}(1 - s_{t-}\kappa_t^{Q'})) / \rho + h_t^a$ is larger than her continuation value without default $\ln(W_{t-}(1 - (s_{t-} + s_{t-}^*)\kappa_t^{Q'})) / \rho + h_t$, which is implied by the enforceability constraint (3). Hence, if $l_t\hat{l} > \bar{l}_t$, the Nash equilibrium at stage 3 is that all bankers default. If $l_t\hat{l} < \bar{l}_t$, we focus on the Nash equilibrium that no banker defaults.

Given the result that if $l_t\hat{l} \leq \bar{l}_t$, no banker defaults at stage 3; otherwise, all bankers default at stage 3, we could specify the payoff function for individual bankers

$$\begin{cases} -\frac{1}{\rho} \ln \left(1 - \frac{b_{t-}\kappa_t^{Q,r}}{(1-l_t\hat{l})(1-s_{t-}\kappa_t^{Q,r})} \right) - \theta_t^i, & \text{if } l_t\hat{l} \geq \bar{l}_t; \\ -\theta_t^i, & \text{otherwise.} \end{cases}$$

The global strategic complementarity is satisfied since the payoff function is increasing in l_t . Proposition 2.1 in Morris and Shin (2002) yields a unique equilibrium at stage 2. In the equilibrium, if bankers receive private signals smaller than l_t^* , they run; otherwise, they do

not run, where θ_t^* is defined by

$$\rho\theta_t^* = - \int_{\frac{\bar{l}}{\bar{i}}}^1 \ln \left(1 - \frac{b_{t-} \kappa_{t-}^{Q,r}}{(1 - \hat{l}) (1 - s_{t-} \kappa_{t-}^{Q,r})} \right) dl,$$

Given creditors' switching strategies at stage 2, the fact that the idiosyncratic shock \hat{l}_t^i is independent across bankers implies that the proportion of run creditors is the same as the proportion of notes redeemed by run creditors by the law of large number. By the same reason, the proportion of run creditors is the same across different shadow banks. Thus, we justify our early conjectures.

For the subgame that starts at stage 1, it is straightforward to show that, in an equilibrium, bankers suspend their shadow banks if their private signals are lower than l_t^* ; otherwise, they do not suspend. This equilibrium has an advantage over other equilibria that it reduces shadow banks' creditors' exposure to the idiosyncratic risk that they might not be able to run at stage 2, which in turn reduces the risk premium that these creditors would ask for in the previous period.

Now, let's consider the stage 5 of period $t-$ given that no shadow bank fails. We can use the same argument as what we have used for the proof of Proposition 3 to show that $(\tilde{r}_{t-}, \bar{s}_{t-}^*)$ is the market clearing note contract. Thus, we complete our proof. ■

Proof of Lemma 2. W_t denotes $\int_0^1 W_t^i di$. In a Markov equilibrium, (2), the optimal choice of bankers, and the defaultable debt market clearing imply that

$$\begin{aligned} dW_t &= W_{t-} (r_{t-} + s_{t-} (R_{t-} - \tau - r_{t-}) + b_{t-} (\tilde{r}_{t-} - r_{t-}) + s_{t-}^* (R_{t-} - \tilde{r}_{t-}) - \rho - \xi) dt \\ &\quad - W_{t-} (s_{t-} + s_{t-}^*) \kappa_t^Q dN_t \\ &= W_{t-} ((r_{t-} + s_{t-} (R_{t-} - \tau - r_{t-}) + s_{t-}^* (R_{t-} - r_{t-}) - \rho - \xi) dt - (s_{t-} + s_{t-}^*) \kappa_t^Q dN_t). \end{aligned}$$

Note bankers exit the economy at the intensity ξ . Next, consider the scaling factor $1/q_t K_t$.

$$d(q_t K_t) = q_{t-} K_{t-} ((\mu_{t-}^q + \mu_{t-}^K) dt - \kappa_t^Q dN_t),$$

and

$$d\left(\frac{1}{q_t K_t}\right) = \left(\frac{1}{q_{t-} K_{t-}}\right) \left(-(\mu_{t-}^q + \mu_{t-}^K) dt + \frac{\kappa_t^Q}{1 - \kappa_t^Q} dN_t\right).$$

Then,

$$\begin{aligned}
d\omega_t &= \omega_{t-}\mu_{t-}^\omega dt - \omega_{t-}\kappa_t^\omega dN_t, \\
\text{where } \mu_{t-}^\omega &\equiv r_{t-} + s_t(R_{t-} - \tau - r_{t-}) + s_{t-}^*(R_{t-} - r_{t-}) - \mu_{t-}^q - \mu_{t-}^K - \rho - \xi, \\
\text{and } \kappa_t^\omega &\equiv \frac{(s_{t-} + s_{t-}^* - 1)\kappa_t^Q}{1 - \kappa_t^Q}.
\end{aligned}$$

■

Proof of Proposition 5. We start with the boundary conditions as ω goes to zero. Given that $\mu^q = \mu^\zeta = \kappa^q = \kappa^\zeta = 0$, (31) are implied by (12) and (14), and (32) is the limit of the consumption good market clearing condition (30). At the same time, we know the limits of $(\mu^\omega, s, s^*, \underline{s}, \kappa^\omega, \kappa^{\omega,r})$.

Next, we show how to advance q and ζ from ω to $\omega + \Delta\omega$. Given ω and $(q(\omega'), \zeta(\omega') : 0 < \omega' \leq \omega)$ as well as $(\mu^\omega, s, s^*, \underline{s}, \kappa^q, \kappa^{q,r}, \kappa^\zeta, \kappa^{\zeta,r}, \kappa^\omega, \kappa^{\omega,r}, \varrho)$ at ω , we can calculate $q(\omega + \Delta\omega)$ and $\zeta(\omega + \Delta\omega)$ by the following procedure.

First, bankers' Euler equation (8) yields the difference between μ^q and r , that is,

$$\mu^q - r = \frac{\lambda(1 - \varrho)\kappa^Q}{1 - (s + s^*)\kappa^Q} + \frac{\lambda\varrho\kappa^{Q,r}}{1 - (s + s^*)\kappa^{Q,r}} + \tau - \left(\frac{a - \delta}{q} + \frac{(q - 1)^2}{2\tau q} \right),$$

and we also know the sum of r and μ^ζ by households' HJB equation (14), that is,

$$\begin{aligned}
r + \mu^\zeta &= \frac{\rho}{1 - b} - \frac{\rho\hat{c}^{1-b}}{(1 - b)\zeta^{1-b}} + \hat{c} \\
&\quad - \underline{s}(\underline{R} - r) - \lambda(1 - \varrho) \frac{((1 - \kappa^\zeta)(1 - \underline{s}\kappa^Q))^{1-\gamma}}{1 - \gamma} \\
&\quad - \lambda\varrho \frac{((1 - \kappa^{\zeta,r})(1 - \underline{s}\kappa^{Q,r}))^{1-\gamma}}{1 - \gamma} + \frac{\lambda}{1 - \gamma},
\end{aligned}$$

where

$$\underline{R} - r = \frac{\lambda(1 - \varrho)\kappa^Q}{(1 - \underline{s}\kappa^Q)^\gamma} (1 - \kappa^\zeta)^{1-\gamma} + \frac{\lambda\varrho\kappa^{Q,r}}{(1 - \underline{s}\kappa^{Q,r})^\gamma} (1 - \kappa^{\zeta,r})^{1-\gamma}$$

by households' Euler equation (13) and $\hat{c} = \rho^{\frac{1}{b}}\zeta^{\frac{b-1}{b}}$. In order to fix (μ^q, μ^ζ, r) at ω , we resort to the consumption good market clearing condition at $\omega + \Delta\omega$. In particular, we pick a triple

(μ^q, μ^ζ, r) and solve q and ζ at $\omega + \Delta\omega$ by Ito's Lemma, i.e.,

$$\begin{aligned} q(\omega + \Delta\omega) &= q(\omega) + \Delta q = q(\omega) + \Delta\omega \frac{q(\omega) \mu^q}{\omega \mu^\omega}, \\ \zeta(\omega + \Delta\omega) &= \zeta(\omega) + \Delta\zeta = \zeta(\omega) + \Delta\omega \frac{\zeta(\omega) \mu^\zeta}{\omega \mu^\omega}. \end{aligned}$$

Given q and ζ , we can solve for $(s, s^*, \underline{s}, \kappa^q, \kappa^{q,r}, \kappa^\zeta, \kappa^{\zeta,r}, \kappa^\omega, \kappa^{\omega,r}, \varrho)$ based on Euler equations (8)–(10) and (13), the enforceability constraint (15), systemic run related equations (20–22), and Ito's Lemma

$$\begin{aligned} \kappa^q &= 1 - \frac{q((\omega + \Delta\omega)(1 - \kappa^\omega))}{q(\omega + \Delta\omega)}, \\ \kappa^\zeta &= 1 - \frac{\zeta((\omega + \Delta\omega)(1 - \kappa^\omega))}{q(\omega + \Delta\omega)}, \end{aligned}$$

where

$$\kappa^\omega = \frac{(s + s^* - 1) \kappa^Q}{1 - \kappa^Q}.$$

Given $(q, \zeta, s, s^*, \underline{s})$ at $\omega + \Delta\omega$, we check if the market clearing condition

$$a(s + s^*)(\omega + \Delta\omega) + \underline{a}s(1 - \omega - \Delta\omega) - \delta - \frac{q^2 - 1}{2\tau} = \left(\rho(\omega + \Delta\omega) + \rho^{\frac{1}{b}} \zeta^{\frac{b-1}{b}} (1 - \omega - \Delta\omega) \right) q$$

holds. This is how we solve (μ^q, μ^ζ, r) at ω . Before proceeding to next iteration, we solve for μ^ω without knowing (μ^q, r) at $\omega + \Delta\omega$ by

$$\begin{aligned} \mu^\omega &= r + s(R - \tau - r) + s^*(R - r) - \mu^q - \mu^K - \rho - \xi \\ &= (s + s^* - 1) \left(\frac{\lambda(1 - \varrho)\kappa^Q}{1 - (s + s^*)\kappa^Q} + \frac{\lambda\varrho\kappa^{Q,r}}{1 - (s + s^*)\kappa^{Q,r}} \right) \\ &\quad + (1 + s^*)\tau + \frac{a - \delta}{q} + \frac{(q - 1)^2}{2\tau q} - \mu^K - \rho - \xi, \end{aligned}$$

where the second equality comes from bankers' Euler equation at $\omega + \Delta\omega$. Thus, we have computed $(\mu^\omega, s, s^*, \underline{s}, \kappa^q, \kappa^\zeta, \kappa^\omega)$ and proceed.

Since bankers will exit the economy with a certain probability, the state variable has an endogenous upper bound denoted by $\underline{\omega}$. By definition, $\mu^\omega = 0$ at $\underline{\omega}$. By Ito's Lemma, $\mu^q = \mu^\zeta = 0$ at $\underline{\omega}$.

Thus, we have described the algorithm of solving for the equilibrium and also showed that the equilibrium can be fully characterized by a system of delay differential equations, i.e., the existence of the Markov equilibrium. ■

B Data

We use monthly data to estimate the chance that a systemic run might happen for a given month. When daily data are available, say VIX, we sample the data point in the middle of the month; when only quarterly data are available, say securitization data, we use the variable’s value at the corresponding quarter as the substitute for its value at each month.

Sharpe Ratio. To follow [He and Krishnamurthy \(2012b\)](#), when calculating the monthly Sharpe ratio, we subtract “predicted GZ credit spread” from “GZ credit spread” and divide the residual term by the “predicted GZ credit spread”. The “predicted GZ credit spread” term that we choose is the one that excludes the effects of term structure and interest rate volatility.

Securitization. We follow [Loutskina \(2011\)](#) to compute the ratio of securitization. The difference is that we focus on securitization done by non-agency security issuers. All data are drawn from the “Flow of Funds Accounts of the United States”. There are 5 loan categories. The details of items for each category are listed in [Table 3](#).

Volatility. We use the Chicago Board Options Exchange Volatility Index as the measure of the stock market volatility since January, 1986.

Table 3: Details of Securitization Data

	Outstanding	Securitized
Home Mortgages	FL383165105	FL673065105
Multifamily Residential Mortgages	FL143165405	FL673065405
Commercial Mortgages	FL383165505	FL673065505
Commercial and Industrial Loans ¹	FL253169255 FL263168005 FL263169255	FL673069505
Consumer Credit	FL153166000	FL673066000

¹ Because item FL253169255 is not available now, we use the ratio of securitization calculated by [Loutskina \(2011\)](#) to estimate the outstanding commercial and industrial loans.